  

Future Direction in Turbulence Modeling:

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  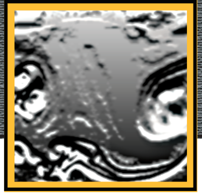

Dynamic Two-way Coupling of Numerical Methods and Physical Models

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MULTI-SCALE MODELING & SIMULATION LABORATORY

m(q)-LES
Adaptive LES with Model Refinement

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Future Directions in CFD Research?

Parallel **adaptive** high order numerical methods

New/improved turbulence models

Future Directions in CFD Research?

Not Parallel **adaptive** high order numerical methods

Not New/improved turbulence models

Future Directions in CFD Research?

Not Parallel **adaptive** high order numerical methods

Not New/improved turbulence models

Reasons:

- Spatial/temporal intermittency of turbulent flows is not used
- Inhomegeneous fidelity
  - *a-priori* large/small scale separation
  - under-resolves energetic structures
  - over-resolves in between them

Future Directions in CFD Research?

Not Parallel **adaptive** high order numerical methods

Not New/improved turbulence models

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Parallel **adaptive** high order numerical methods

New/improved turbulence models

New **direction/philosophy/paradigm**:

Parallel **adaptive** high order numerical methods

New/improved turbulence models

Future Directions in CFD Research?

New **direction/philosophy/paradigm**:

Direct physics-based **coupling** of

Parallel **adaptive** high order numerical methods

&

New/improved turbulence models

that takes advantage of spatio-temporal intermittency of turbulent flows

What does direct coupling bring?

- the active control of the fidelity/accuracy of the simulation
- near optimal spatially adaptive computational mesh
- the "desired" flow-physics is captured by considerably smaller number of spatial modes
- considerably smaller Reynolds scaling exponent, $Re^\alpha, \ \alpha < 9/4$
- robust general mathematical framework for spatial/temporal *model*-refinement (*m*-refinement) that can be extended to LES with AMR approach
- mathematical framework for epistemic uncertainty quantification

Wavelet thresholding filter:

$$\overline{u}_i^{>\epsilon}(\mathbf{x}) = \sum_{\mathbf{l}\in\mathcal{L}^0} c^0_{\mathbf{l}}\,\phi^0_{\mathbf{l}}(\mathbf{x}) + \sum_{j=0}^{+\infty}\sum_{\mu=1}^{2^n-1}\sum_{\substack{\mathbf{k}\in\mathcal{K}^j \\ |d^j_{\mathbf{k}}|\geq\epsilon\|\mathbf{u}\|}} d^{\mu,j}_{\mathbf{k}}\,\psi^{\mu,j}_{\mathbf{k}}(\mathbf{x})$$

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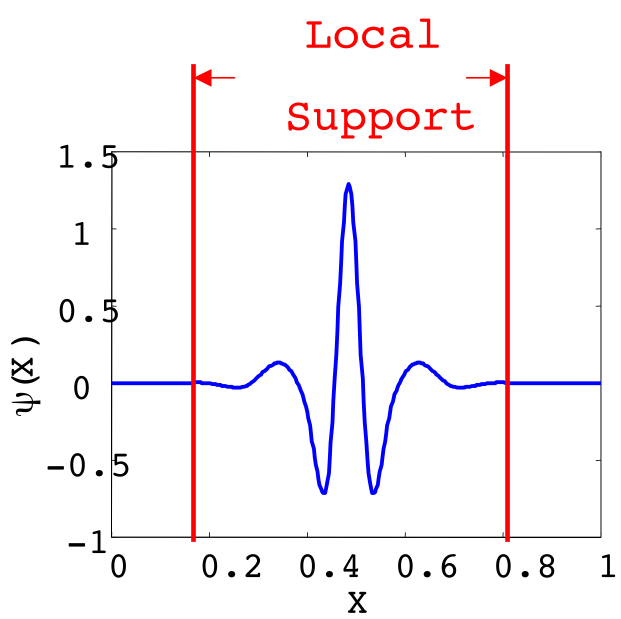
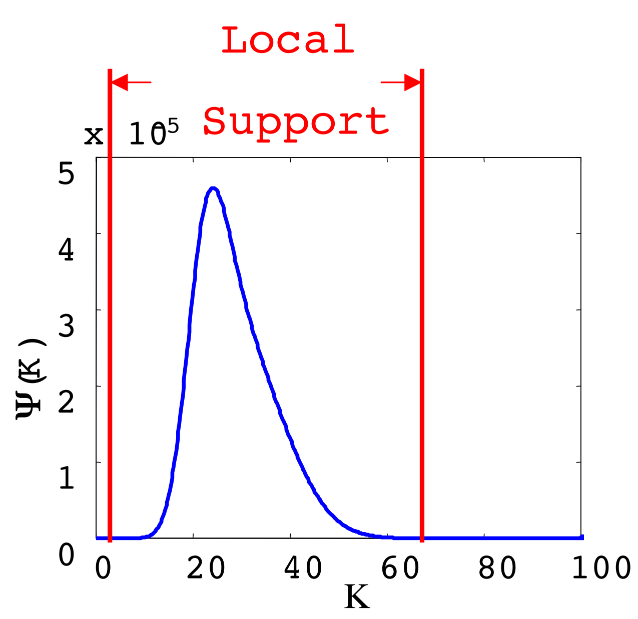
Physical Space



Wave Number Space



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$$\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}}^{>\epsilon}(\mathbf{x},t) + \overline{\mathbf{u}}^{\leq\epsilon}(\mathbf{x},t)$$

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Choice of ϵ:

- WDNS - $\epsilon \ll 1$
- CVS* - $\epsilon \approx \epsilon_{\mathrm{opt}}$
- SCALES† - $\epsilon > \epsilon_{\mathrm{opt}}$

Coherent Vortex SImulation (CVS): Farge M, Schneider K, Kevlahan N. Phys. Fluids 11:2187–201, 1999.

†Stochastic Coherent Adaptive Large Eddy Simulations (SCALES): Goldstein, D.E. and Vasilyev, O.V., Phys. Fluids 16: 2497-2513, 2004.

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Wavelet-based Turbulence Modeling Hierarchy

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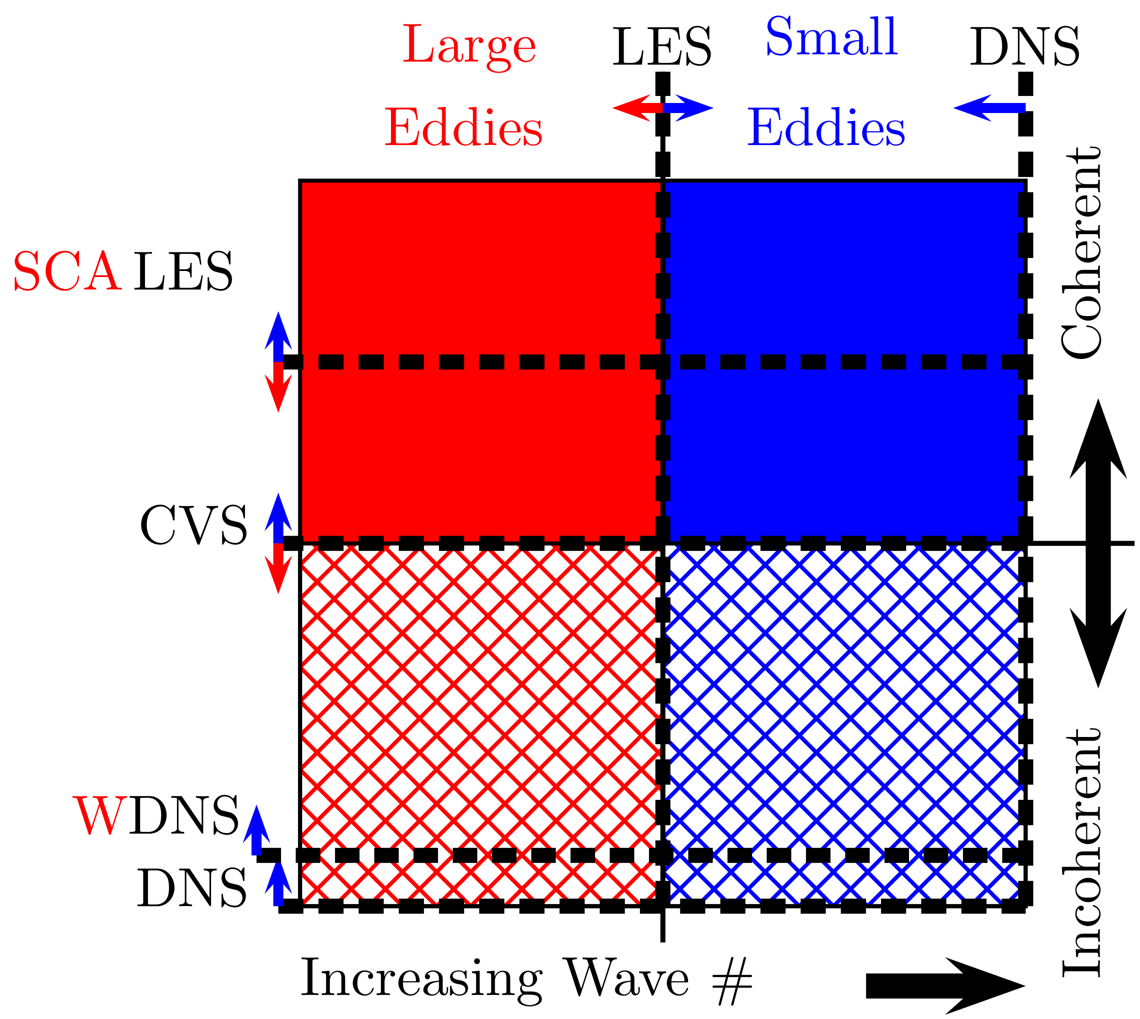

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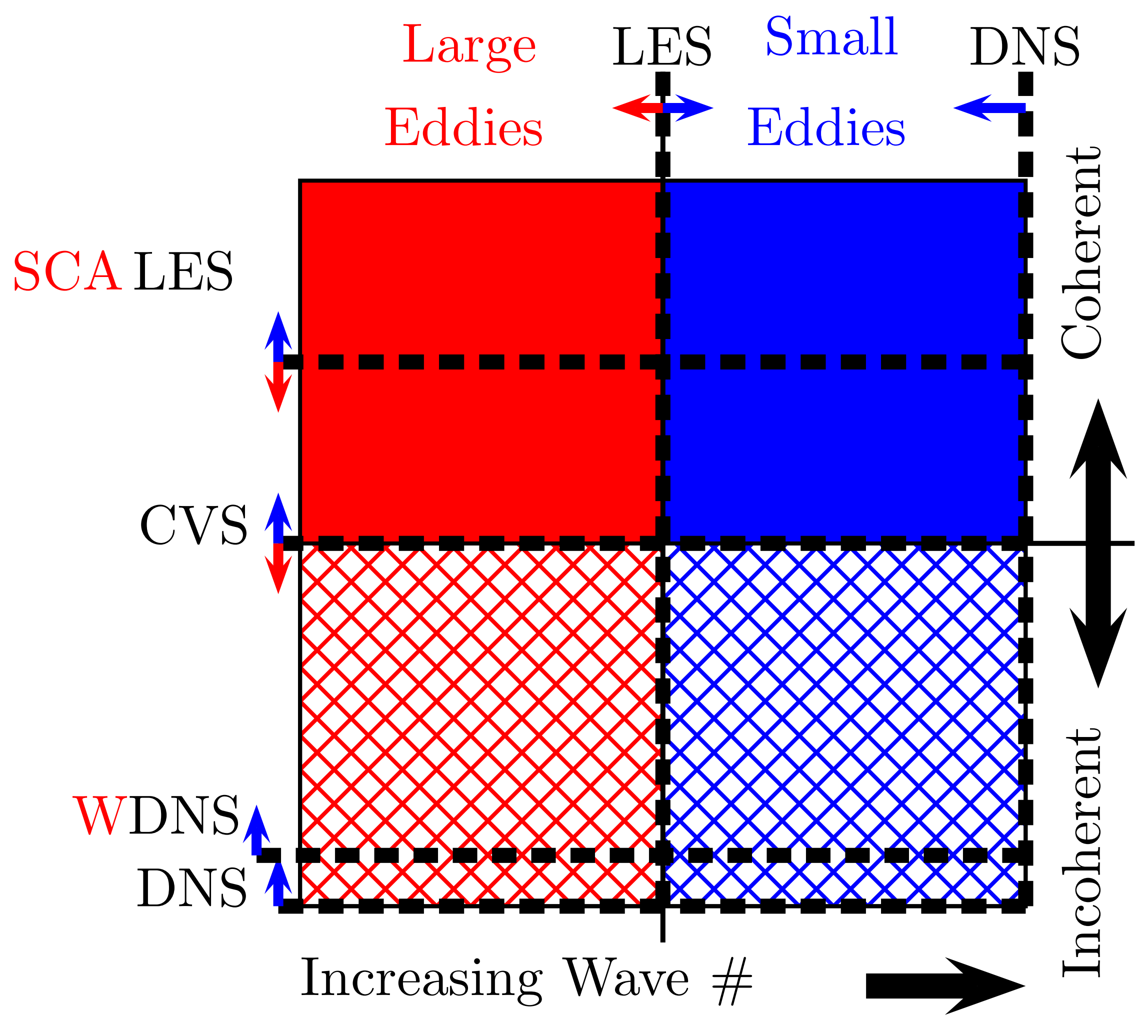

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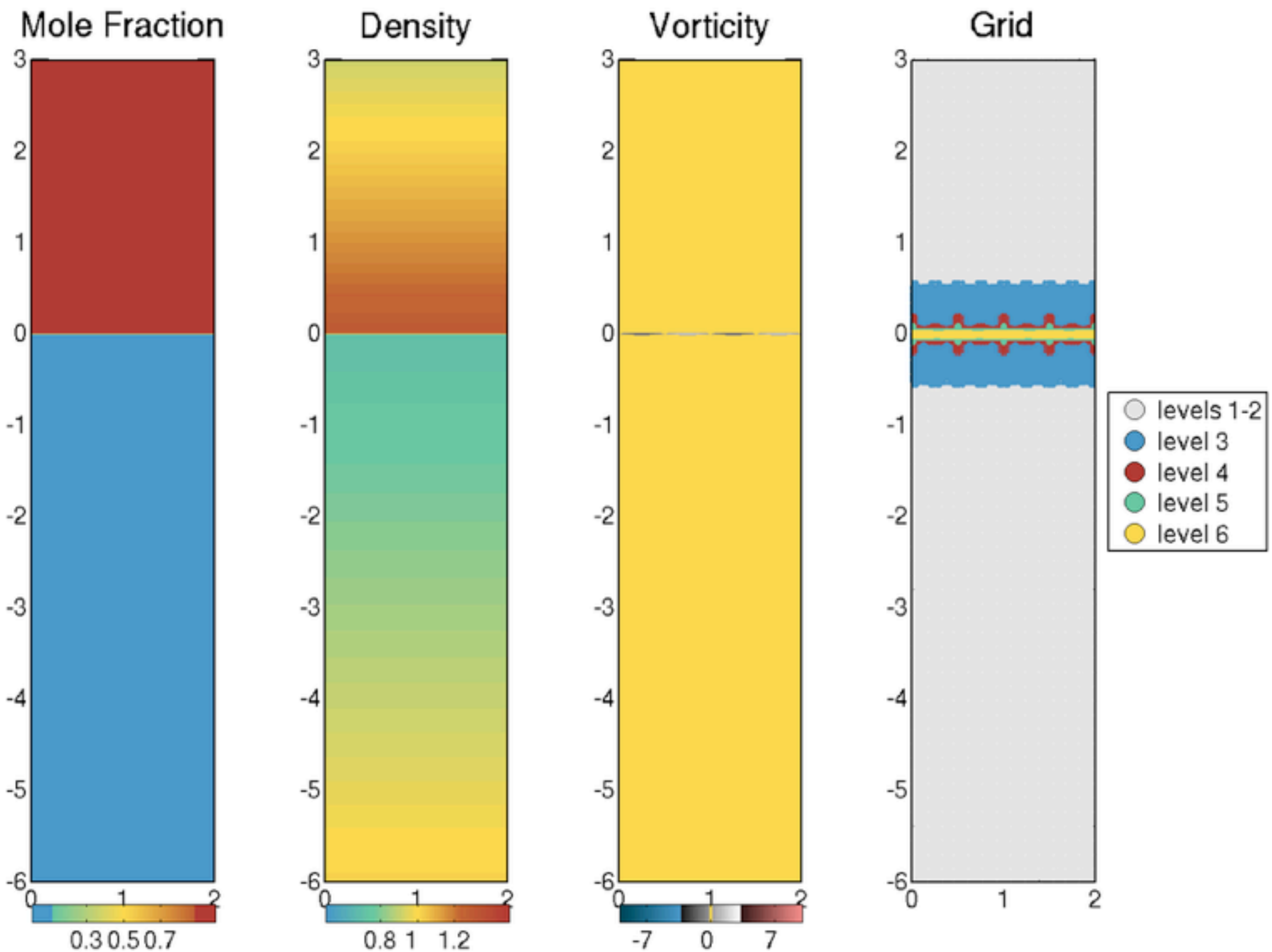
Simulate the evolution of the most energetic coherent vortices (track them), while modeling the effect of the subgrid scales.

$$\frac{\partial\overline{u}_i^{>\epsilon}}{\partial t} + \frac{\partial\overline{u}_i^{>\epsilon}\overline{u}_j^{>\epsilon}}{\partial x_j} = -\frac{\partial\overline{p}^{>\epsilon}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2\overline{u}_i^{>\epsilon}}{\partial x_j\partial x_j} + \frac{\partial\tau_{ij}}{\partial x_j}$$

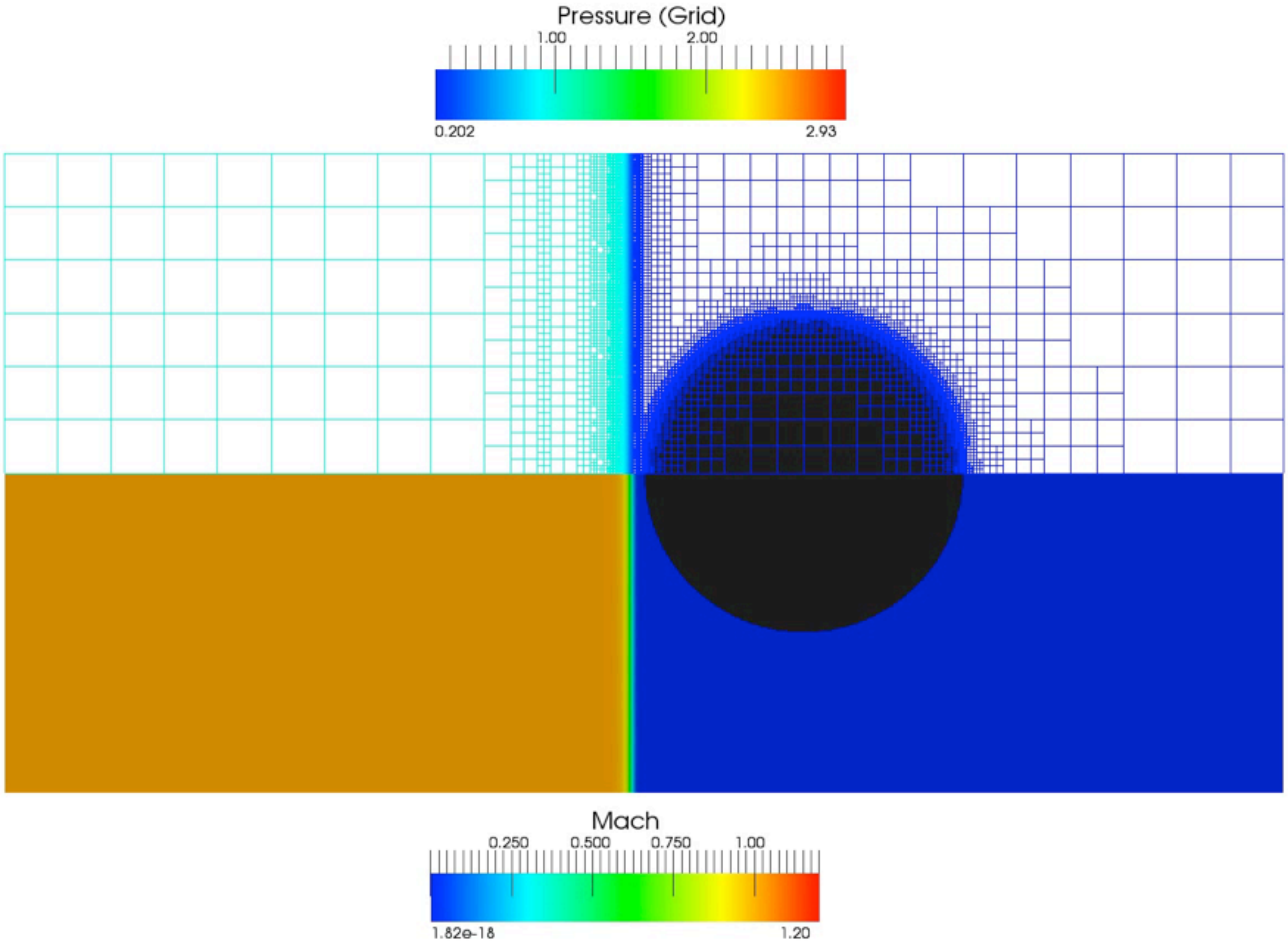
Adaptive Wavelet Collocation Method

Adaptive Wavelet Collocation Method (AWCM)

