Transition and turbulence in MHD at very strong magnetic fields

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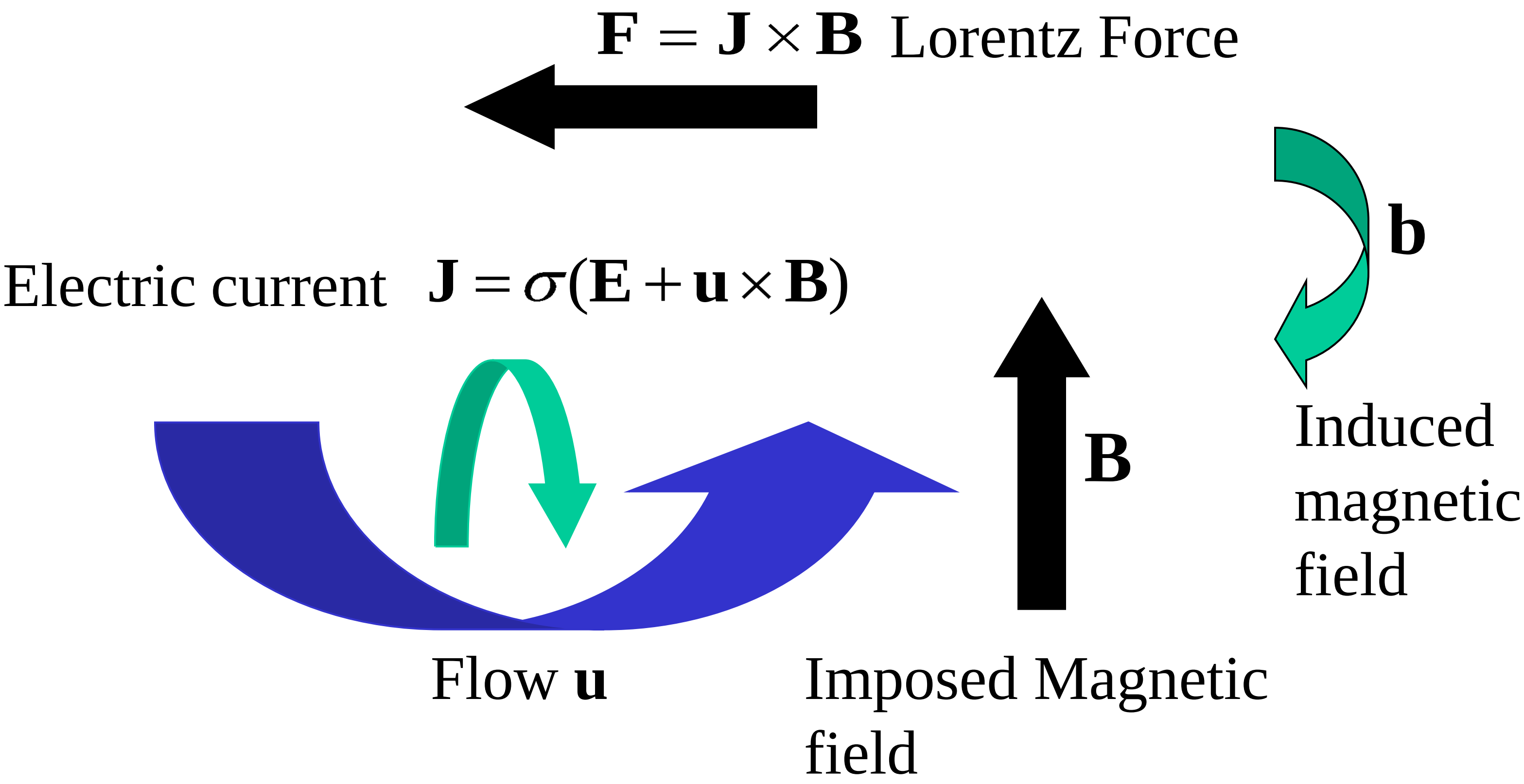

$\mathbf{F} = \mathbf{J} \times \mathbf{B}$ Lorentz Force
b
Electric current $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B})$
B
Induced magnetic field
Flow **u**
Imposed Magnetic field

MHD Approximation

Assumption:

Instantaneous propagation of electromagnetic radiation, $L/\tau <<$c. τ, L – typical time and length scales

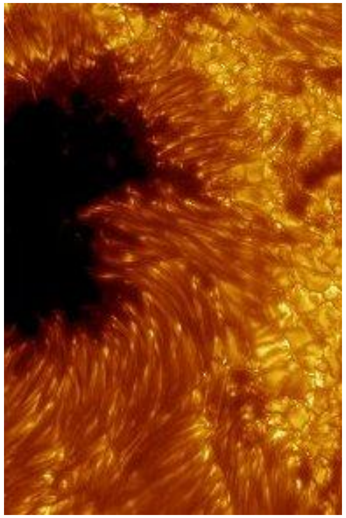
Neglect:

Displacement currents, $\vartheta\mathbf{u}$ in Ohm's law, $\vartheta\mathbf{E}$ in electromagnetic force