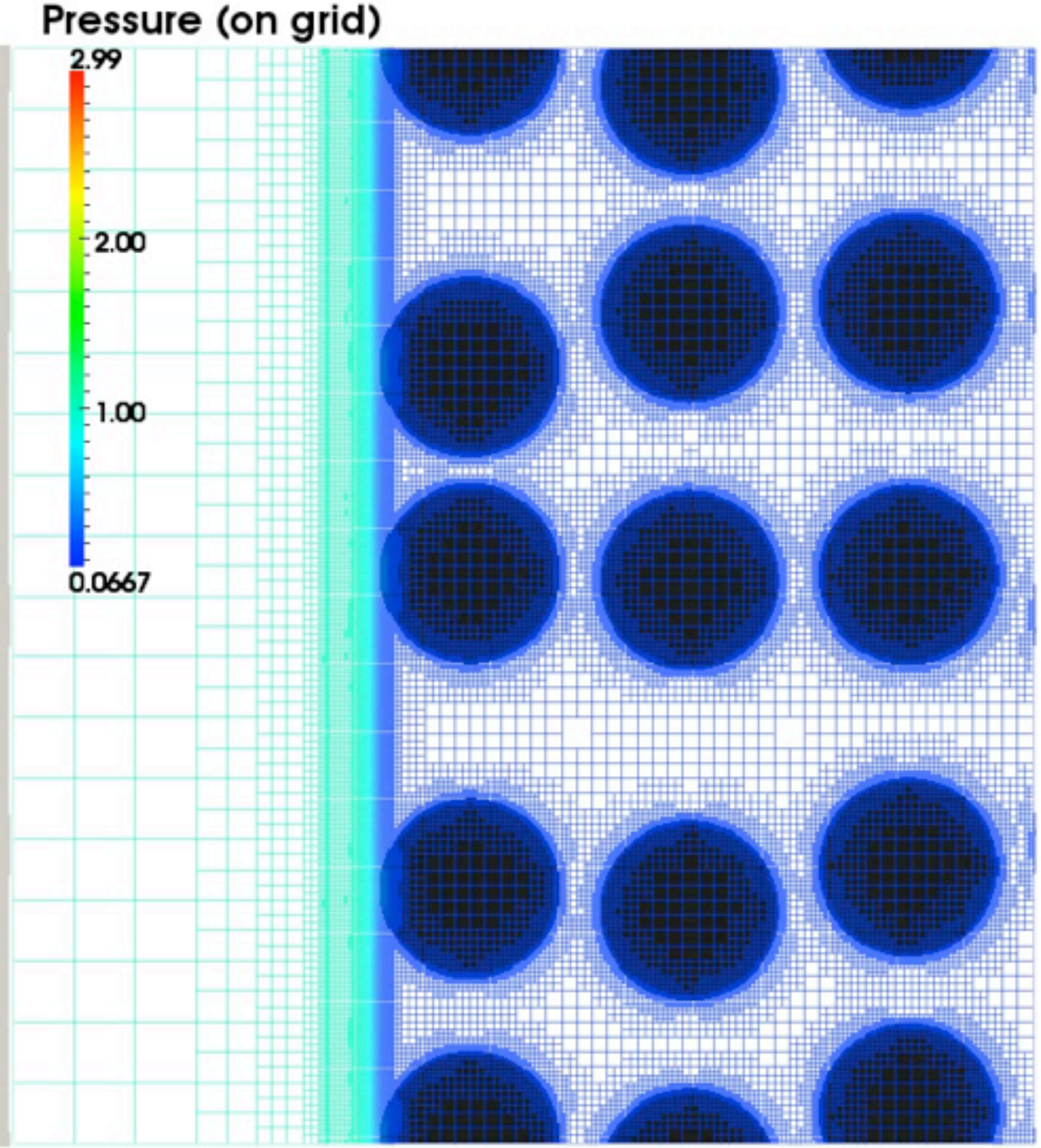
Single-mode
Rayleigh-Taylor Instability
(incompressible limit)



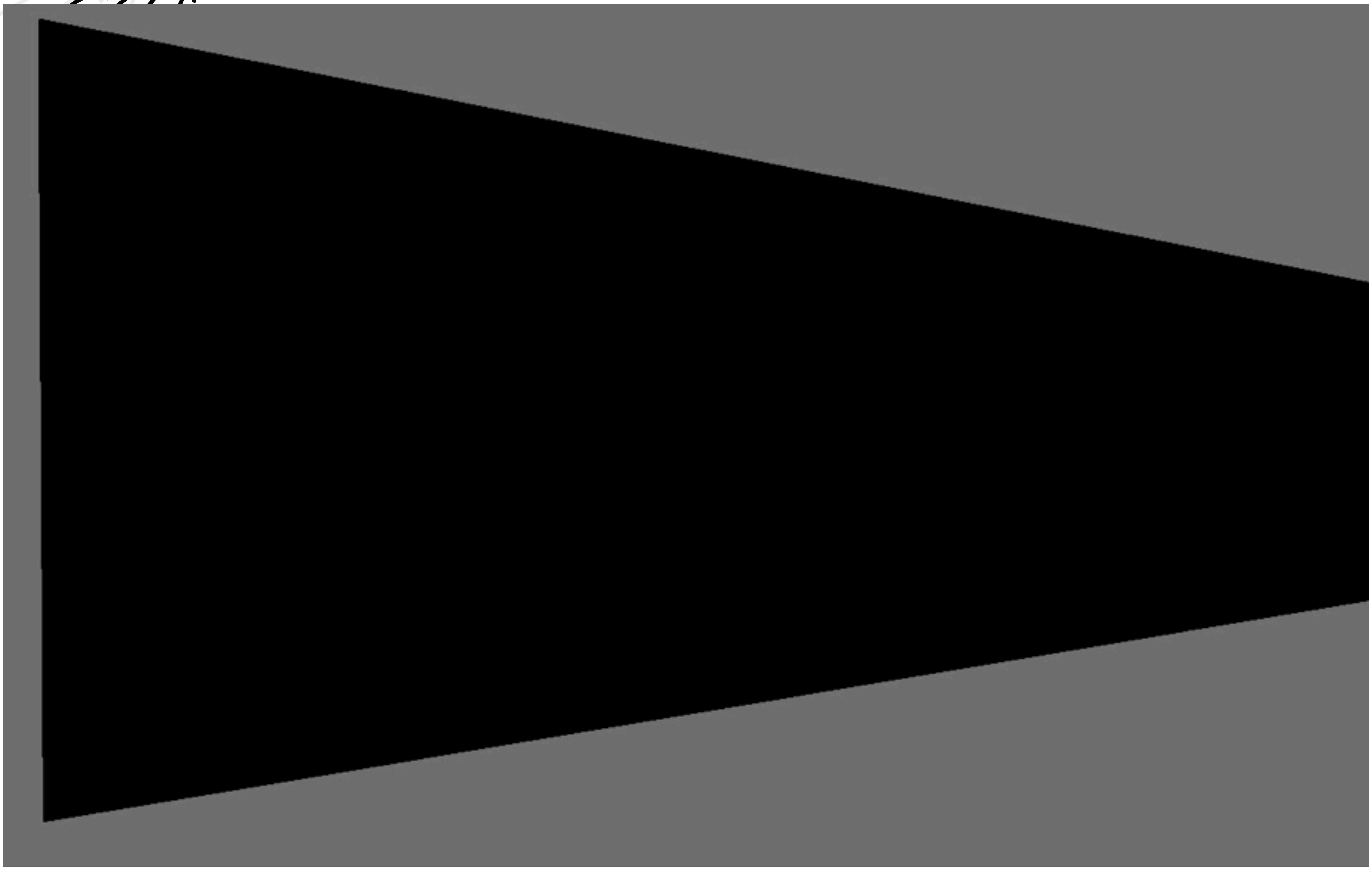
Shock Wave Propagation over the Cylinder



Shock Wave Propagation through the Cylinder Array



Acoustic Timescale Detonation Initiation



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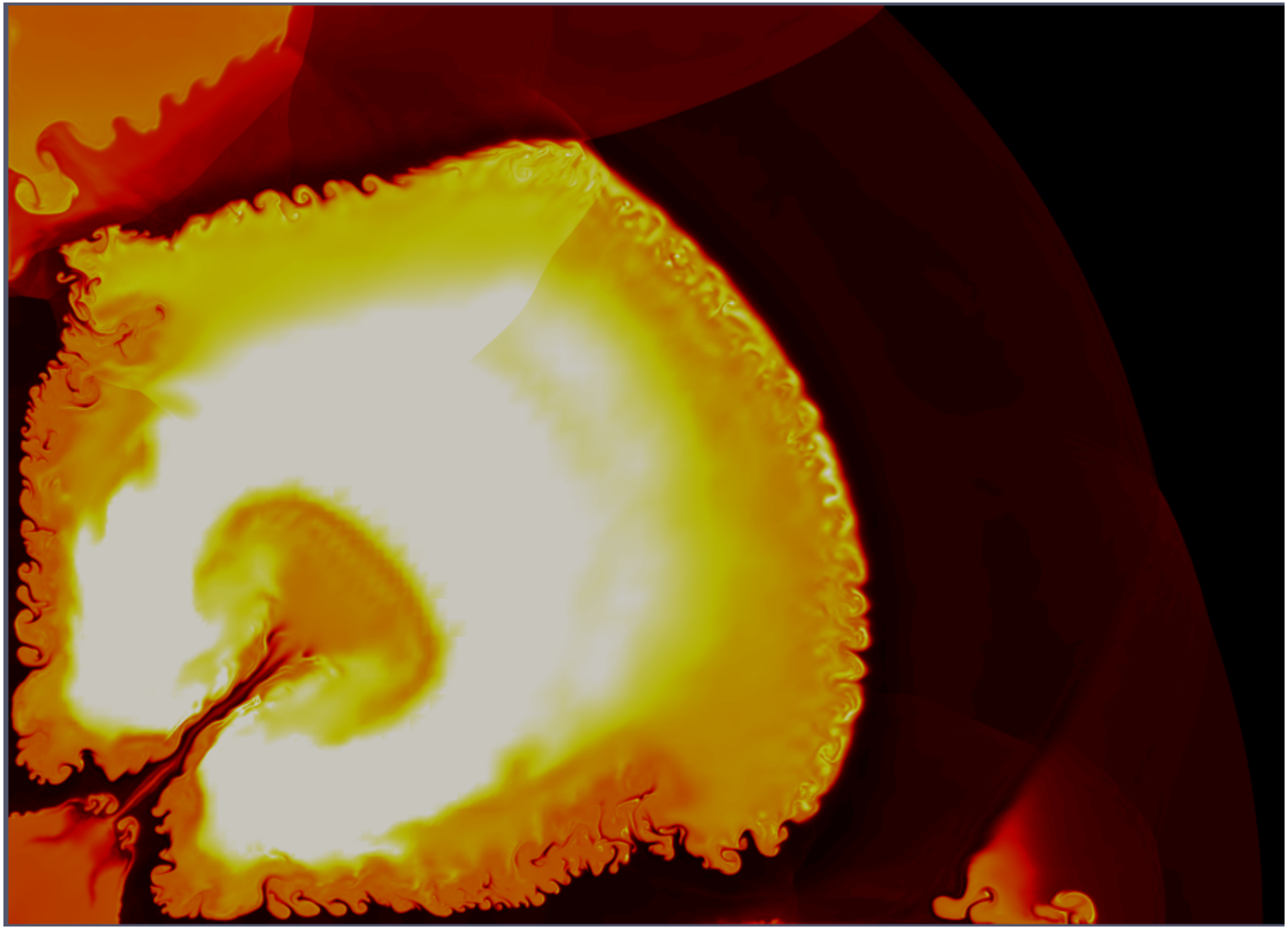

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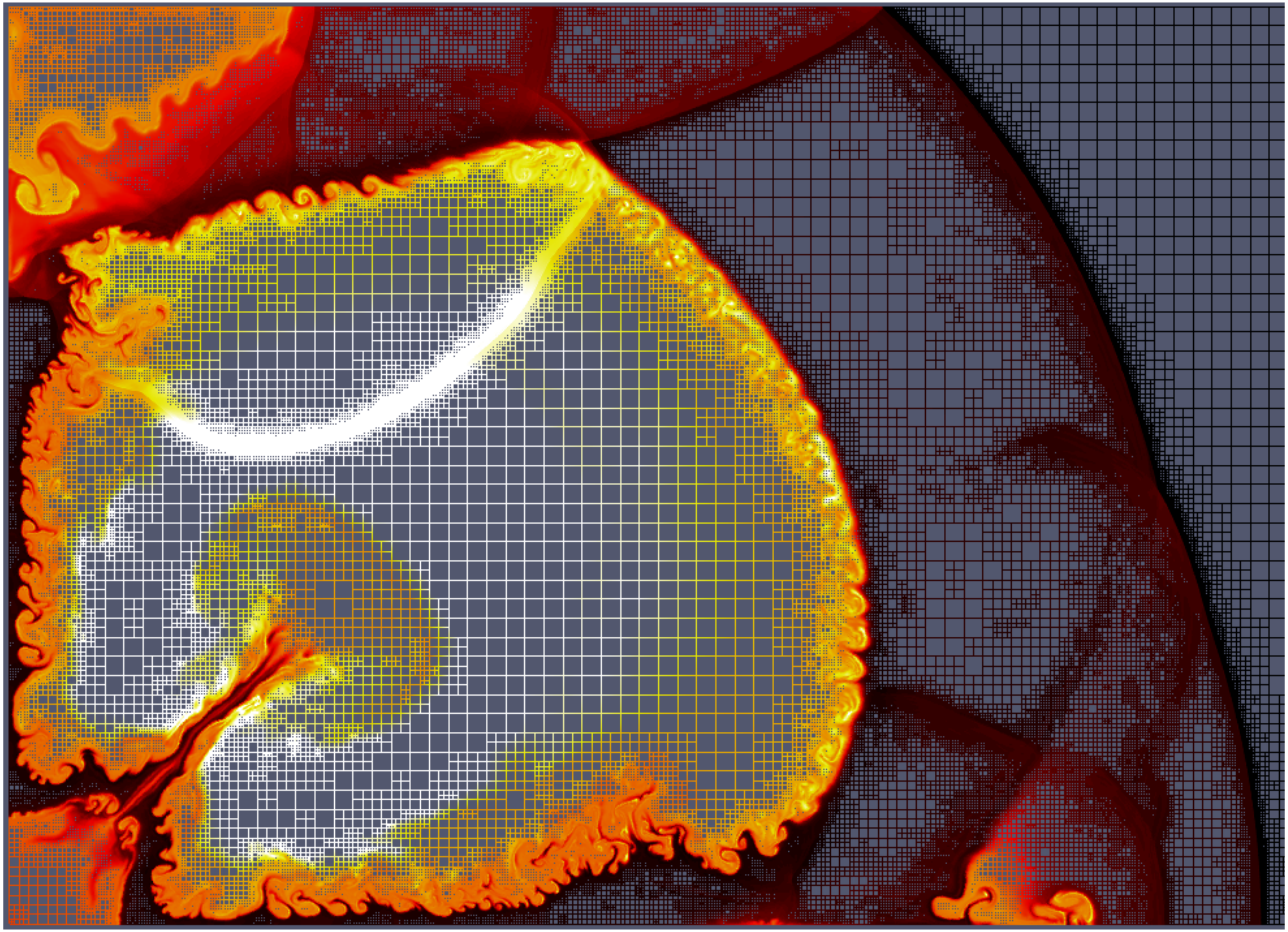

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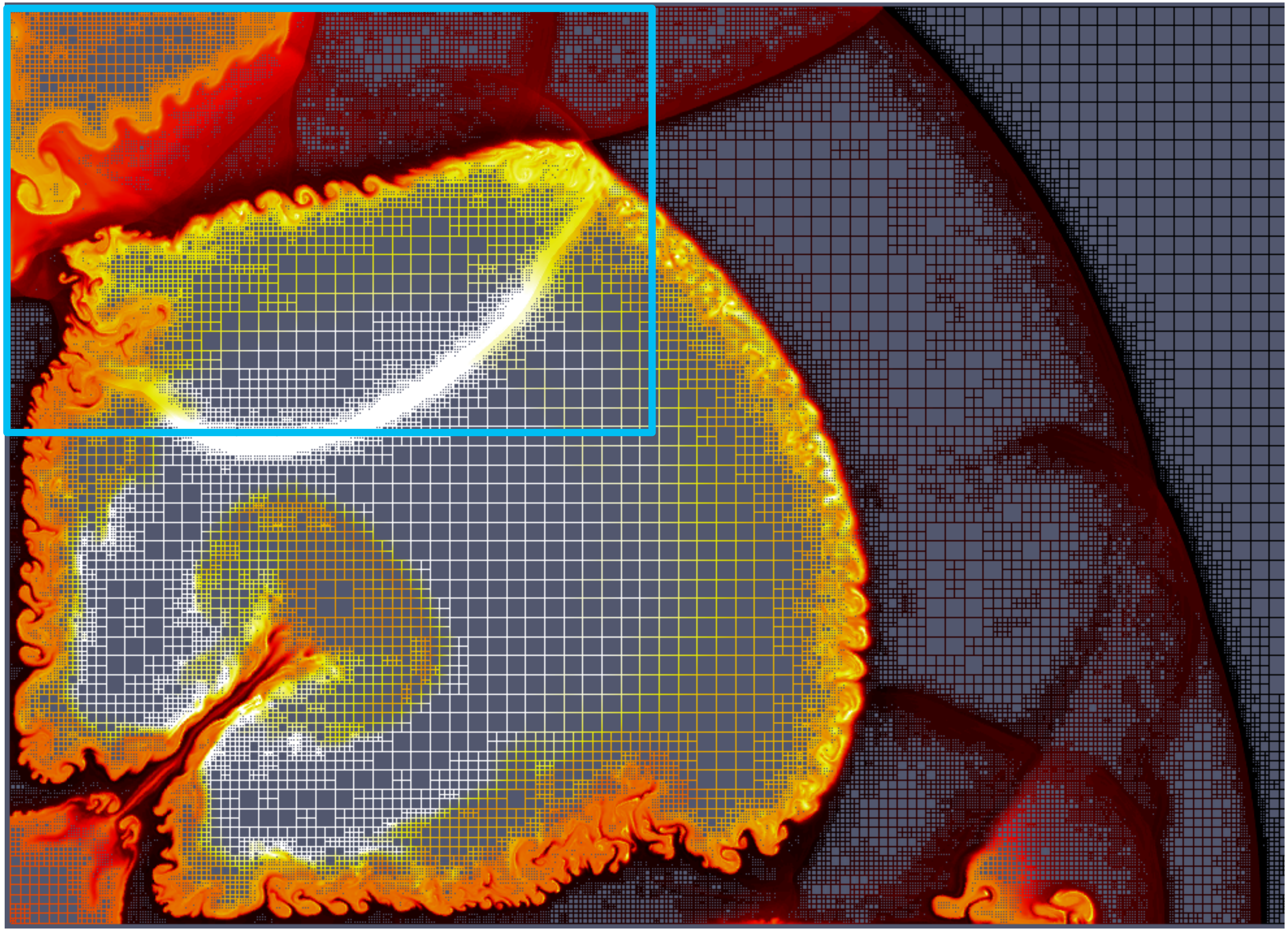

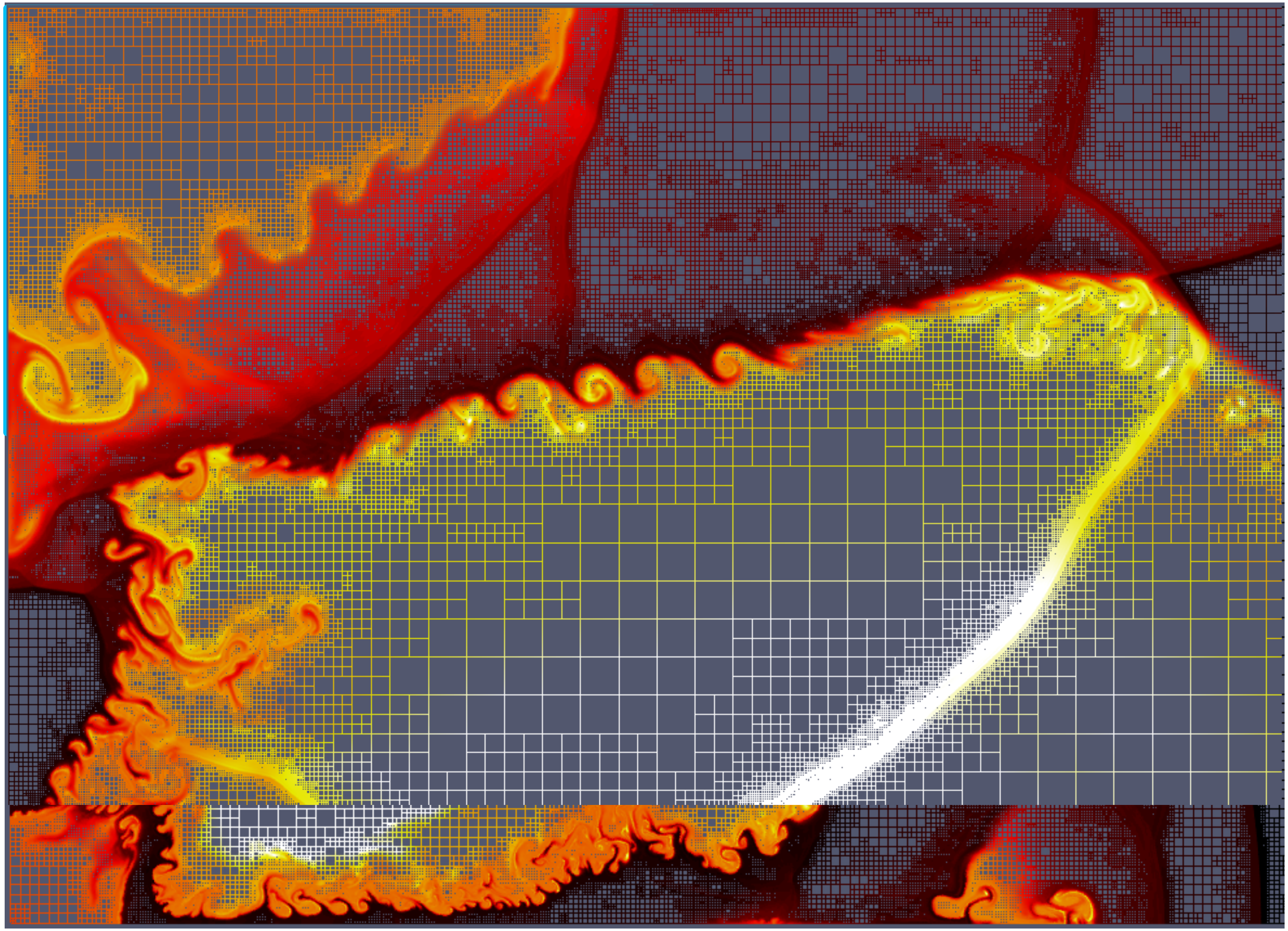

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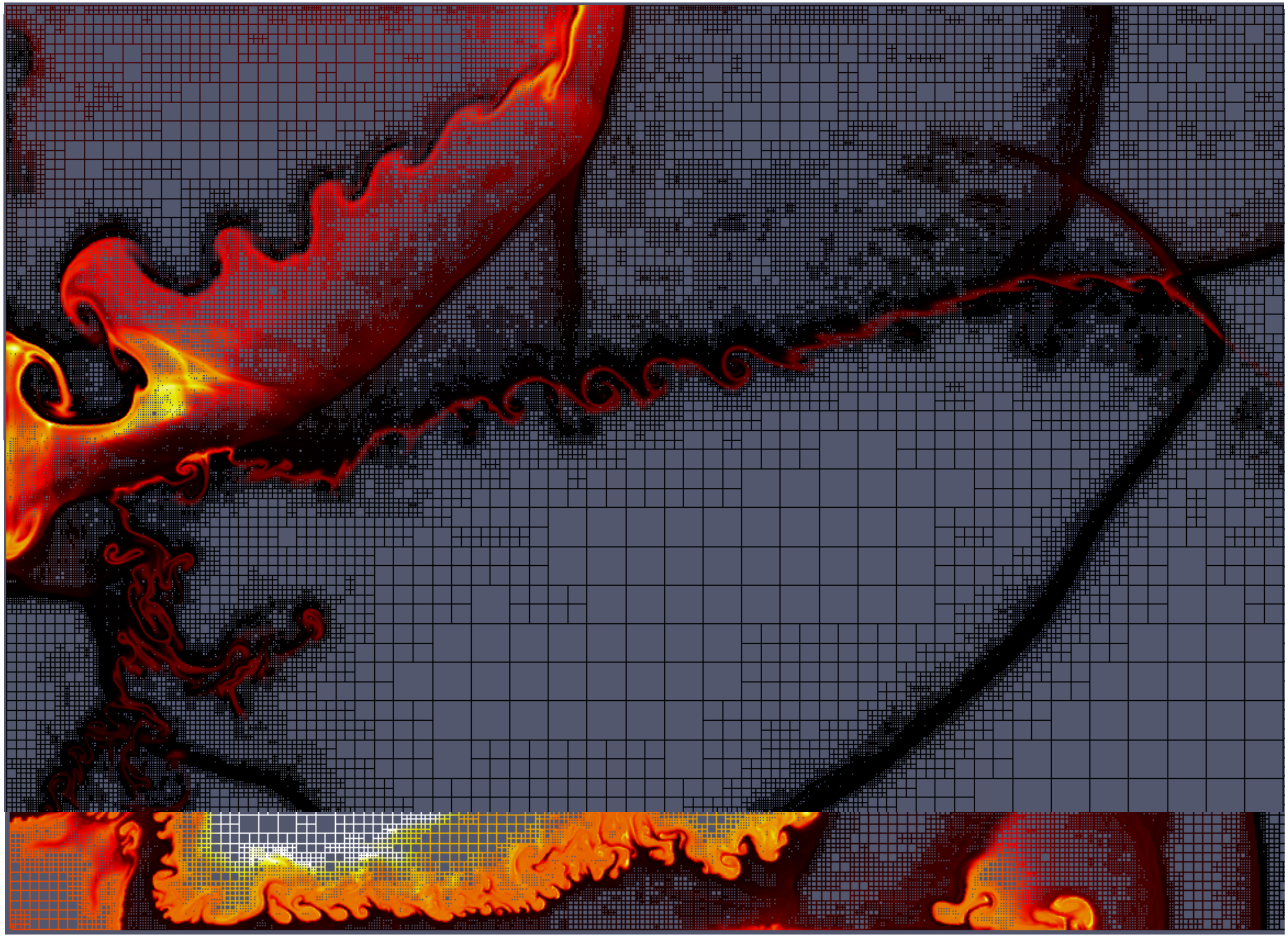

Hierarchical Variable Fidelity Multiscale Turbulence Modeling

Kinetic Energy Based: $$\mathcal{F} = \frac{k_{\mathrm{sgs}}}{k_{\mathrm{res}} + k_{\mathrm{sgs}}}$$

SGS dissipation Based: $$\mathcal{F} = \frac{\Pi}{\varepsilon_{\mathrm{res}} + \Pi}$$

Turbulence Resolution

Kinetic Energy Based: $$\mathcal{F} = \frac{k_{\text{sgs}}}{k_{\text{res}} + k_{\text{sgs}}}$$

SGS dissipation Based: $$\mathcal{F} = \frac{\Pi}{\varepsilon_{\text{res}} + \Pi}$$

Fidelity of the simulation is a function of Turbulence Resolution

Objective - control the level of fidelity

Kinetic Energy Based: $$\mathcal{F} = \frac{k_{\mathrm{sgs}}}{k_{\mathrm{res}} + k_{\mathrm{sgs}}}$$

SGS dissipation Based: $$\mathcal{F} = \frac{\Pi}{\varepsilon_{\mathrm{res}} + \Pi}$$

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Homogeneous Turbulence:

LES with $\mathcal{F}_{KE}$ fixed complexity $\sim Re^0 = 1$

LES with $\mathcal{F}_D$ fixed complexity $\sim Re^{9/4}$

Spatial Variable Thresholding

Scales Dependency

$\epsilon\downarrow \quad \Longrightarrow \quad$ Buildup scales $\quad \varepsilon_{\text{res}}\uparrow$

$\epsilon\uparrow \quad \Longrightarrow \quad$ Losing some scales $\quad \Pi\uparrow$

Goal: $\epsilon\downarrow$ wherever $\Pi > \Pi_{\text{Goal}}$

In General:

Wavelet-Threshold-Filtered Velocity Depends on:

1) Threshold Level
2) Velocity Scale

$$\overline{u}_i^{>\epsilon} \sim \epsilon \ \& \ \|u_i\|$$

Idea:

Threshold is determined on-the-fly By Tracking areas of Locally Significant:

1) SGS Dissipation or
2) Any other Physical quantity

Fully Adaptive Wavelet Thresholding Filter

$$\epsilon = \epsilon\,(\Pi)$$

Lagrangian "Variable Thresholding" SCALES

If ϵ changed in spatial space

Then @ next time-step that flow-structures will move in space, it will face to either a smaller or greater ϵ

Recommended Solution:

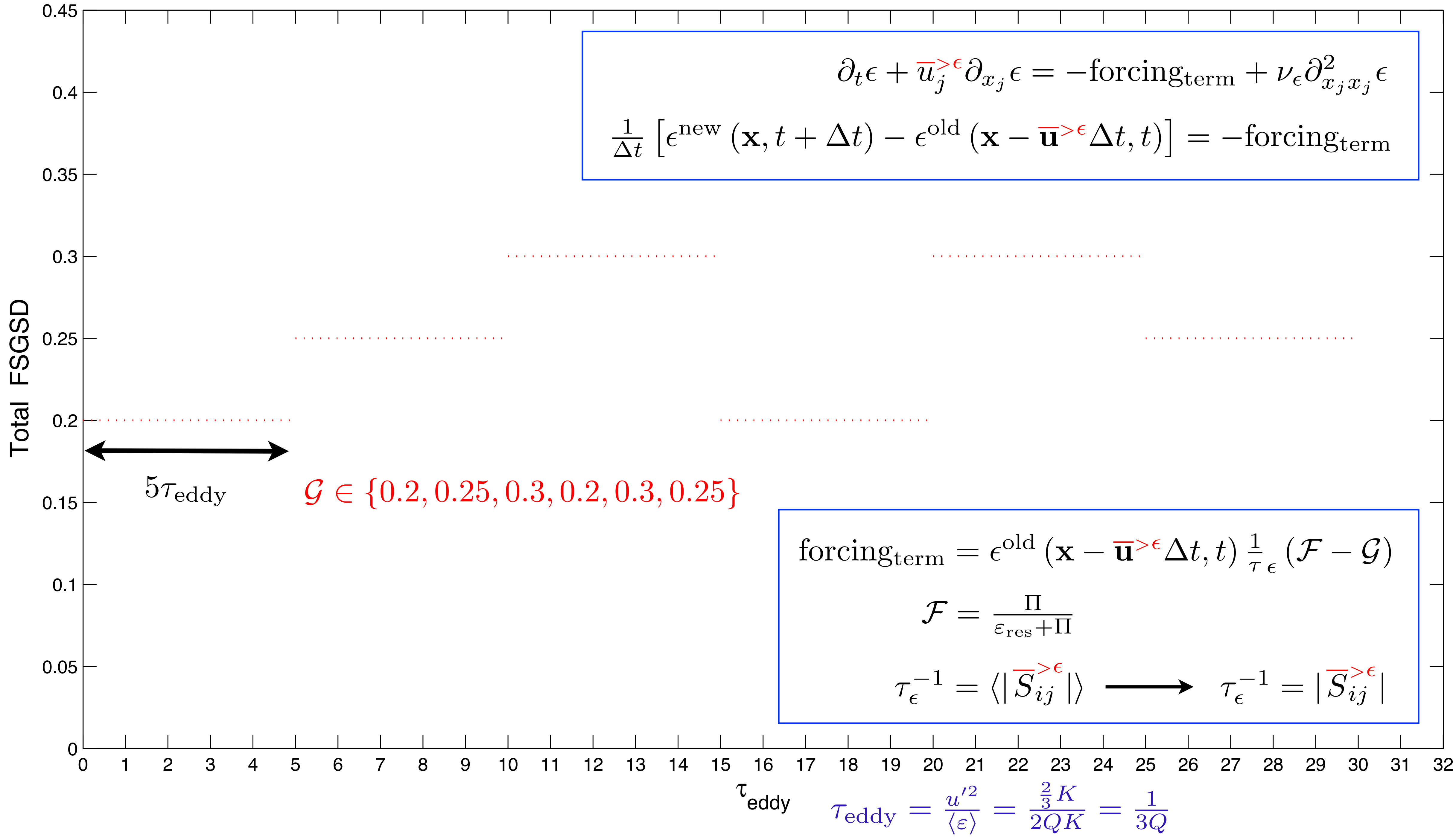
Track ϵ within a Lagrangian frame by "Lagrangian Path-Line Diffusive Averaging" Approach (Similarly to Vasilyev et al., [JOT, 9(11), 2008] Lagrangian SGS SCALES])

$$\partial_t \epsilon + \overline{u}_j^{>\epsilon} \partial_{x_j} \epsilon = -\mathrm{forcing}_{\mathrm{term}} + \nu_\epsilon \partial^2_{x_j x_j} \epsilon$$

Similarly to Meneveau et al. [JFM, 319, 1996] : Linear Averaging Along Characteristics
Diffusion Term can be ignored Because "Linear Averaging" itself will create required diffusion.
Lagrangian Path-Line Diffusive Averaging Evolution equation for

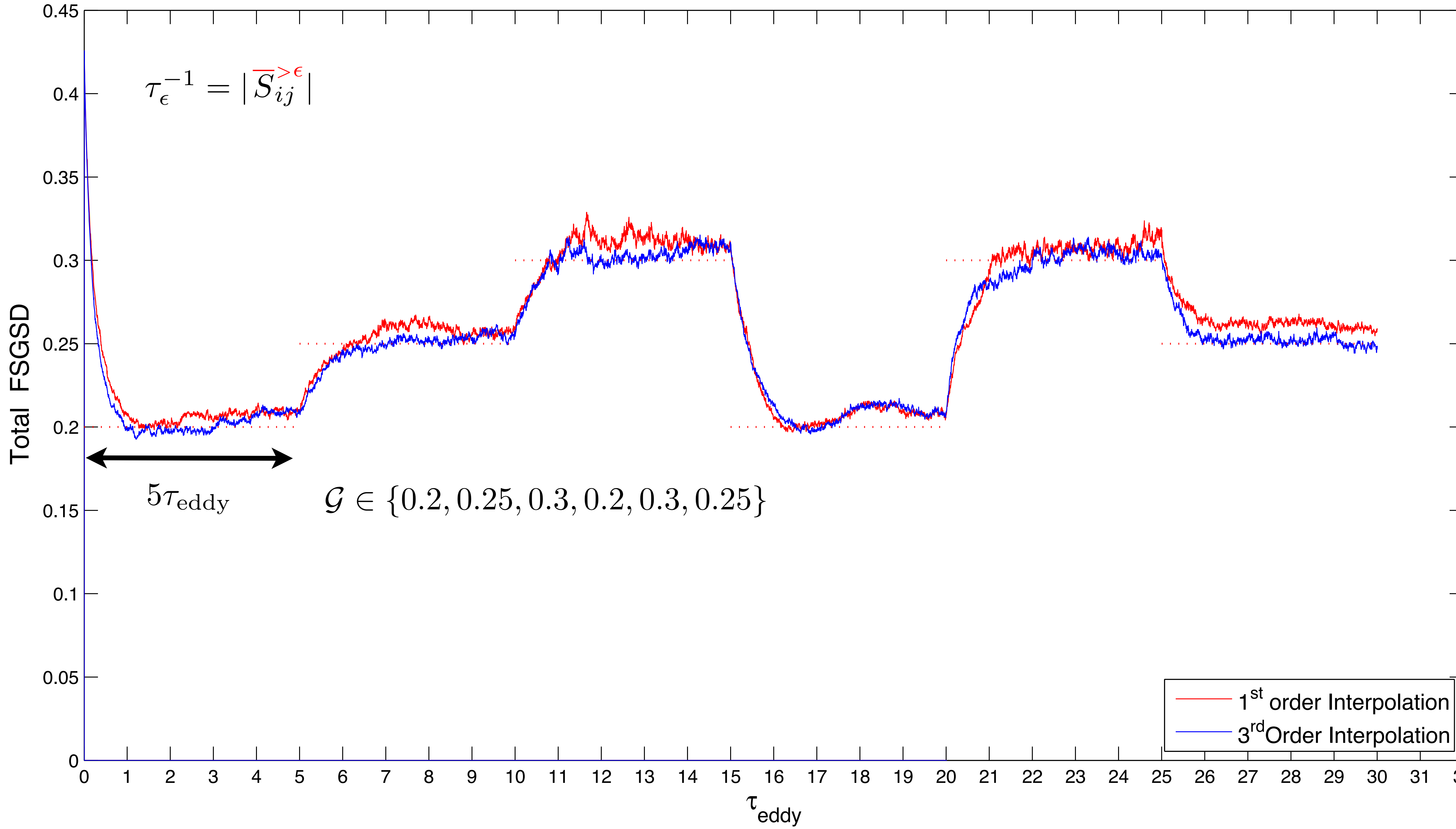
$$\frac{1}{\Delta t}\left[\epsilon^{\mathrm{new}}\left(\mathbf{x}, t+\Delta t\right) - \epsilon^{\mathrm{old}}\left(\mathbf{x} - \overline{\mathbf{u}}^{>\epsilon}\Delta t, t\right)\right] = -\mathrm{forcing}_{\mathrm{term}}$$

$$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\text{res}} \rangle + \langle \Pi \rangle}$$

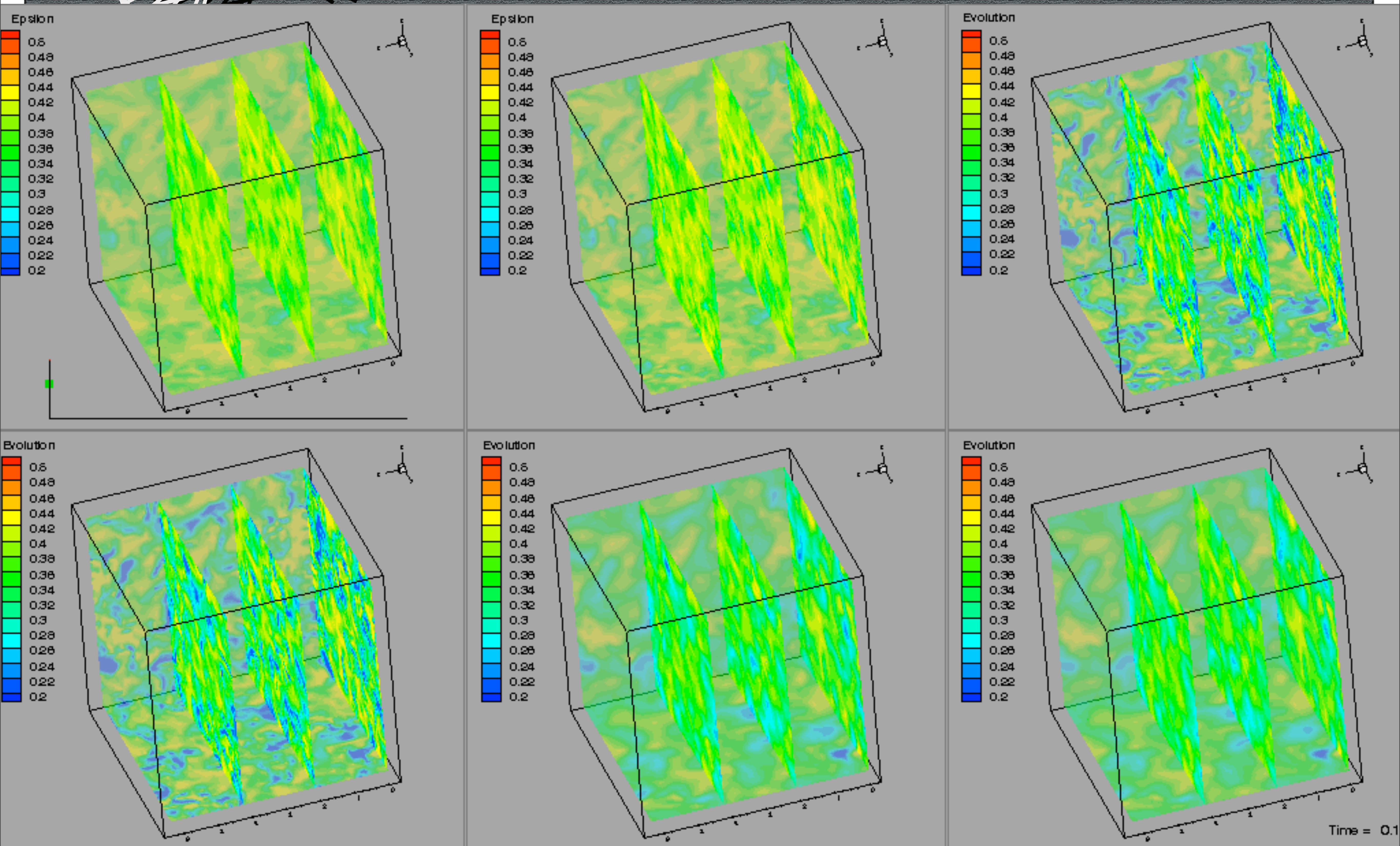


$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\text{res}} \rangle + \langle \Pi \rangle}$

1st & 3rd Order Interpolation Approach



Hybrid CVS / SCALES – Threshold Animation



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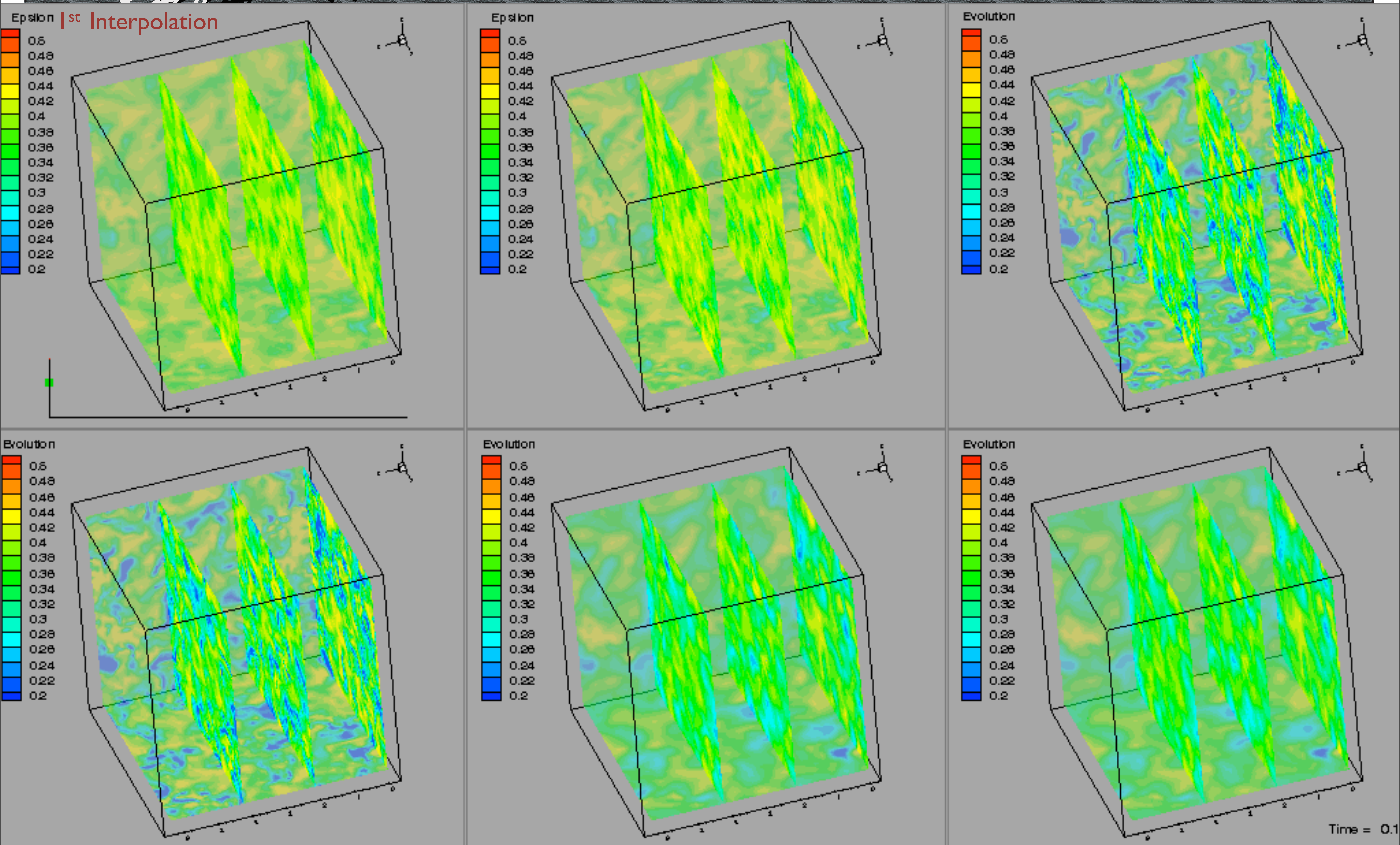