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\sigma\mu_0}\nabla^2 \mathbf{B} \qquad \nabla \cdot \mathbf{j} = 0$$

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \qquad \mathbf{F} = \mathbf{j} \times \mathbf{B}$$

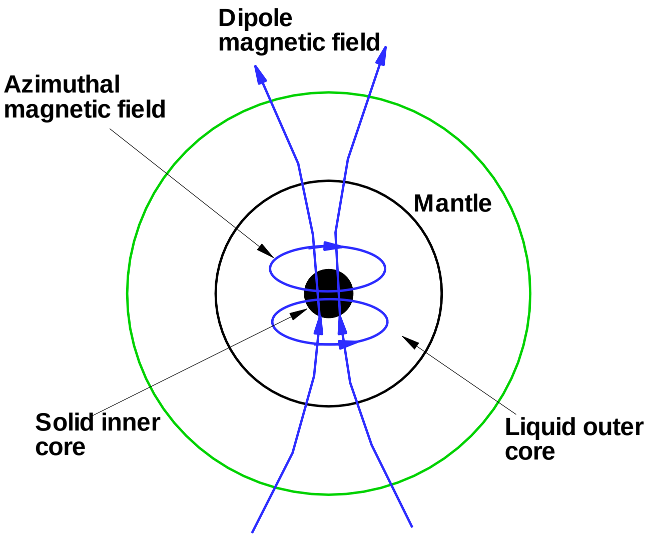
MHD flows



Sunspots

Nature 2002

Planetary dynamo



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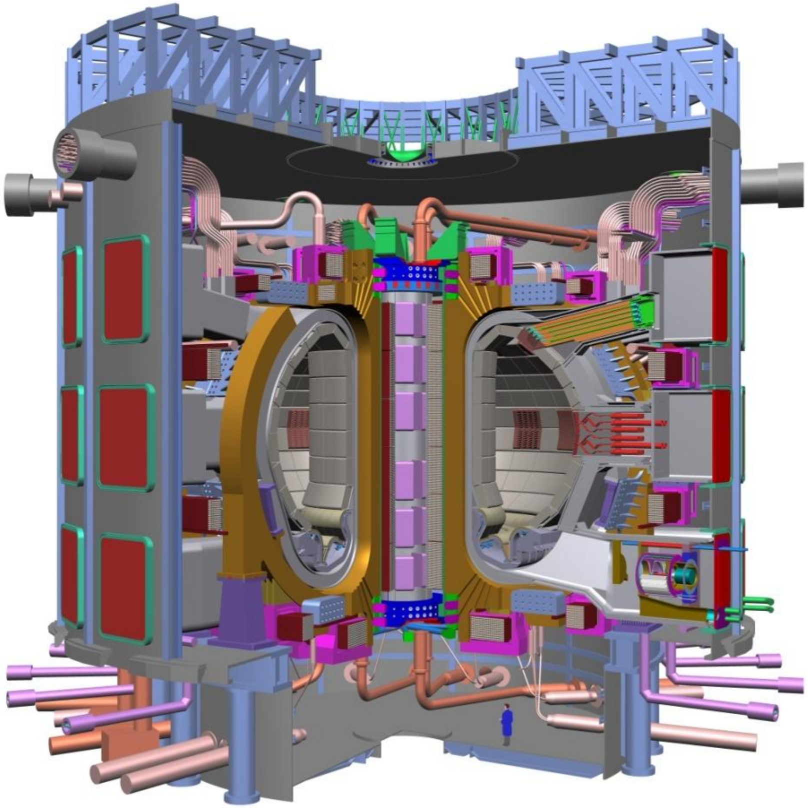
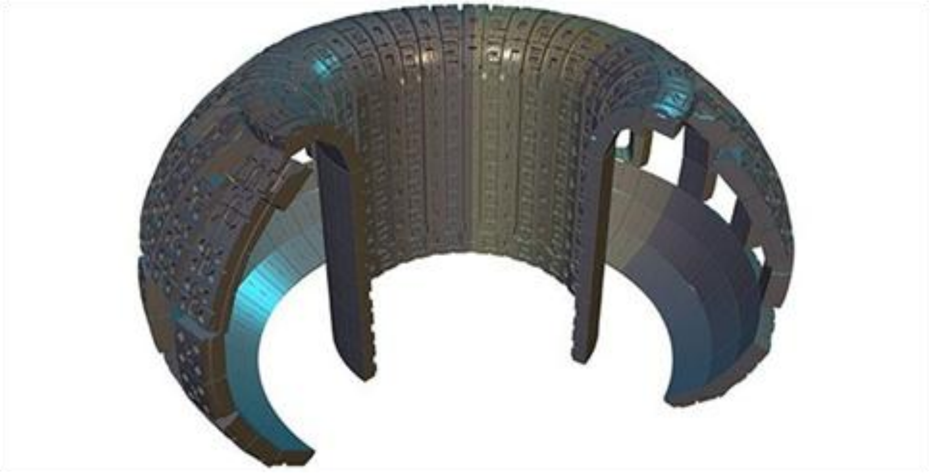
Magnetic confinement fusion