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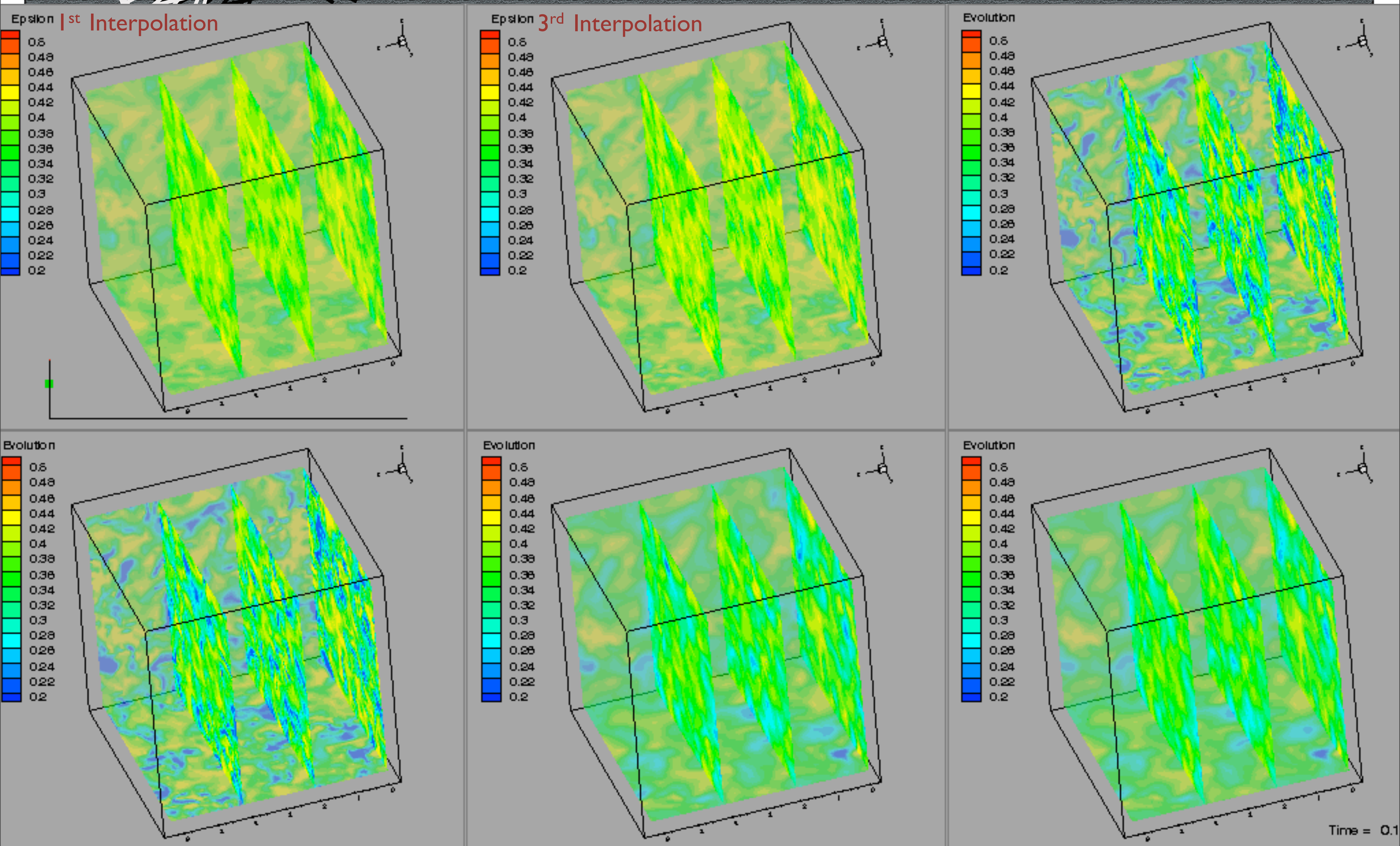

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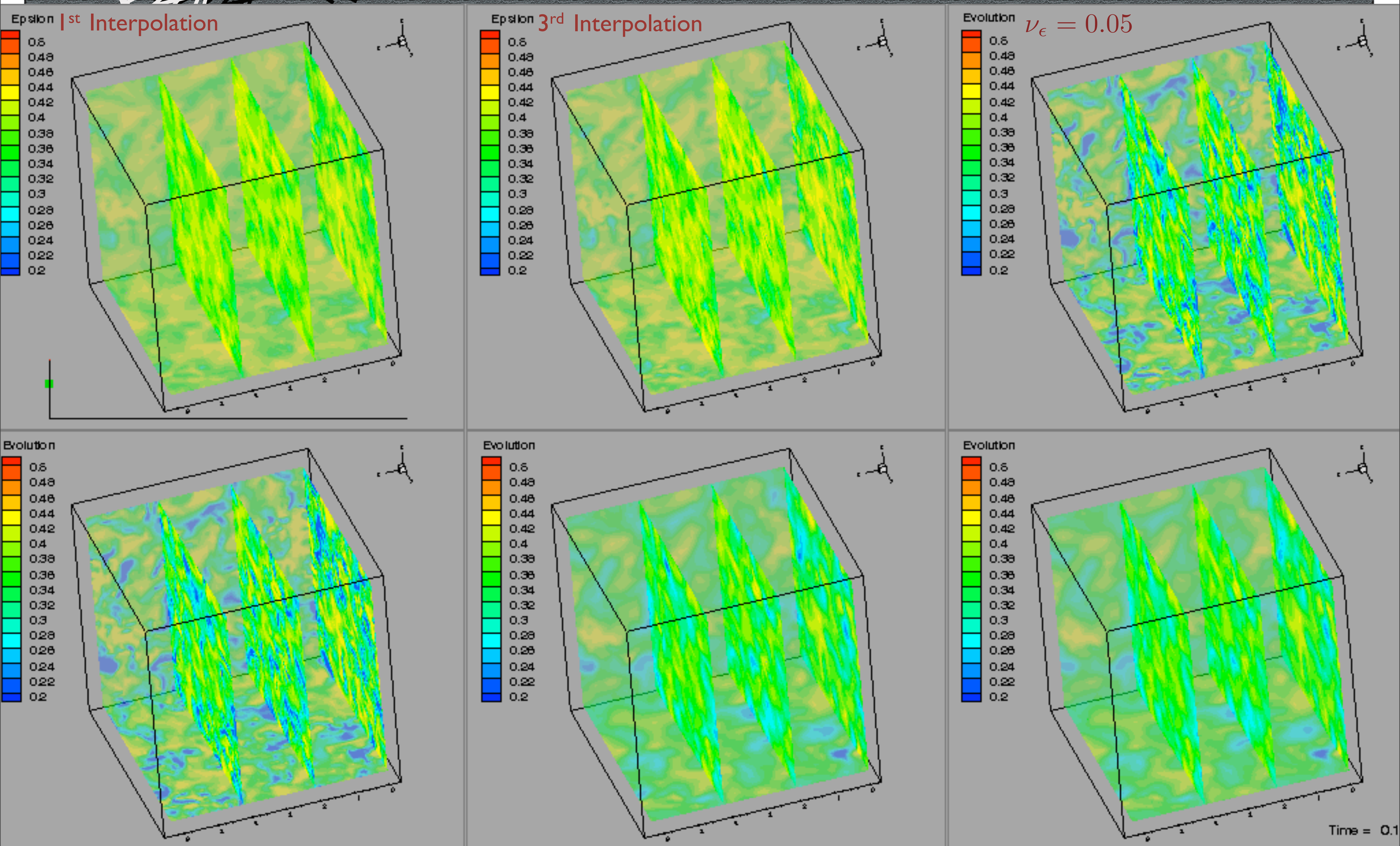

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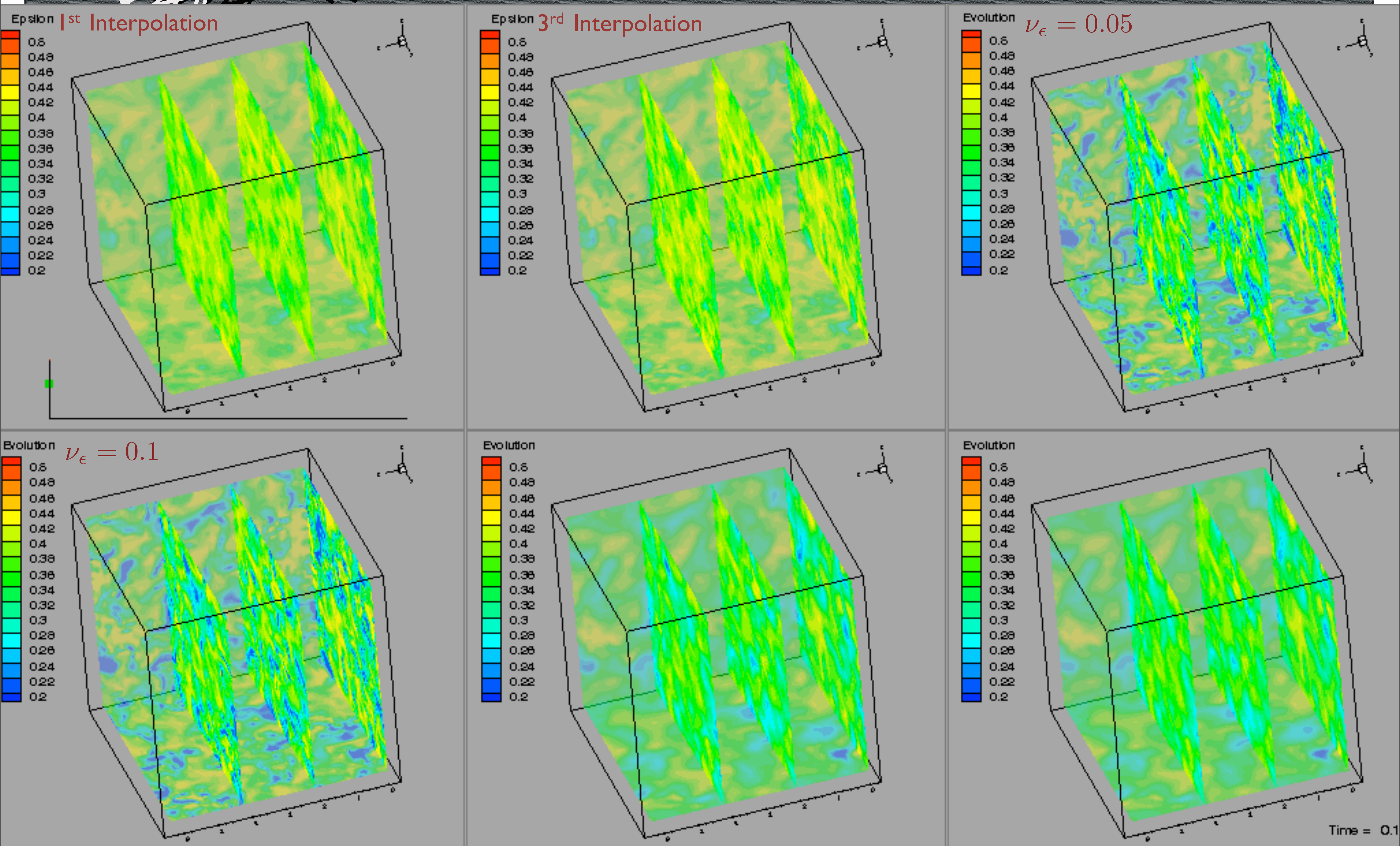

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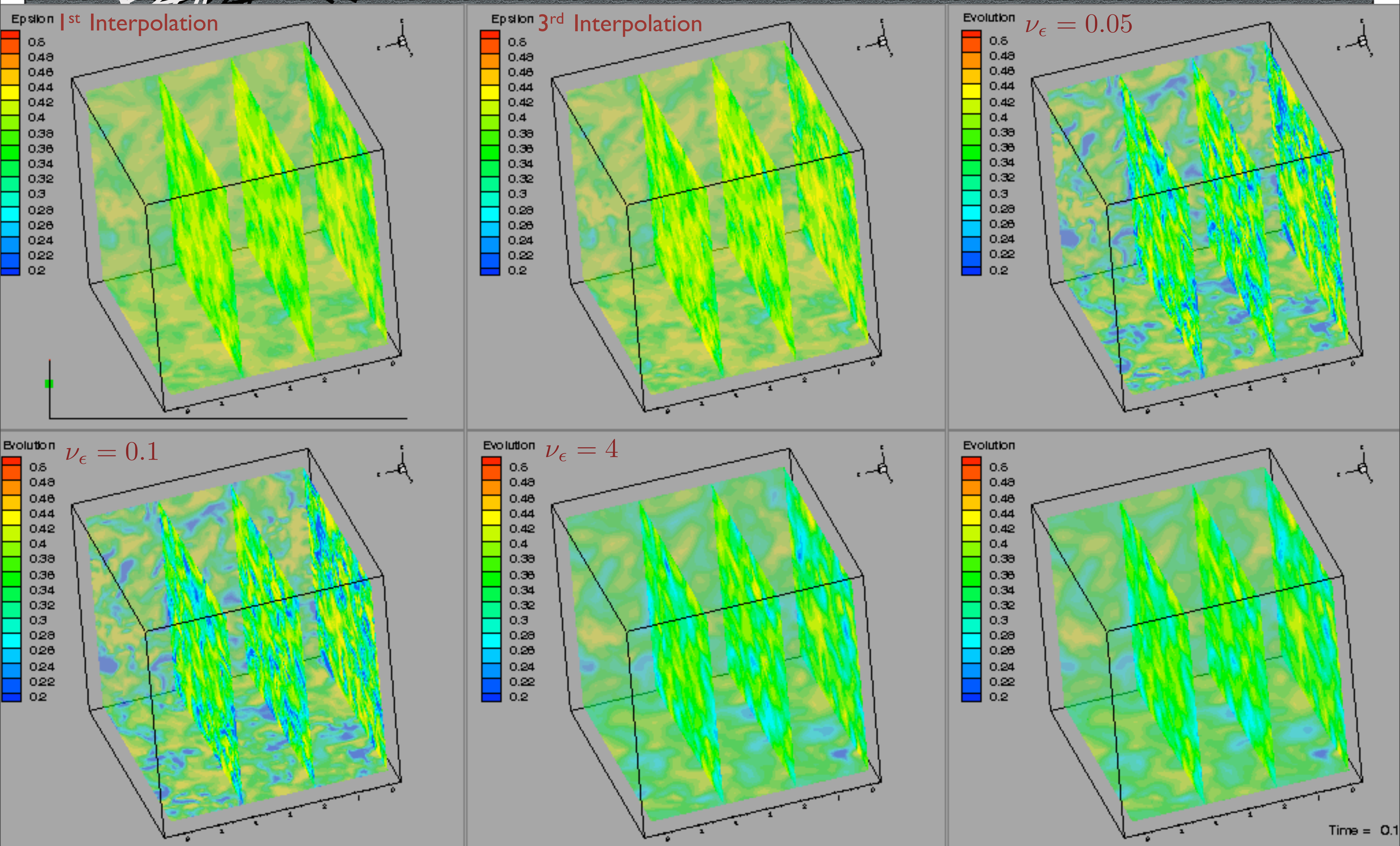

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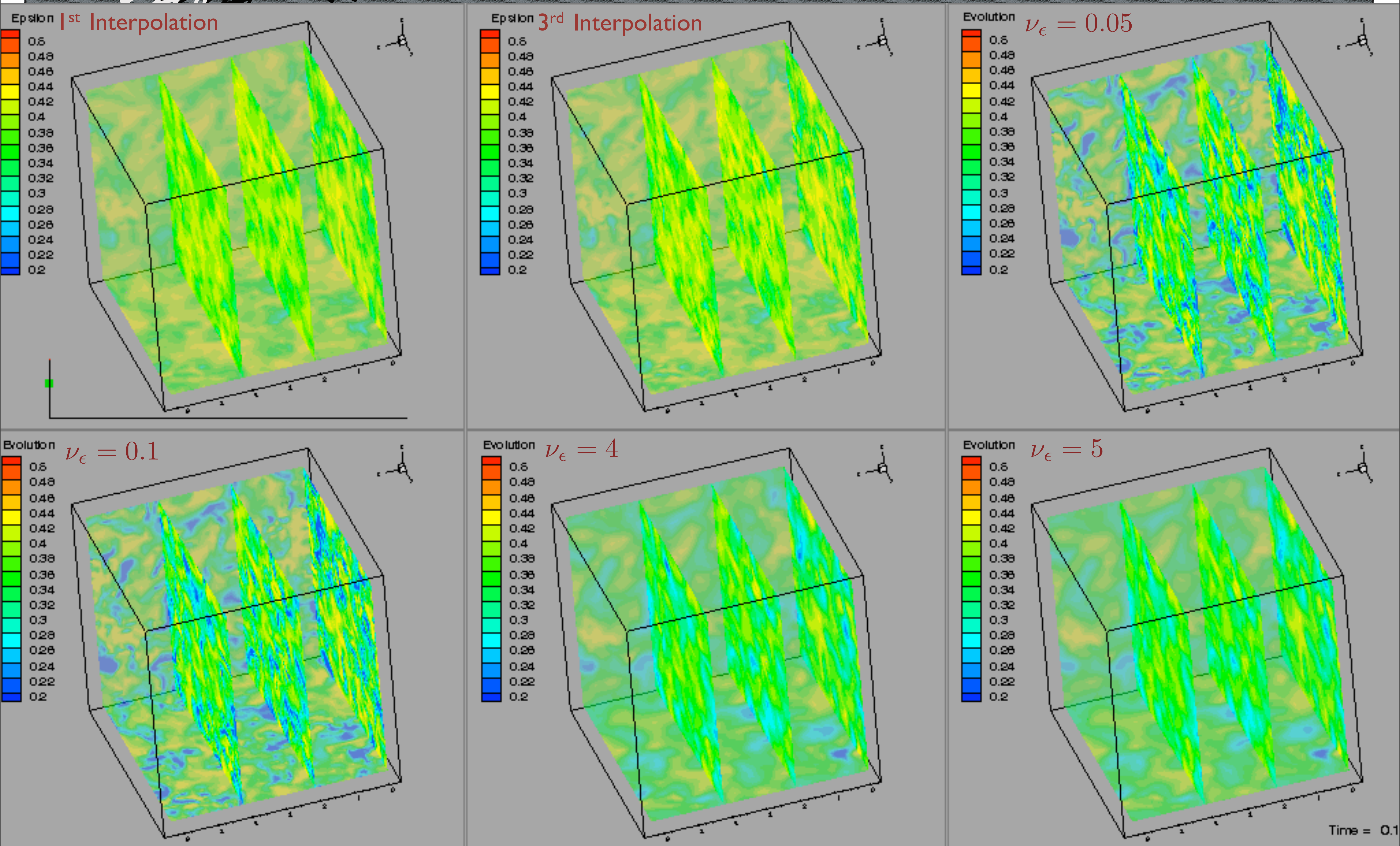

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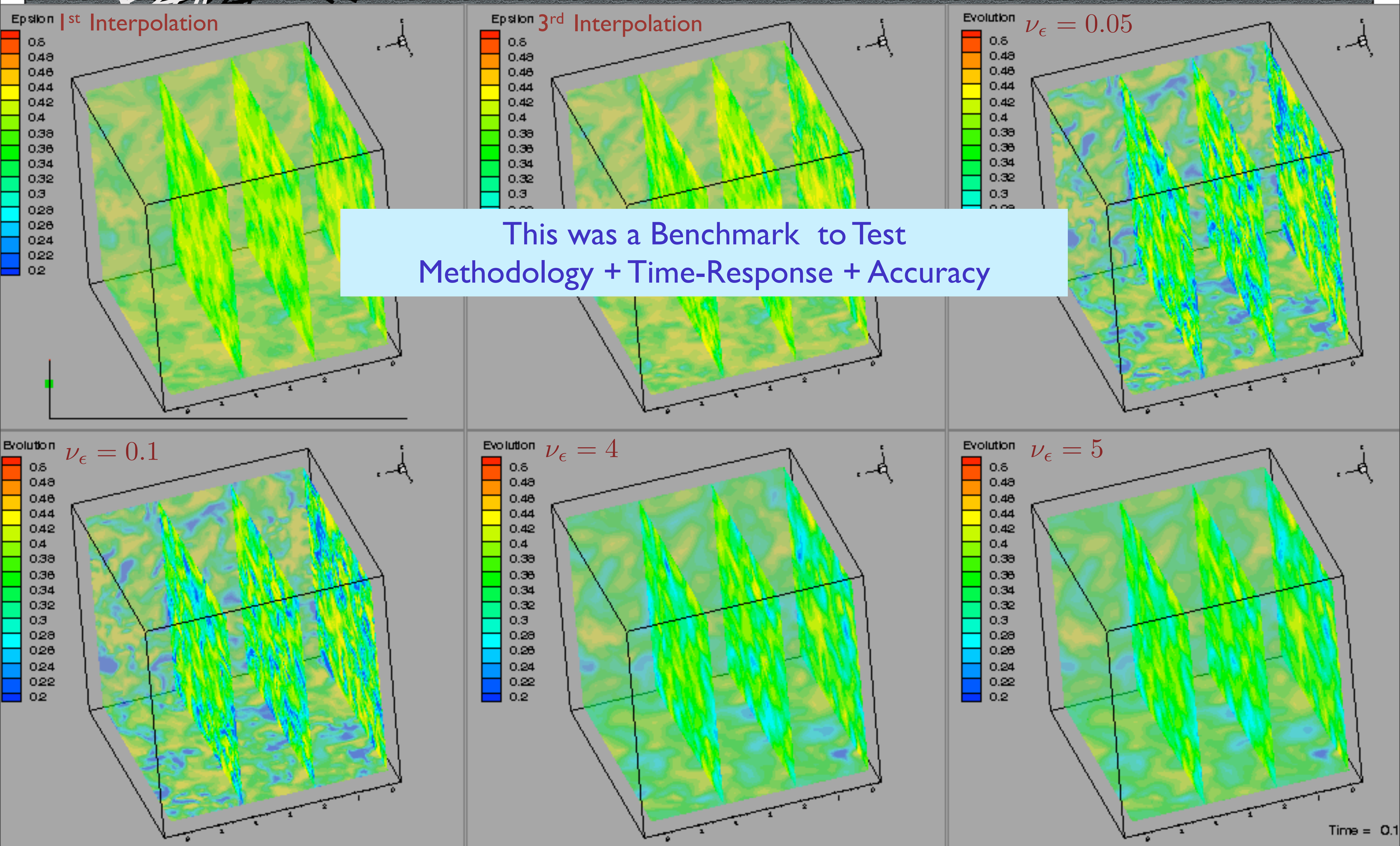

This was a Benchmark to Test
Methodology + Time-Response + Accuracy

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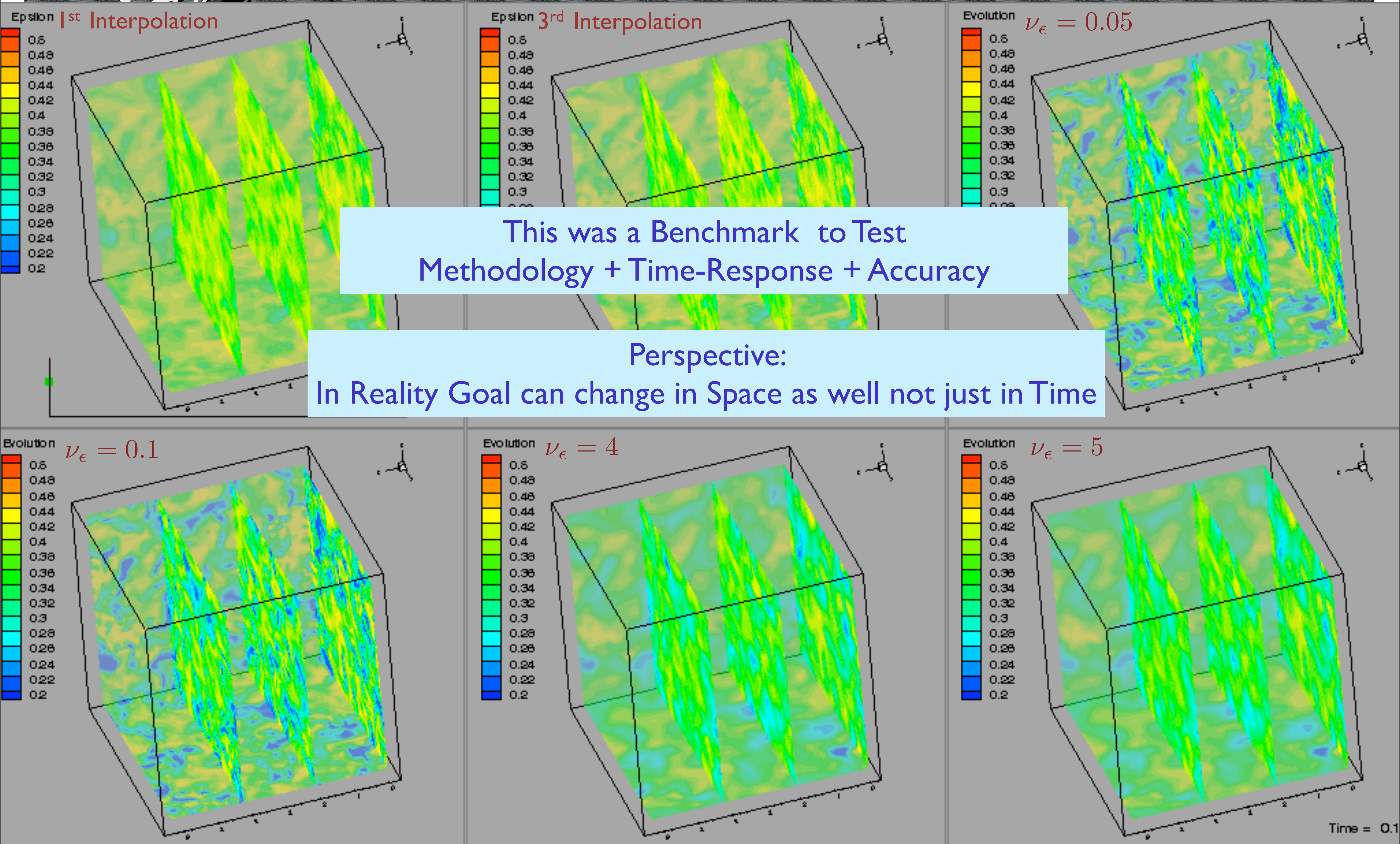

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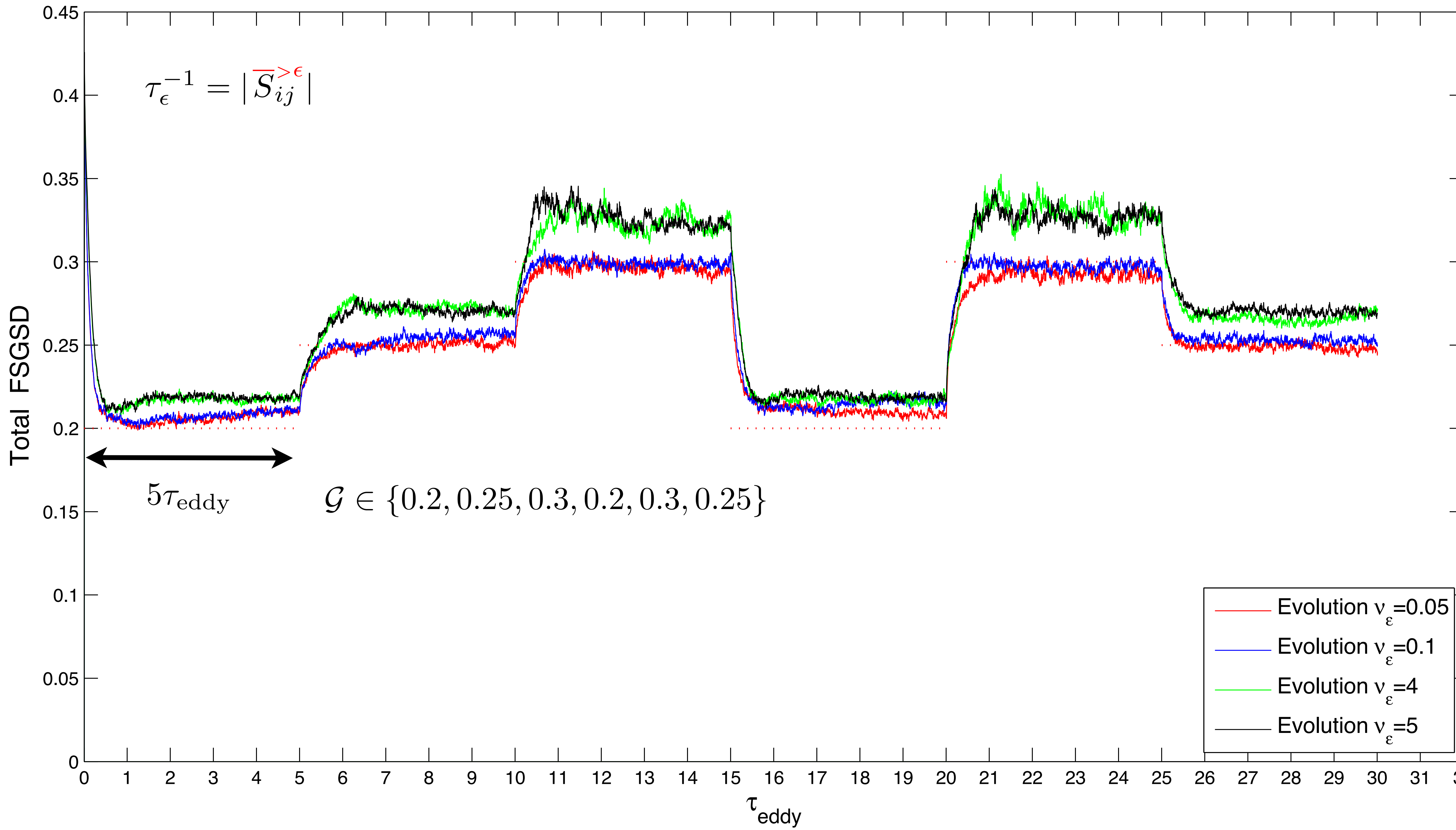
Perspective:
In Reality Goal can change in Space as well not just in Time

Hybrid CVS & SCALES (Hierarchical Multiscale Adaptive Variable Fidelity) – Time Varying Goal Benchmark

$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\rm res} \rangle + \langle \Pi \rangle}$

$\nu_\epsilon \in \{0.05, 0.1, 4, 5\}$

Solving Evolution Equation Directly



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Reynolds Scaling and its Dependence on “Desired” Captured Flow Physics

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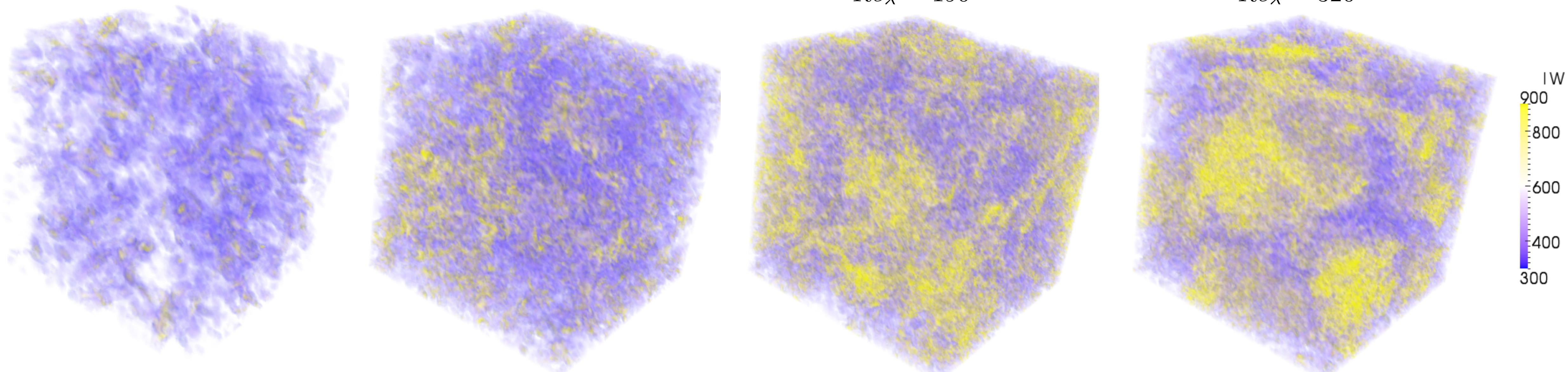
Time-Averaged Energy Spectra – CVS and SCALES

Linear Forcing Coefficient : $Q = 6.\bar{6}$

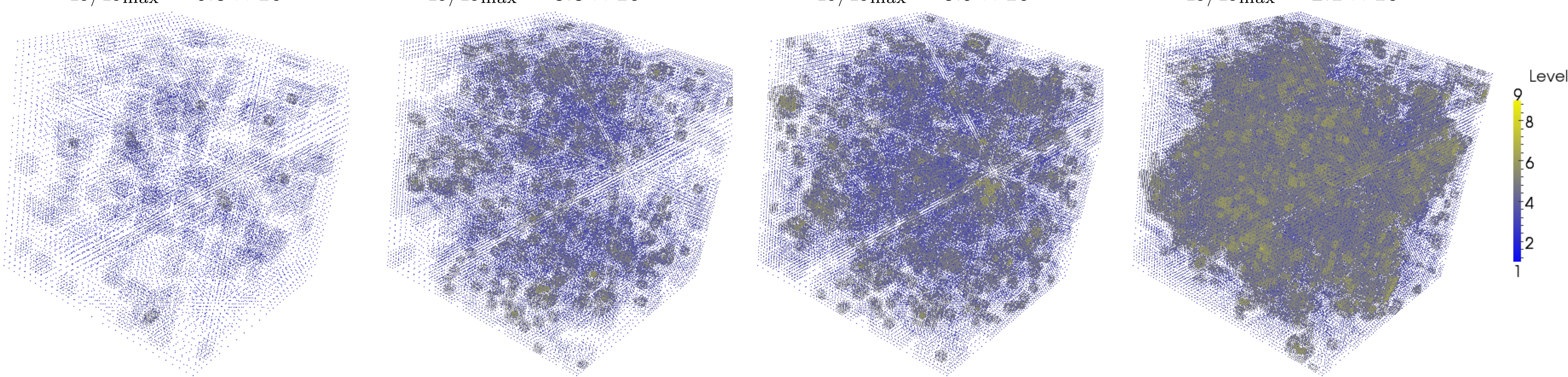
Taylor micro-scale Reynolds number : $Re_\lambda \cong 70, 120, 190, 320$ $\nu = 0.09, 0.035, 0.015, 0.006$

Adaptive Grid corresponds to $256^3, 512^3, 1024^3, 2048^3$ (at highest level of resolution) $J_{\max} = 6, 7, 8, 9$ $\epsilon = 0.2, 0.43$

$Re_\lambda \cong 70$ $Re_\lambda \cong 120$ $Re_\lambda \cong 190$ $Re_\lambda \cong 320$



$\mathcal{D}/\mathcal{D}_{\max} = 6.8 \times 10^{-3}$ $\mathcal{D}/\mathcal{D}_{\max} = 3.3 \times 10^{-3}$ $\mathcal{D}/\mathcal{D}_{\max} = 8.9 \times 10^{-4}$ $\mathcal{D}/\mathcal{D}_{\max} = 2.1 \times 10^{-4}$



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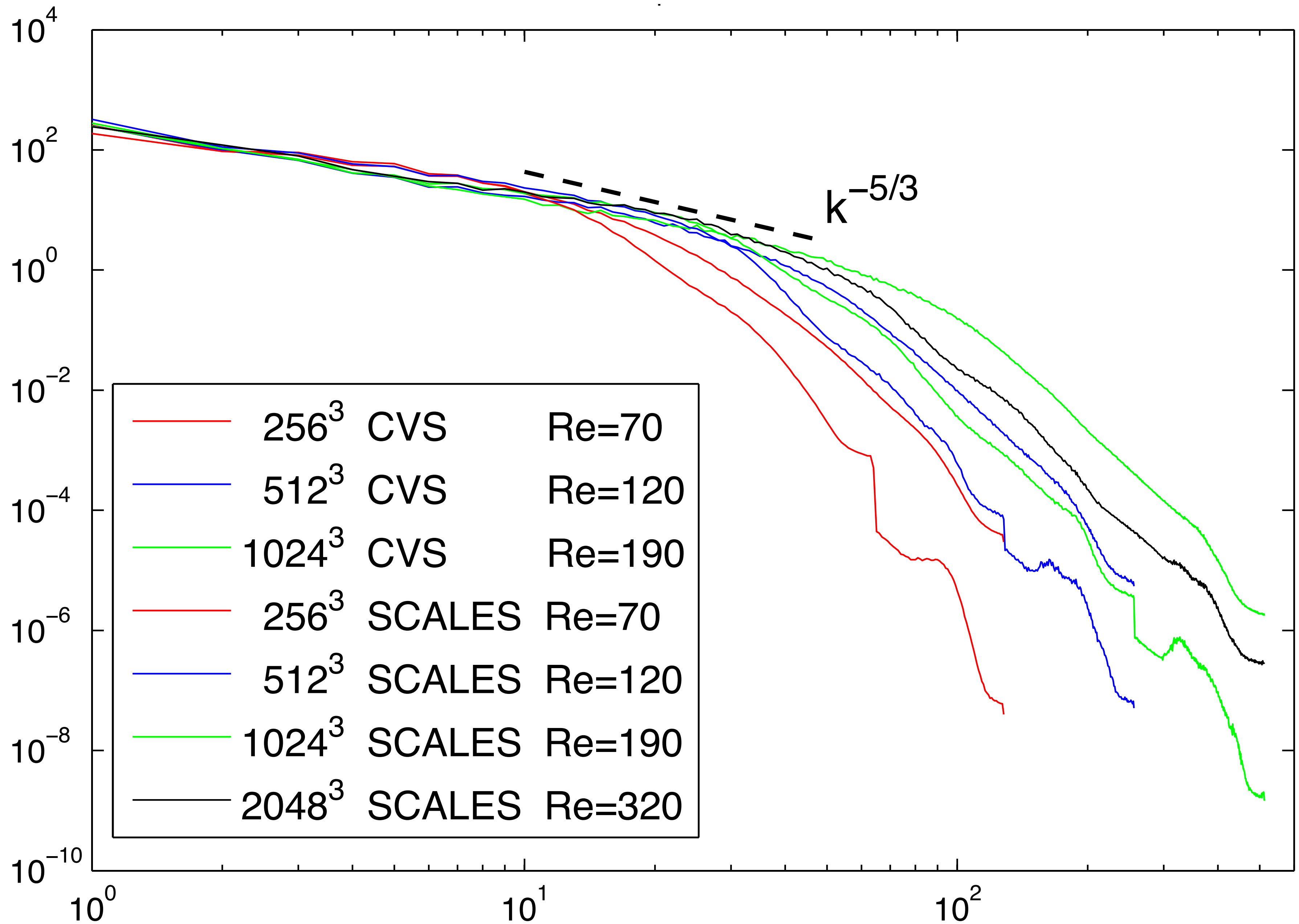

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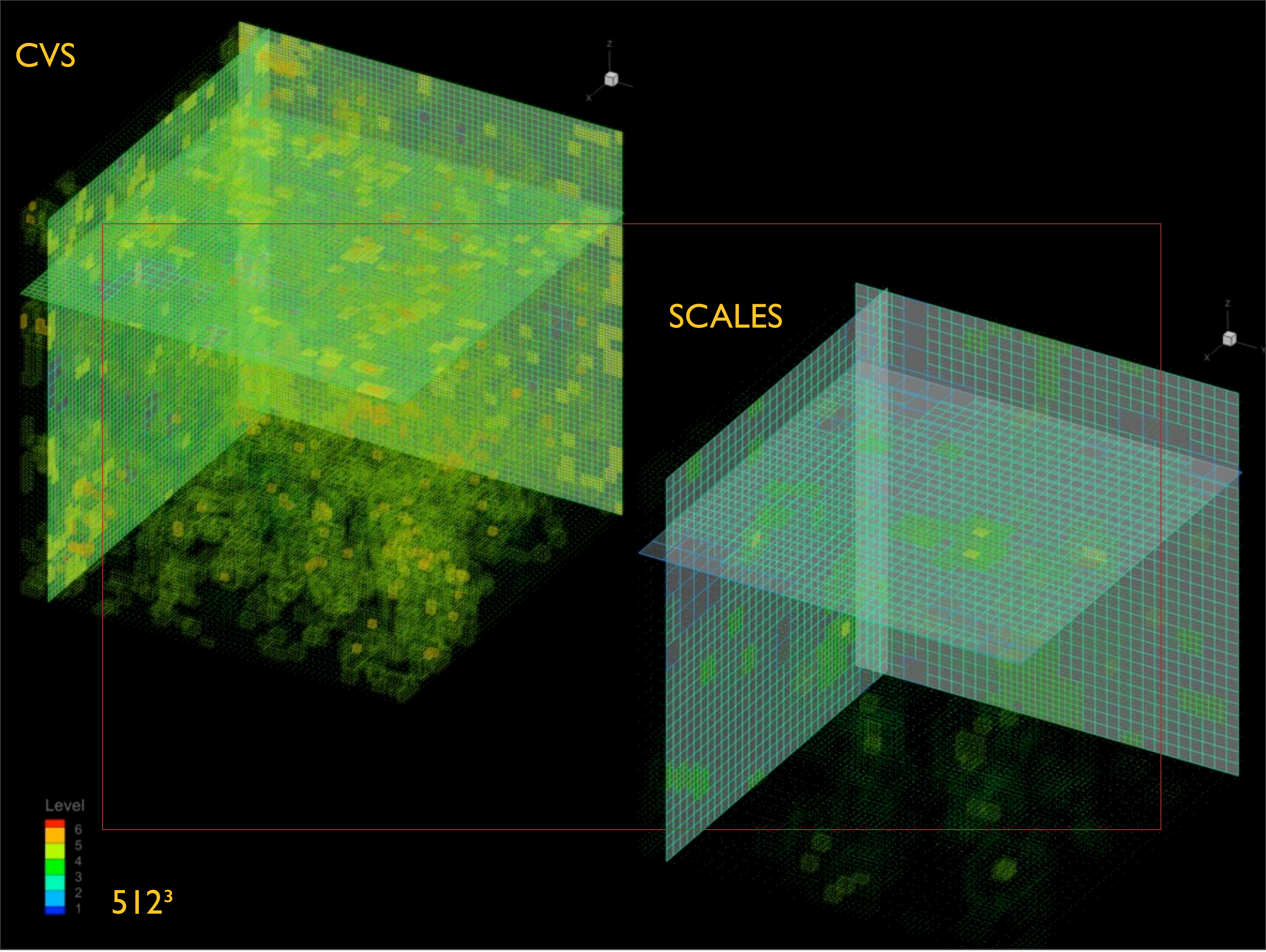
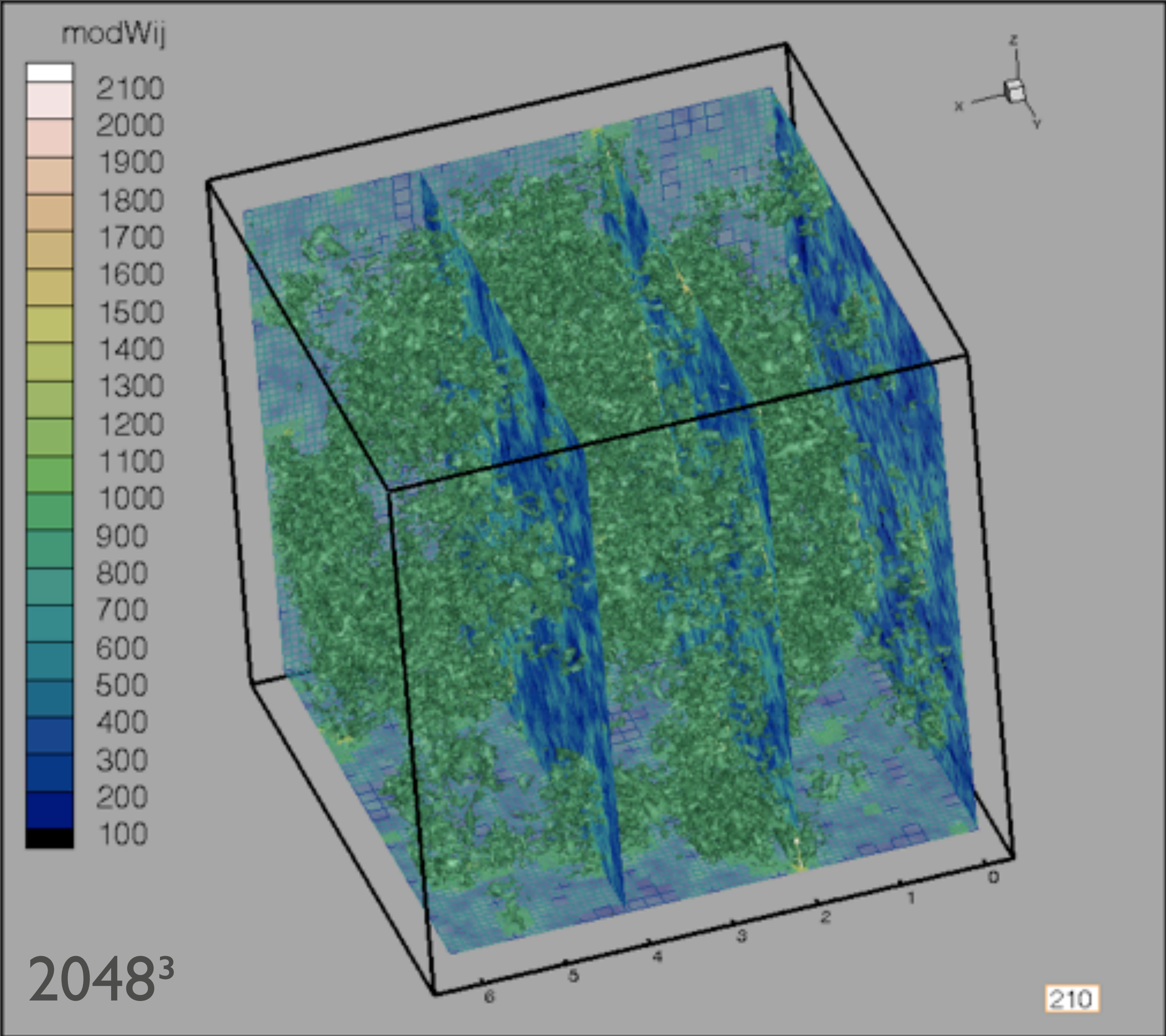
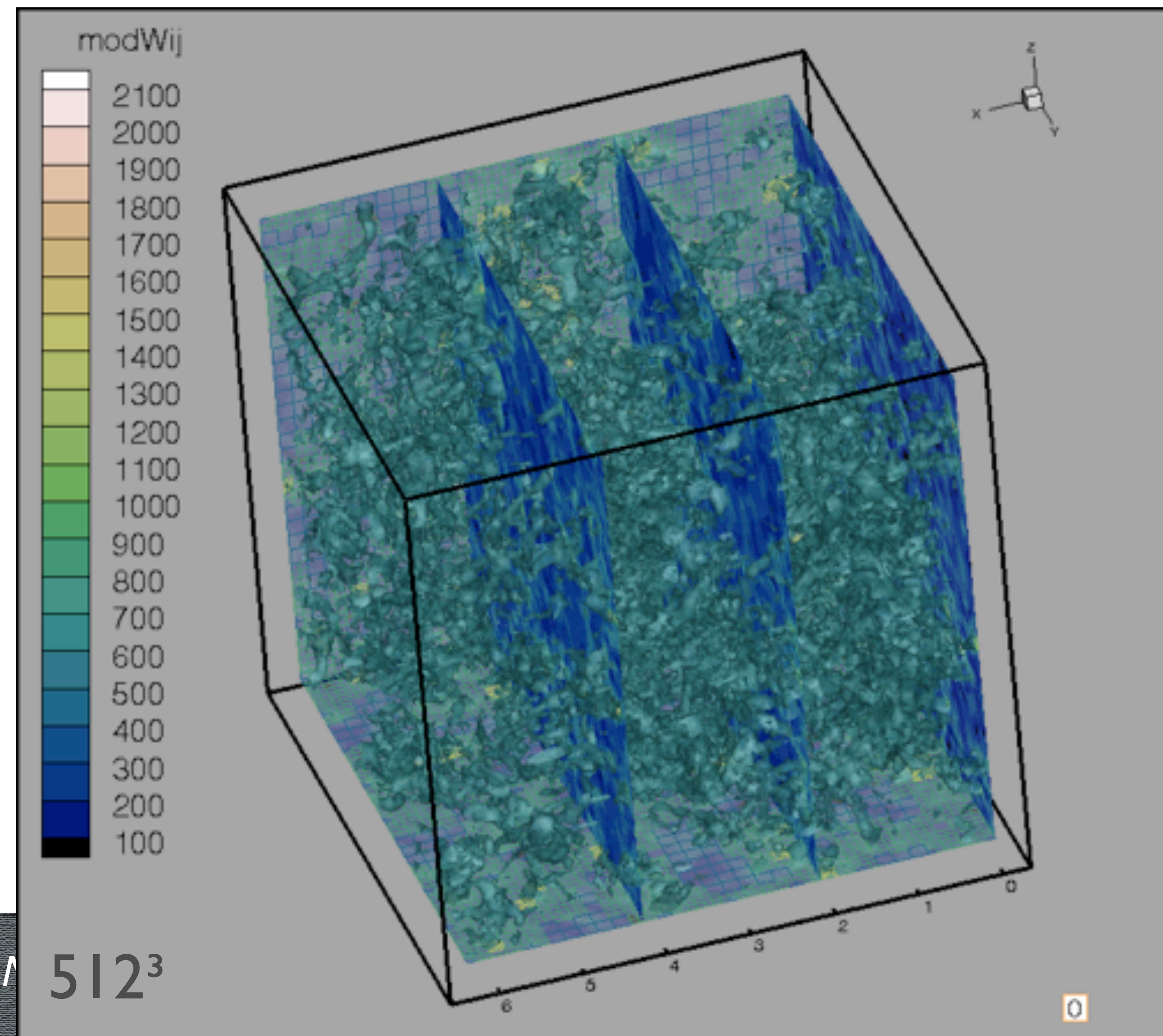
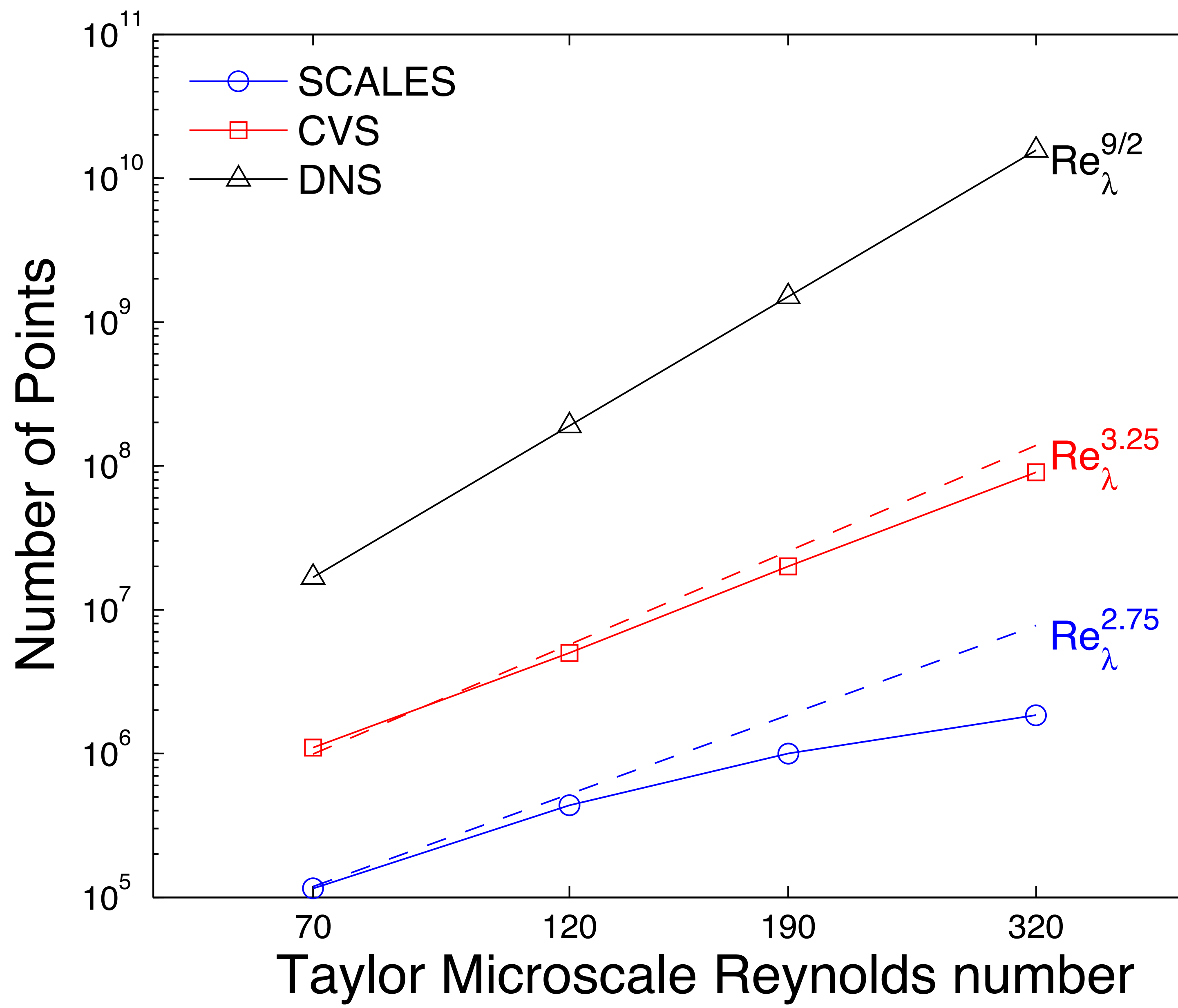

CVS
SCALES
Level
6
5
4
3
2
1
512^3





Wednesday, August 8, 12



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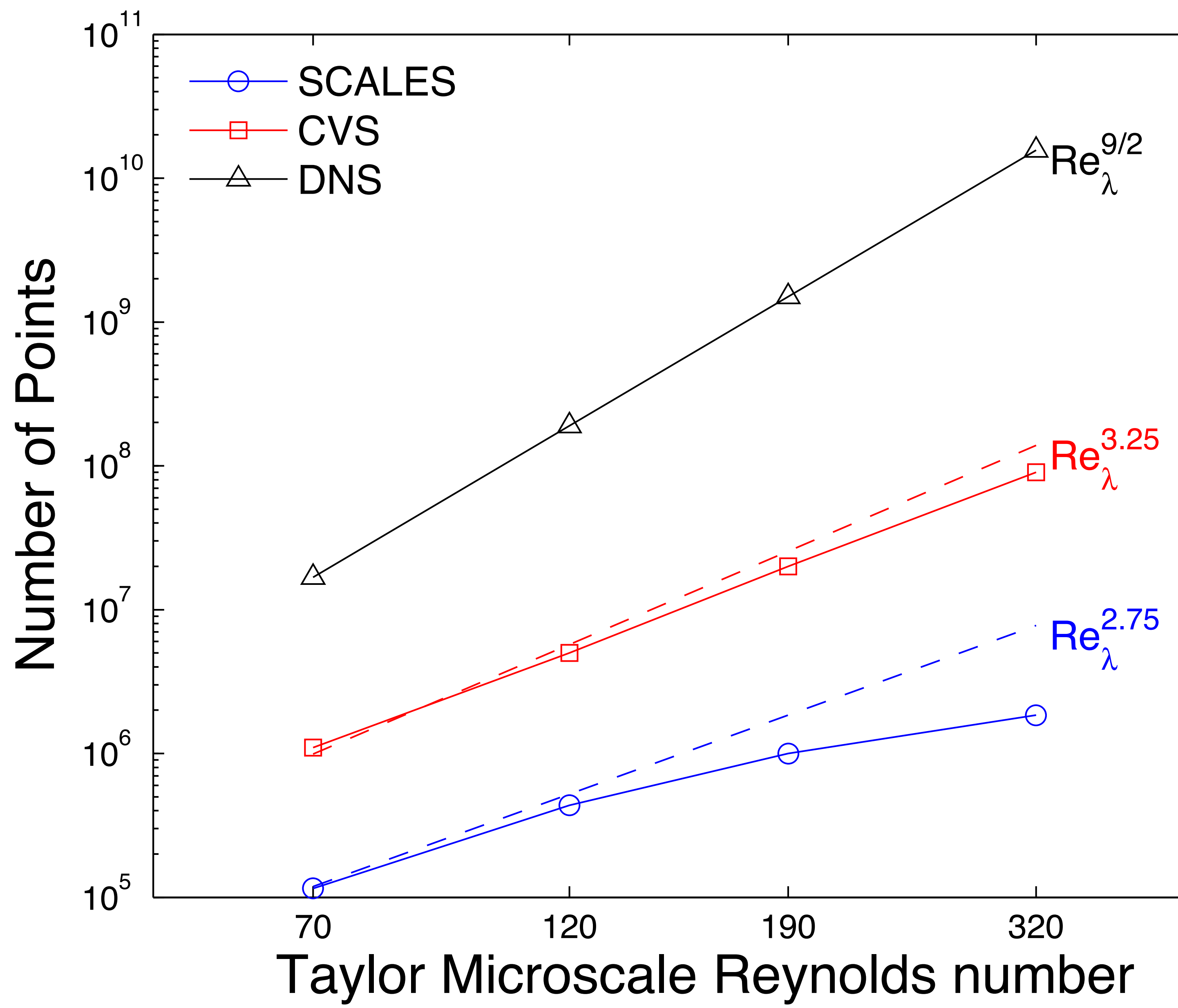
Computational Complexity –



$$4.5 - 3.25 = 1.25$$

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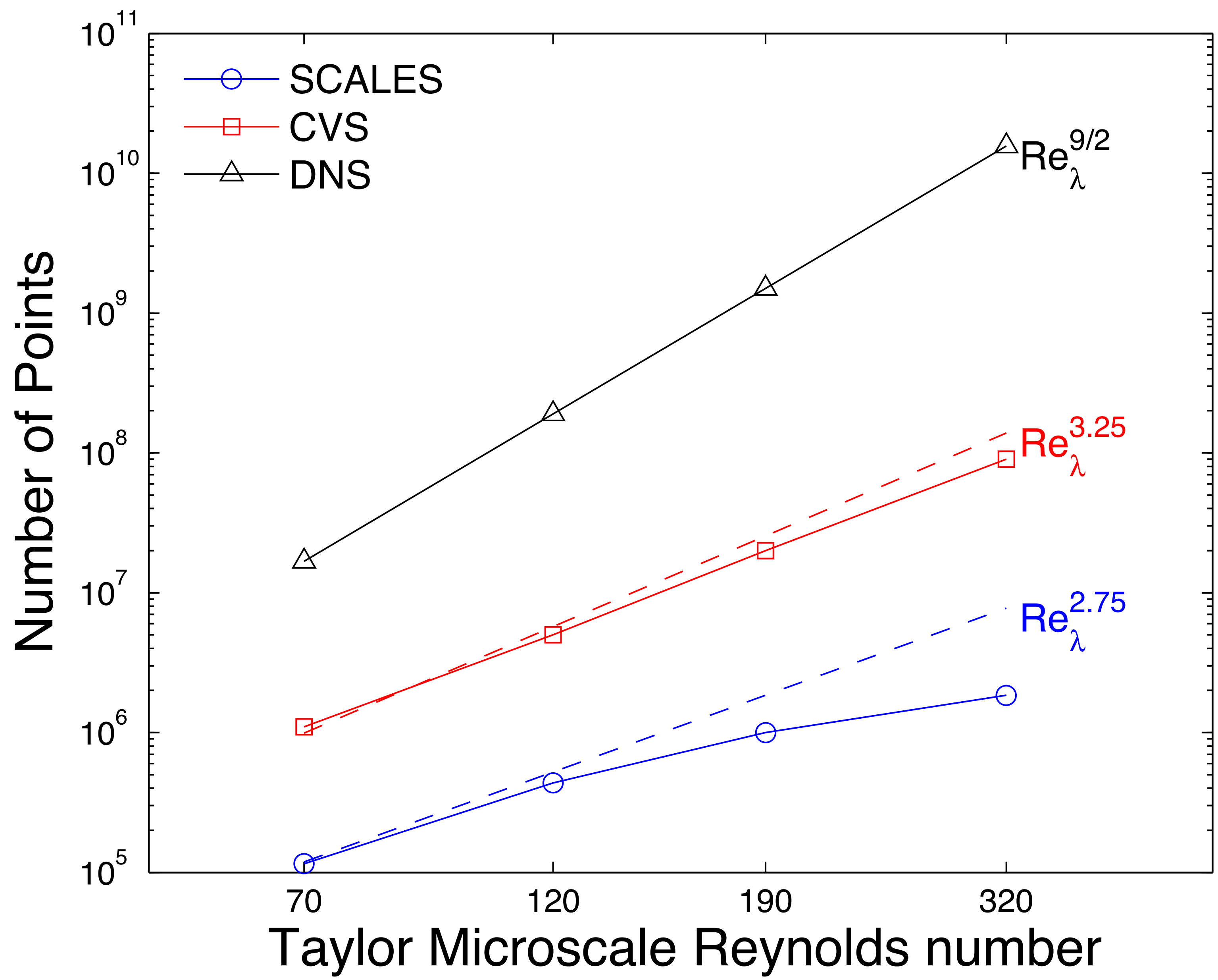

$$4.5 - 3.25 = 1.25$$

$$4.5 - 2.75 = 1.75$$

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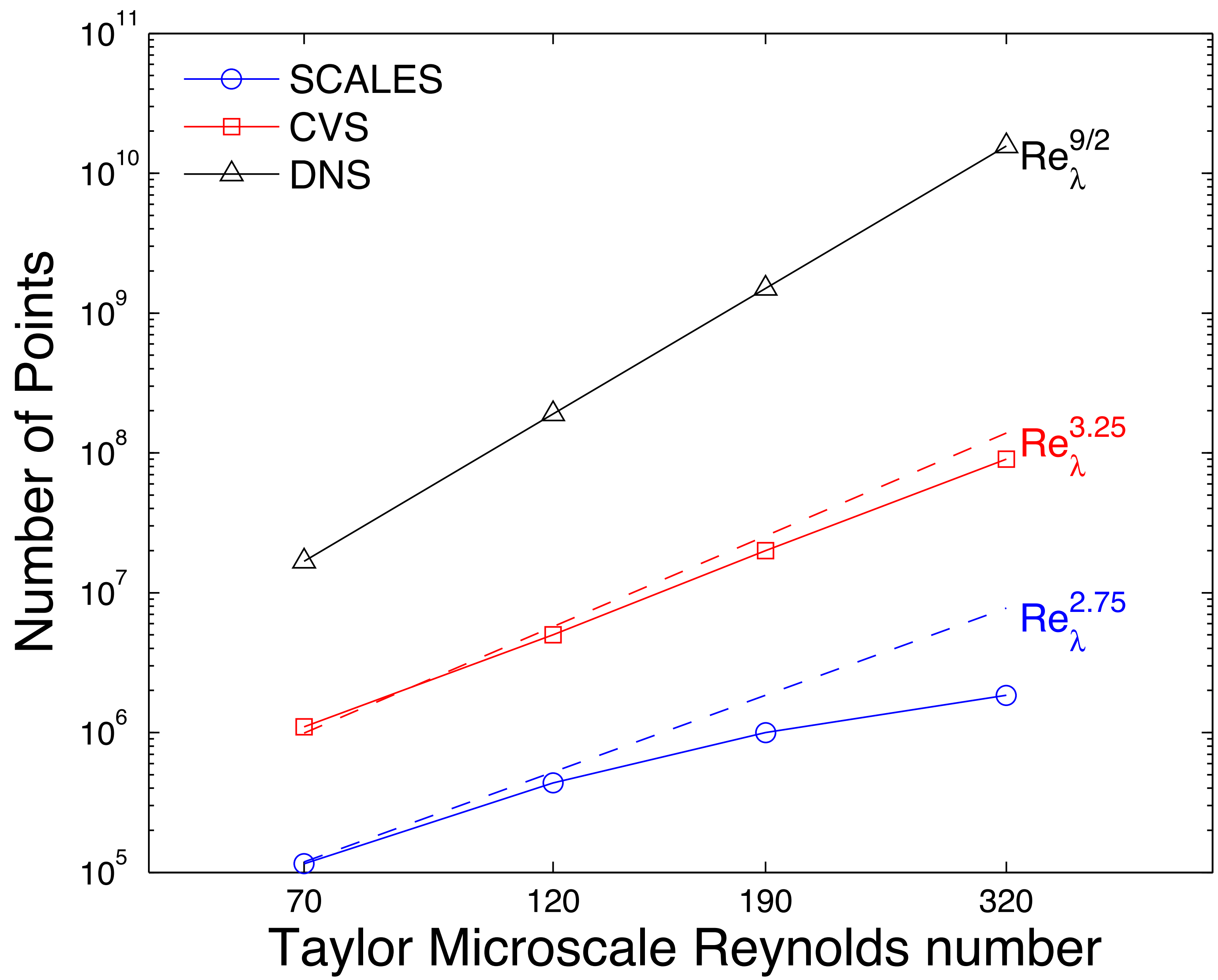
Computational Complexity – Fractal Dimension

Spatial DOF† : $Re^{3D_F/(D_F+1)}$

$D_F \leq 3$



$D_{F_{\mathrm{CVS}}} \lesssim \frac{13}{11} = 1.\overline{18}$

$D_{F_{\mathrm{SCALES}}} \lesssim \frac{11}{13} = 0.\overline{846153}$

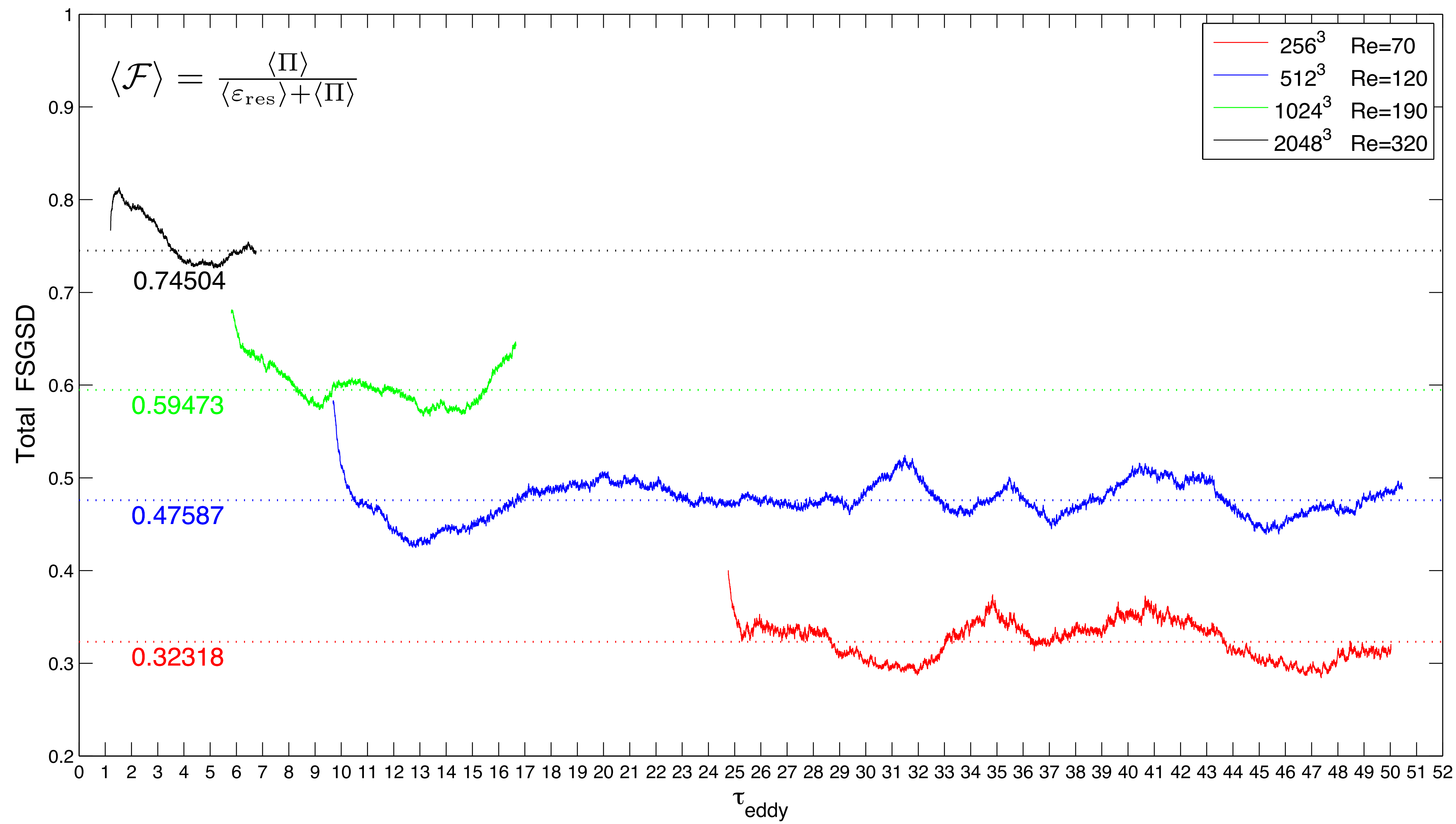
$D_{F_{\mathrm{SCALES}}} < 1$

†*Paladin G, Vulpiani A, 1987. Phys. Rev. A 35:1971–1973*

Linear Forcing Coefficient : $Q = 6.\bar{6}$

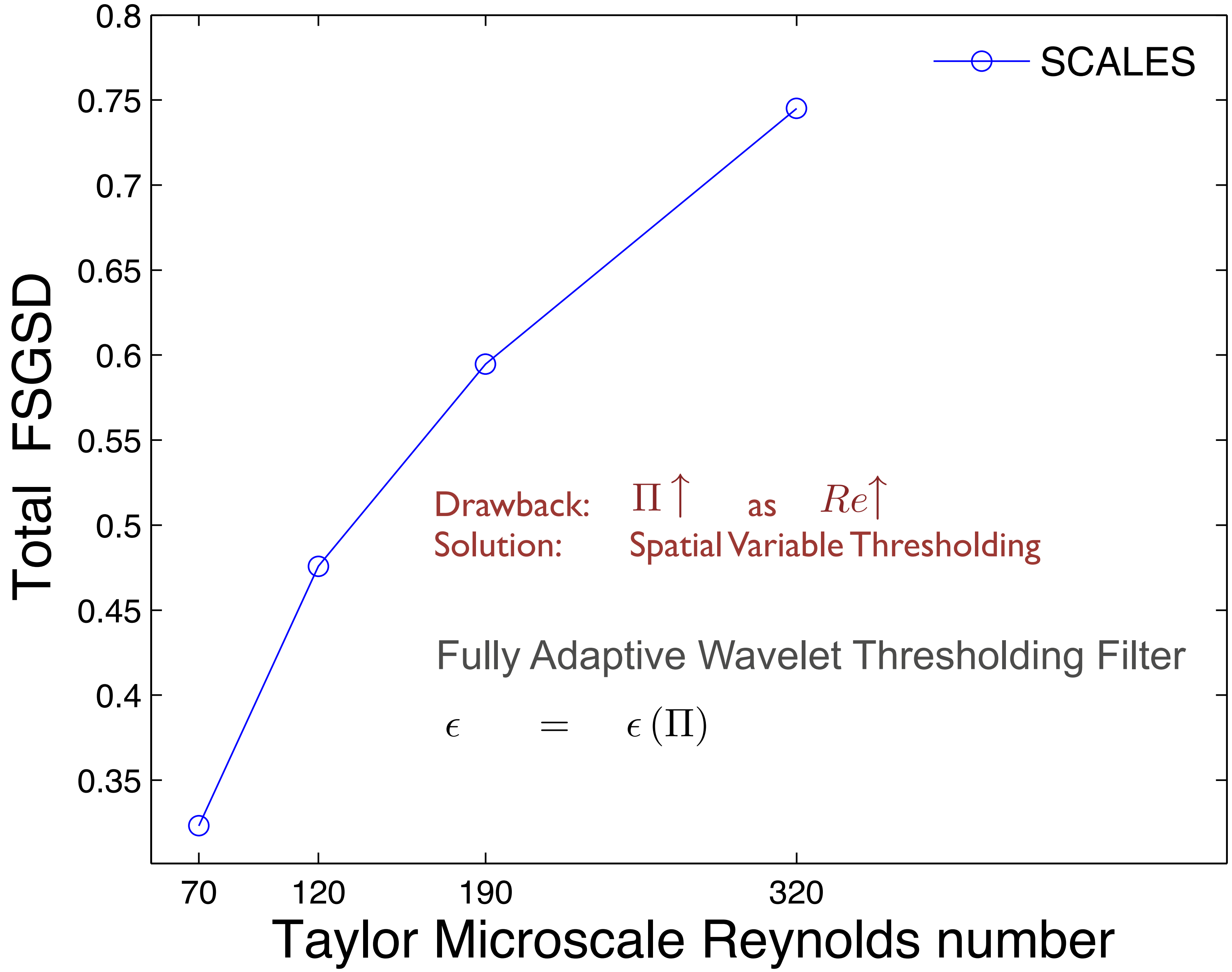
Taylor micro-scale Reynolds number : $Re_\lambda \cong 70, 120, 190, 320$ $\nu = 0.09, 0.035, 0.015, 0.006$

Adaptive Grid corresponds to $256^3, 512^3, 1024^3, 2048^3$ (at highest level of resolution) $J_{\max} = 6, 7, 8, 9$ $\epsilon = 0.43$



Complexity – How %Fraction-SGSD Scales as Reynolds?

$$\overline{\langle \mathcal{F} \rangle} = \overline{\left(\frac{\langle \Pi \rangle}{\langle \varepsilon_{\rm res} \rangle + \langle \Pi \rangle} \right)}$$



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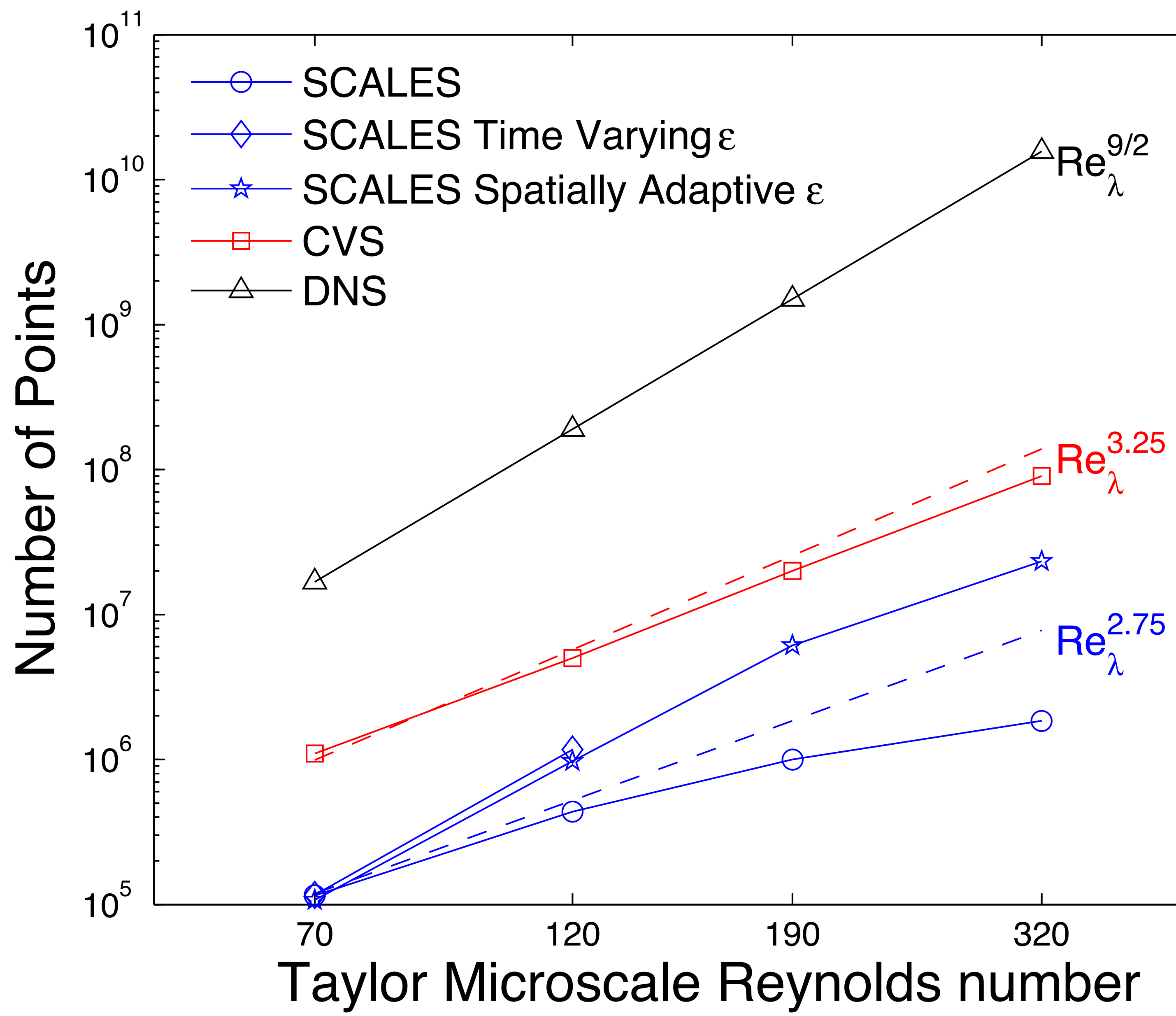

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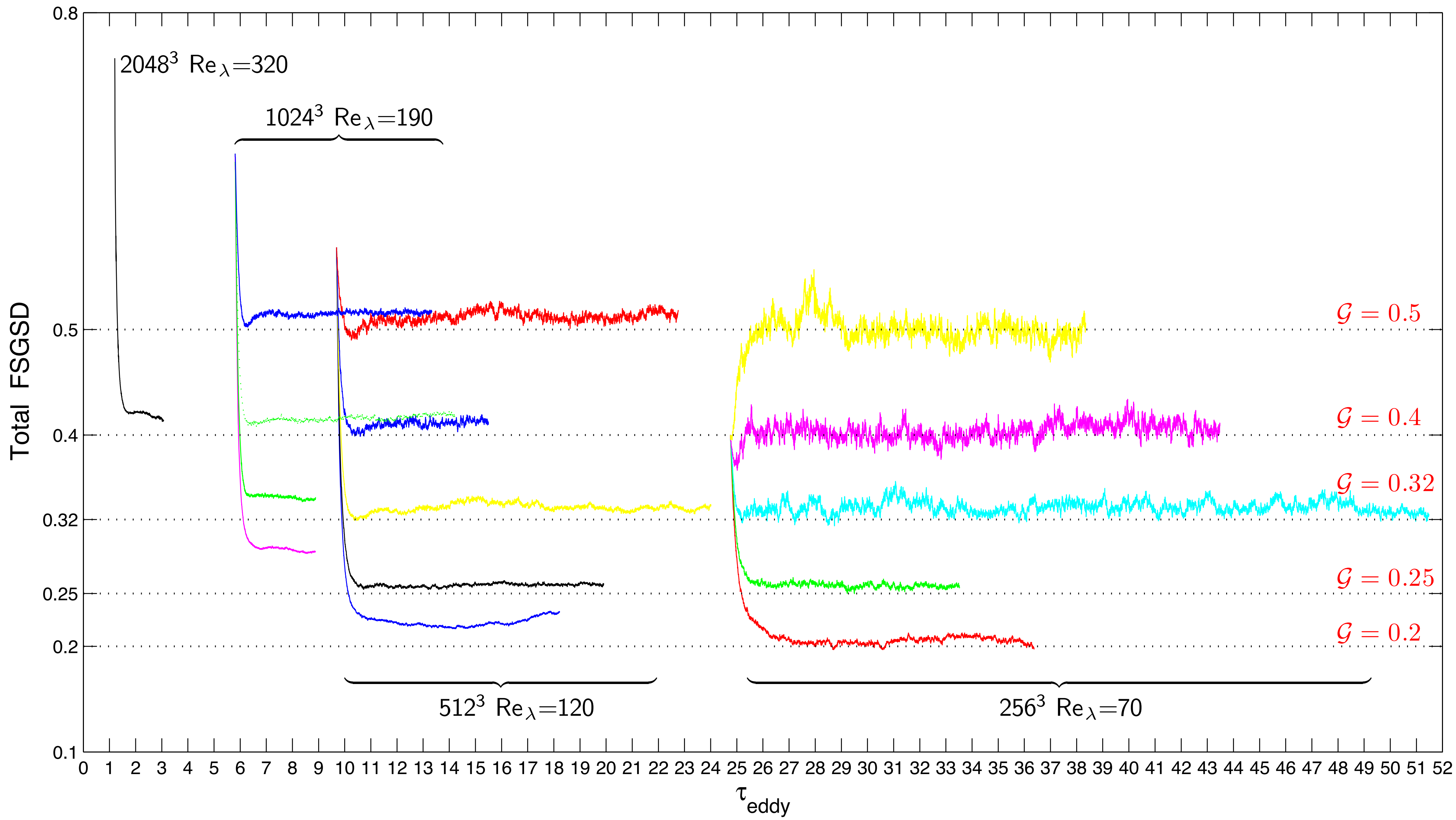
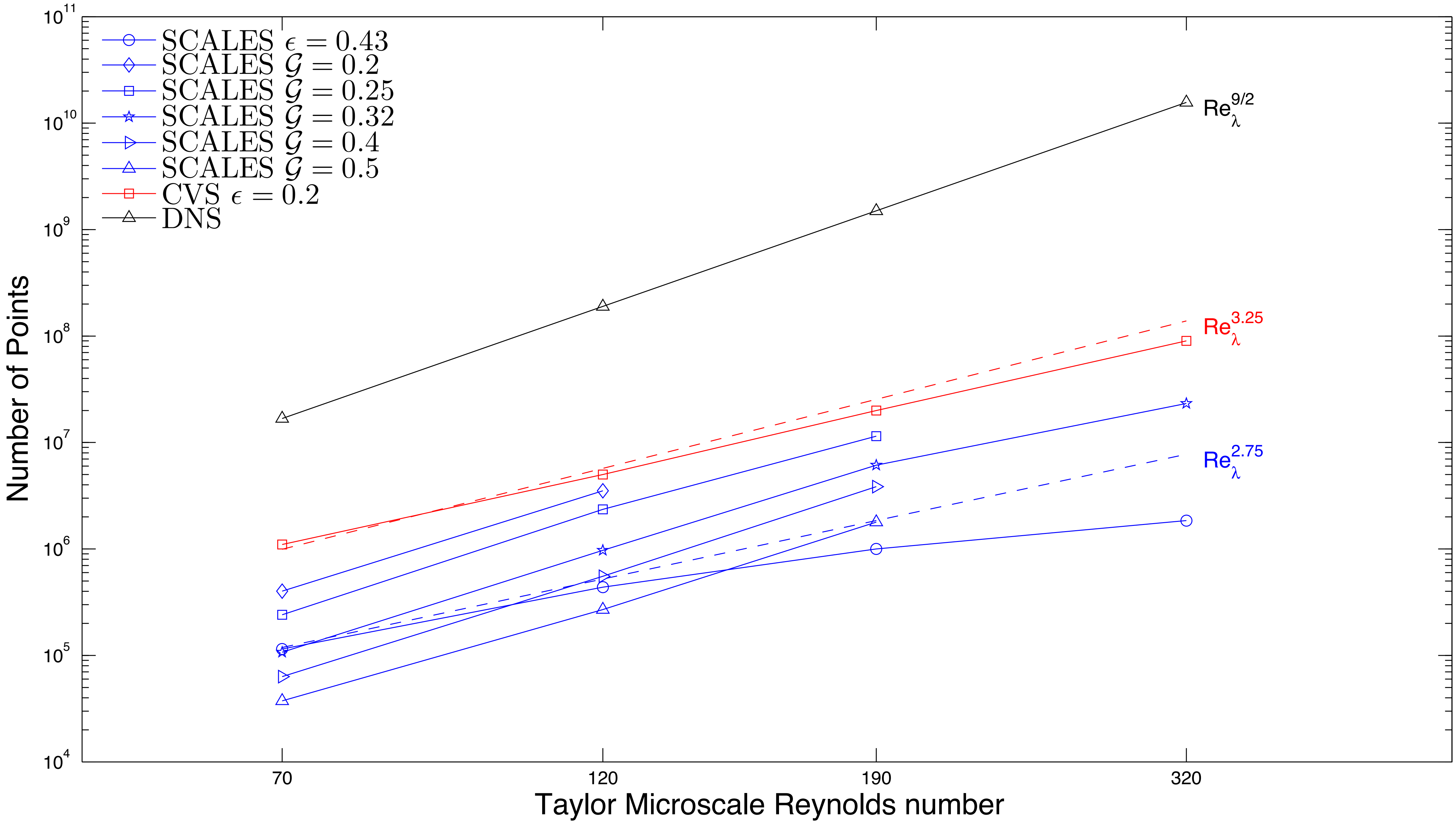
$$\langle \mathcal{F} \rangle = \frac{\langle \Pi \rangle}{\langle \varepsilon_{\mathrm{res}} \rangle + \langle \Pi \rangle}$$





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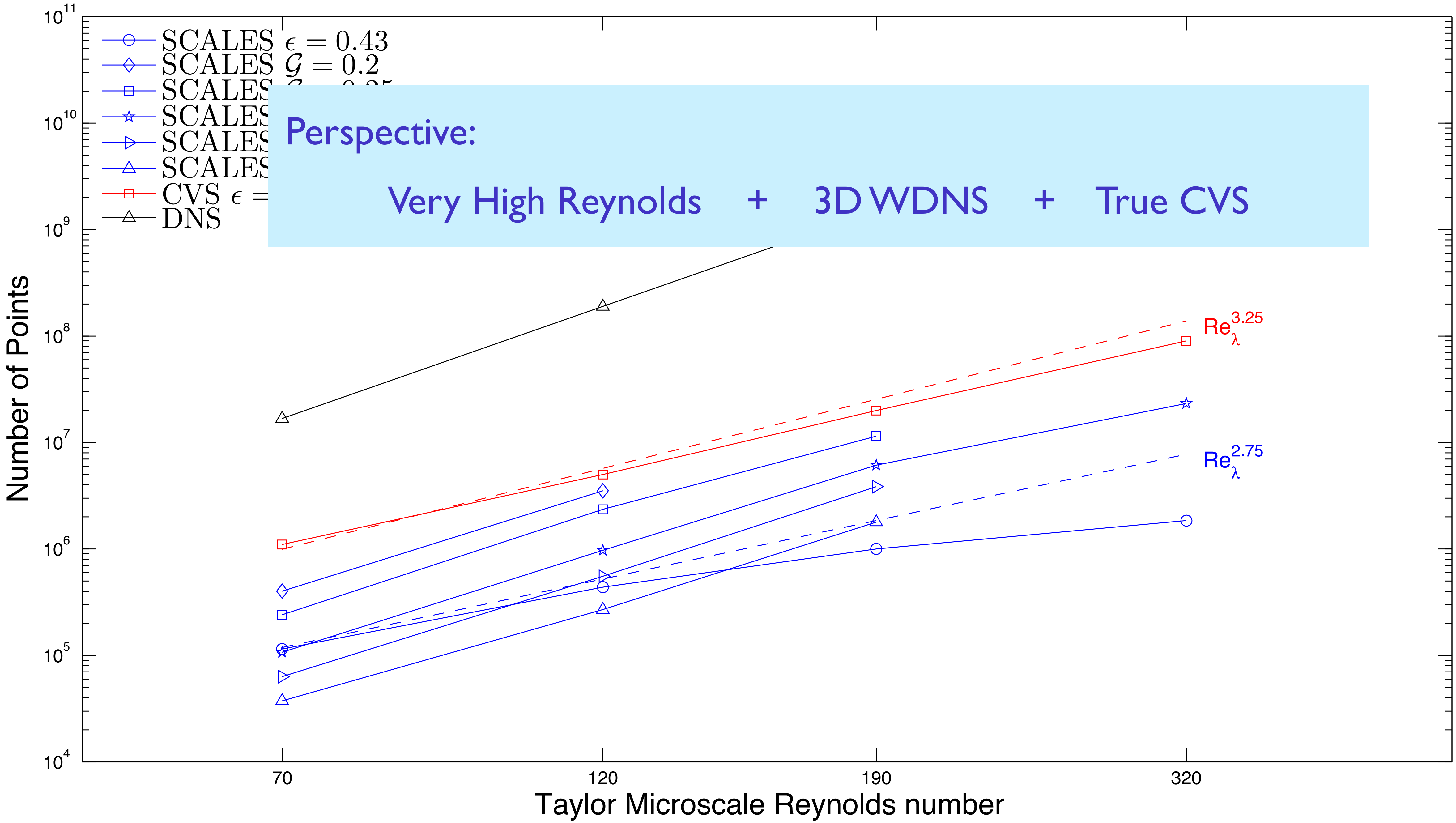
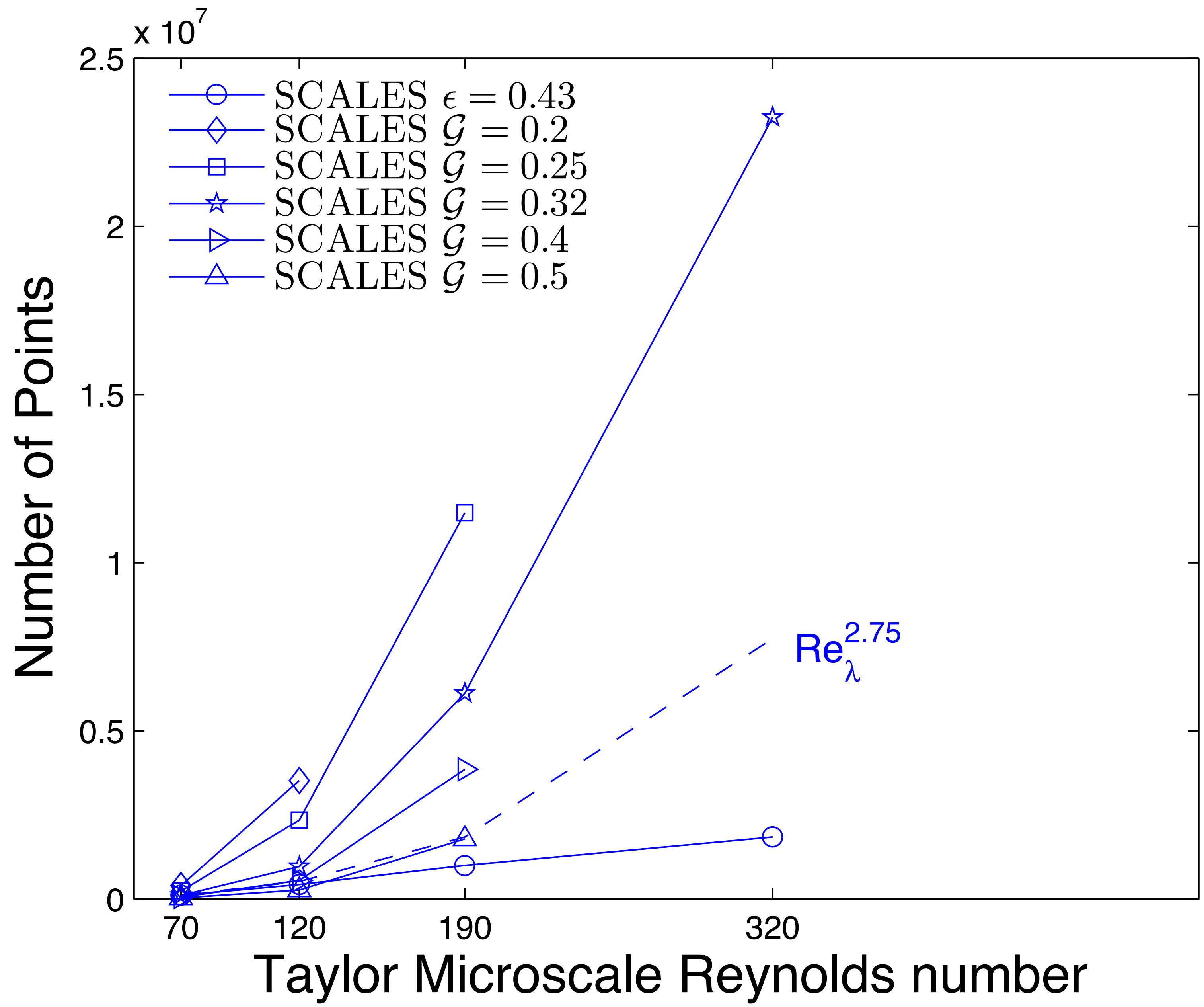
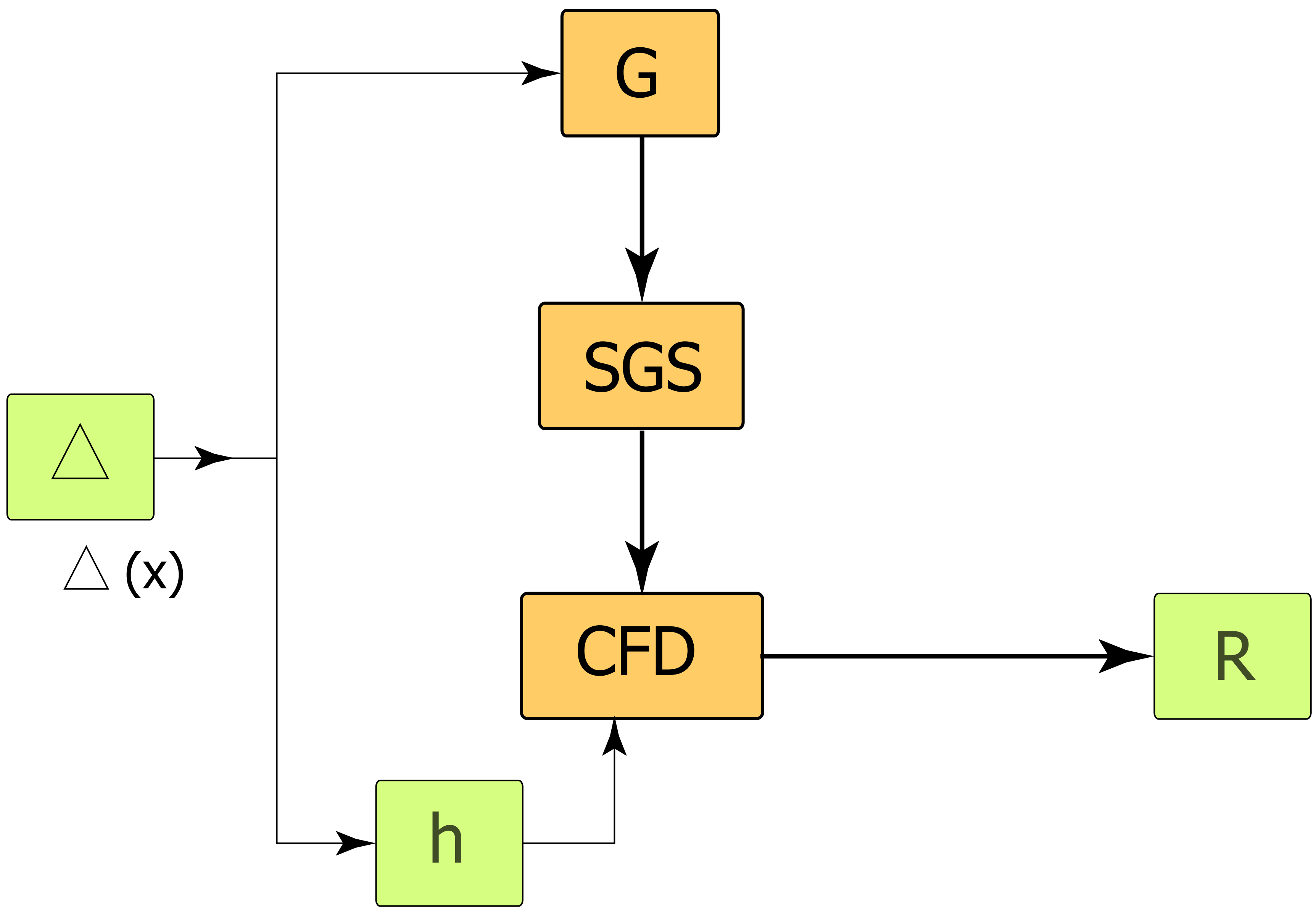


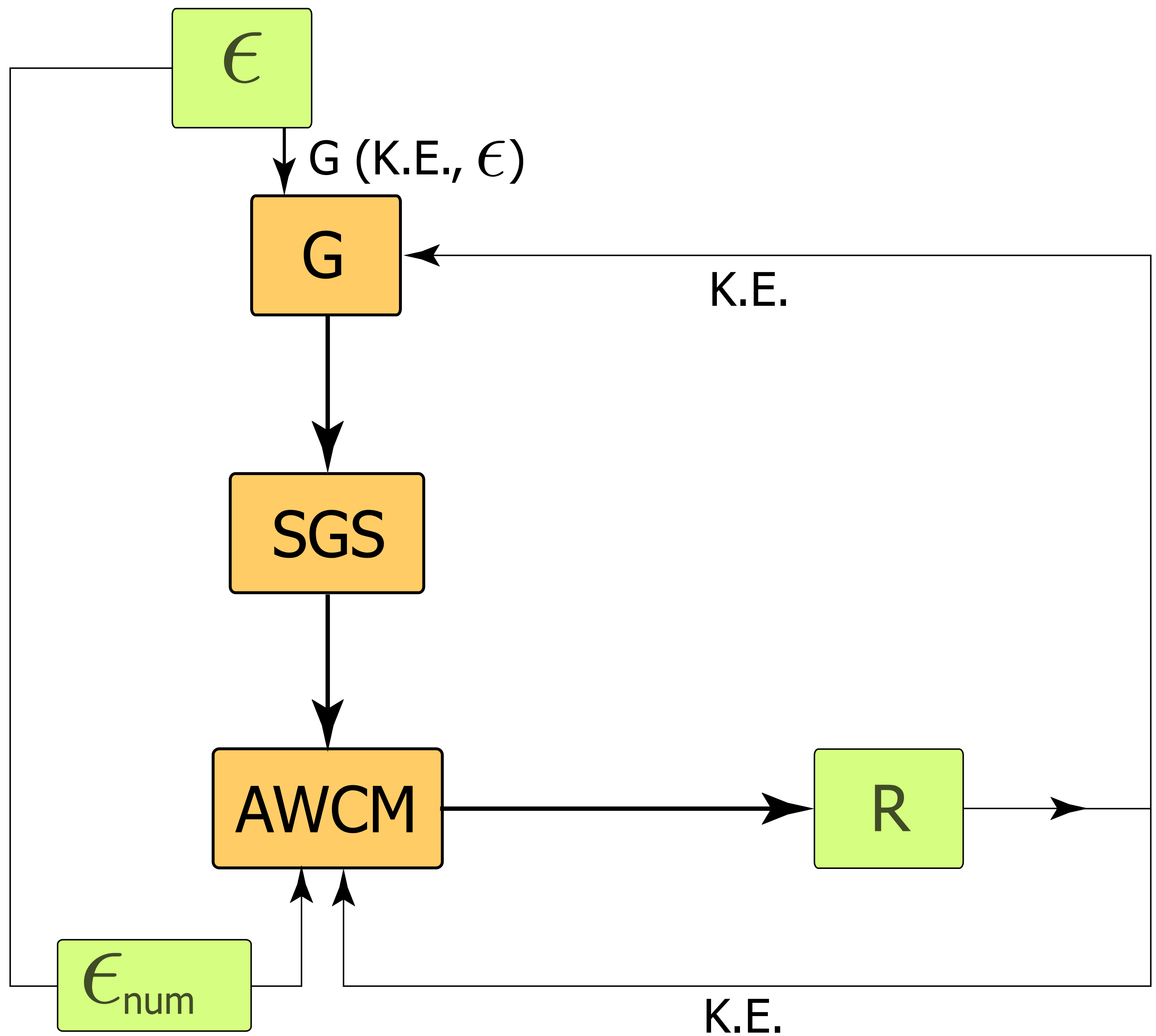
Perspective:

Very High Reynolds + 3D WDNS + True CVS

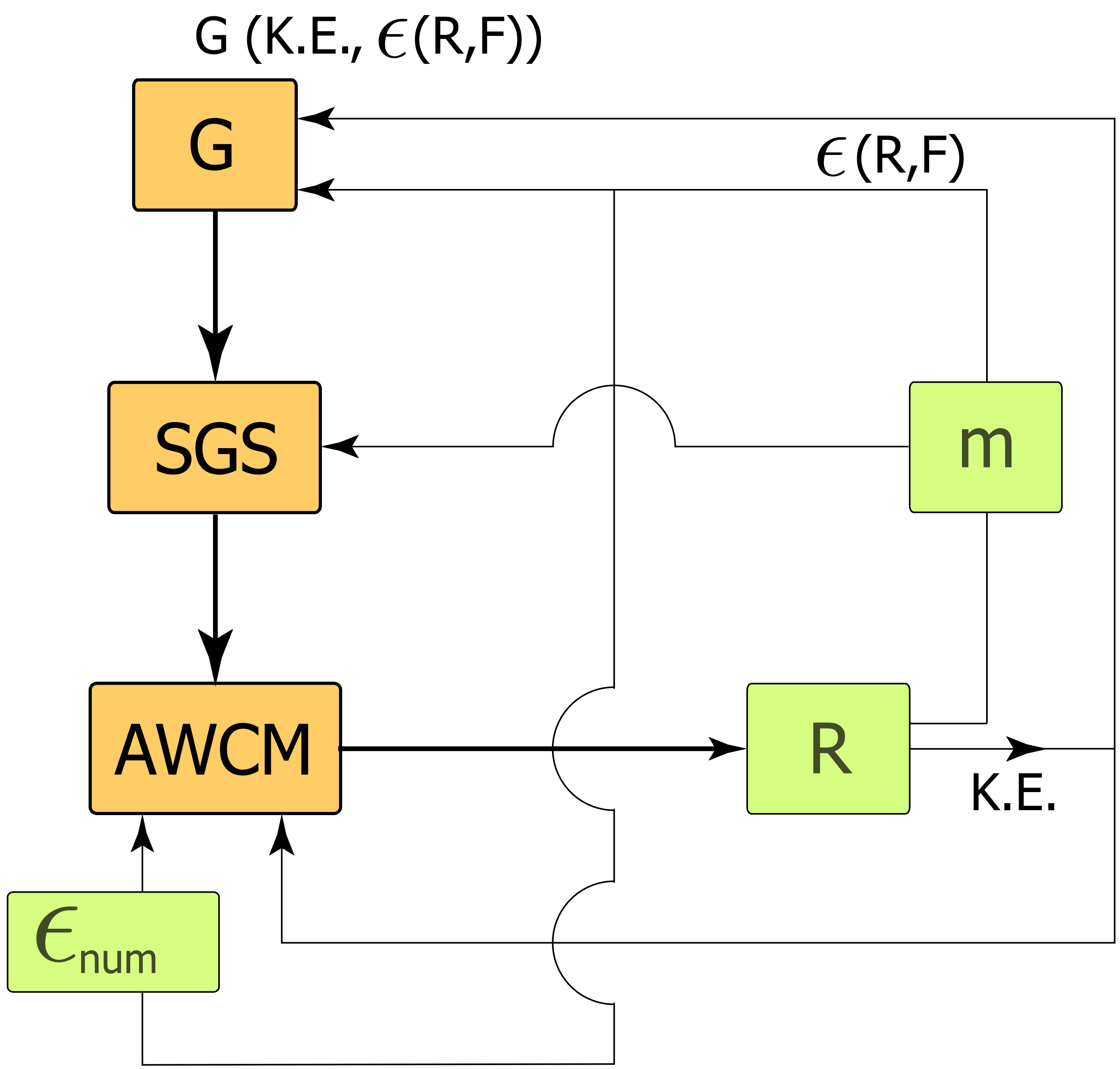

x 10^7
2.5
2
1.5
1
0.5
0
Number of Points
SCALES $\epsilon = 0.43$
SCALES $\mathcal{G} = 0.2$
SCALES $\mathcal{G} = 0.25$
SCALES $\mathcal{G} = 0.32$
SCALES $\mathcal{G} = 0.4$
SCALES $\mathcal{G} = 0.5$
$Re_\lambda^{2.75}$
70
120
190
320
Taylor Microscale Reynolds number

How to Incorporate Dynamic Coupling into existing LES


G
SGS
△
△ (x)
CFD
R
h

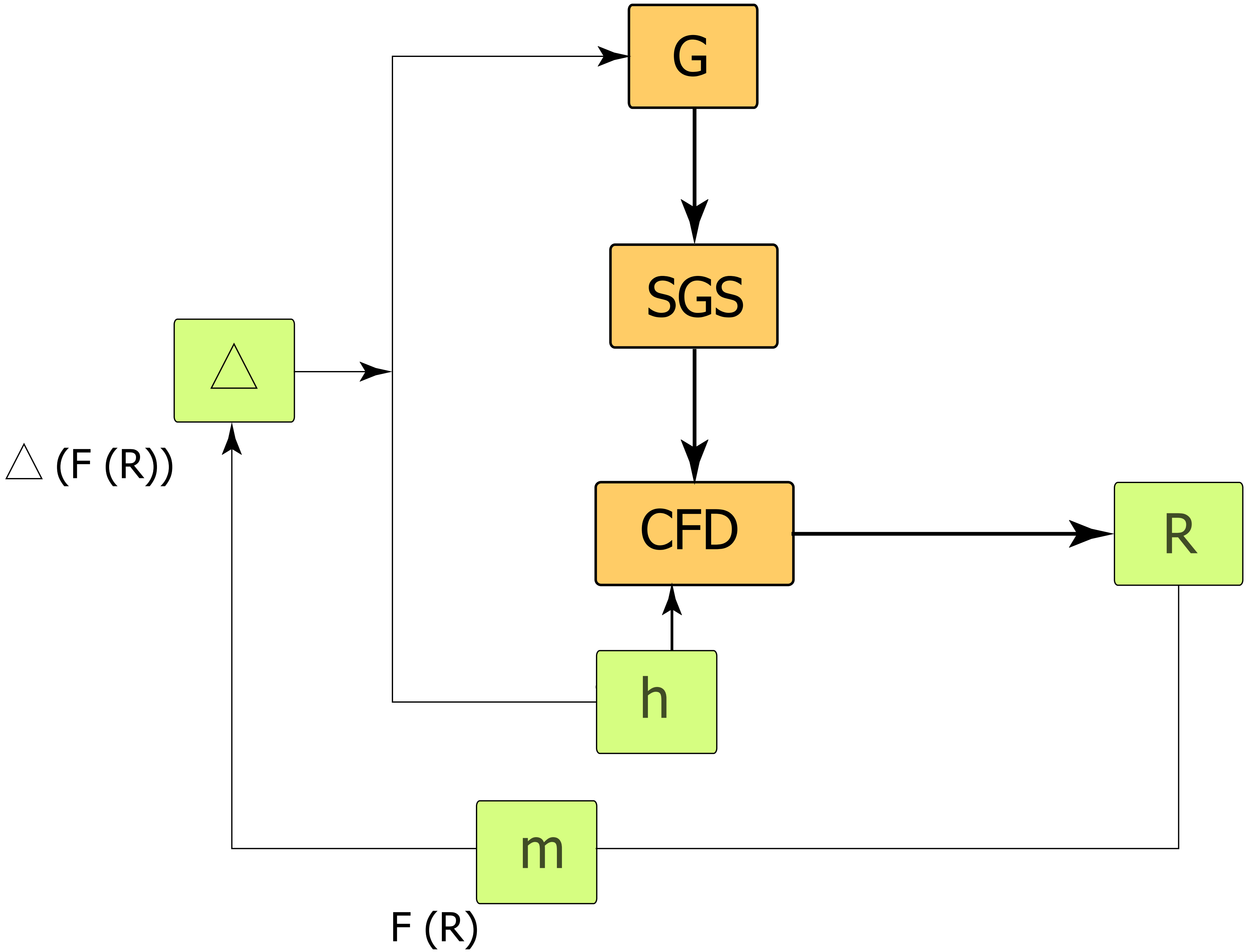


ϵ
G (K.E., ϵ)
G
K.E.
SGS
AWCM
R
ϵ_{num}
K.E.



G (K.E., ϵ(R,F))
G
ϵ(R,F)
SGS
m
AWCM
R
K.E.
ϵnum

model refinement is not limited to SCALES – m-LES



Conclusions

Demonstrated:

- the active control of the fidelity/accuracy of the simulation
- near optimal spatially adaptive computational mesh for the user-defined fidelity
- the “desired” flow-physics is captured by considerably smaller number of spatial modes
- considerably smaller Reynolds scaling exponent, that depends on the captured flow physics (KE or SGS dissipation)
- robust general mathematical framework for spatial/temporal *model*-refinement (*m*-refinement) that can be extended to AMR approach

Perspectives

The proposed philosophy/paradigm of *dynamic coupling* of AMR and turbulence modeling is the **FUTURE**!