Metallurgical applications

- Control of nozzle jet in continuous steel casting
- Crystal growth
- Primary aluminum production in Hall-Héroult cells
- Induction heating and stirring
- Vacuum arc remelting
- Magnetic valves
- Electromagnetic pumps
- Electromagnetic flow meters
- …

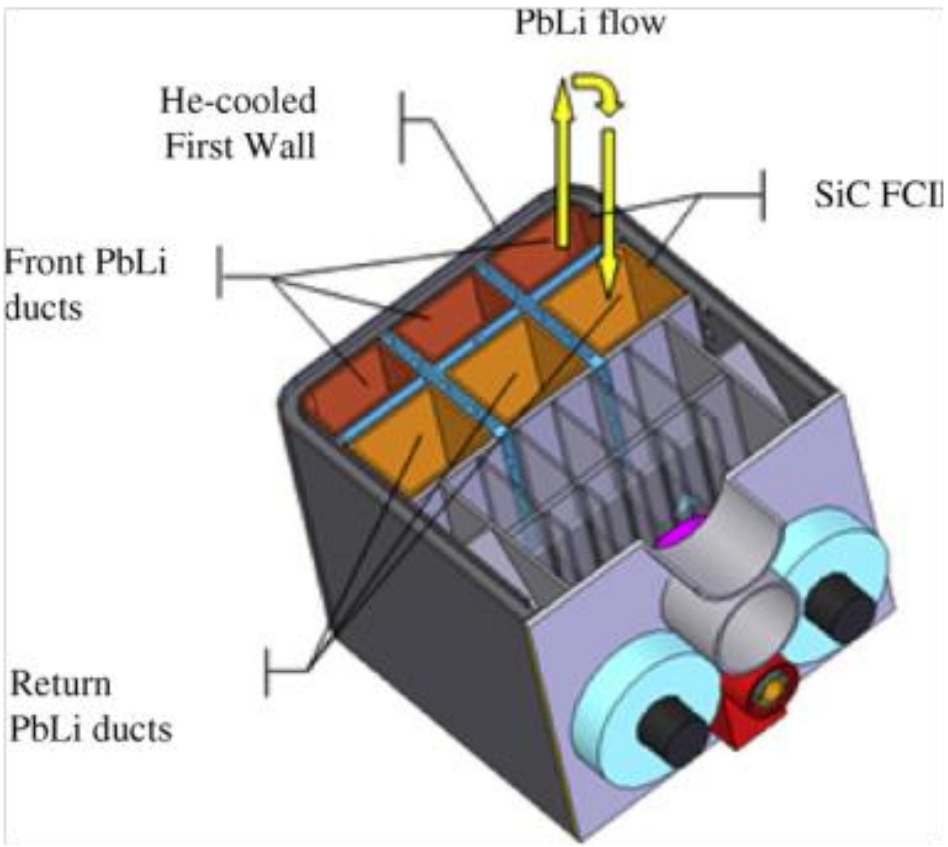
Fusion enabling technology

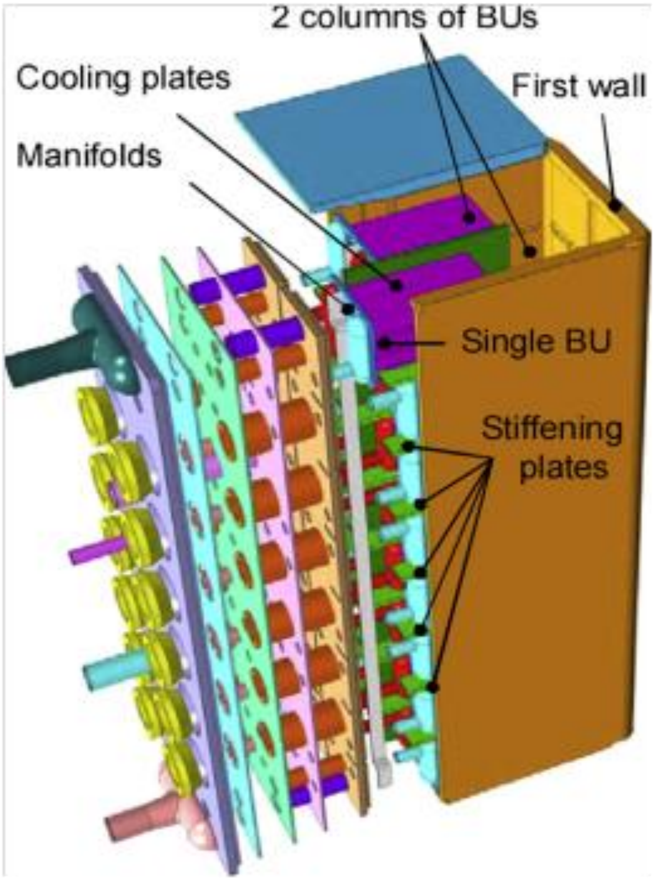




Cooling/breeding blankets and divertors for TOKAMAK reactors

Flows of liquid metals (Li, Li-Pb, FLiBe) in strong magnetic fields





Magnetic Reynolds number

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\mathrm{Re}_m} \nabla^2 \mathbf{B}$$

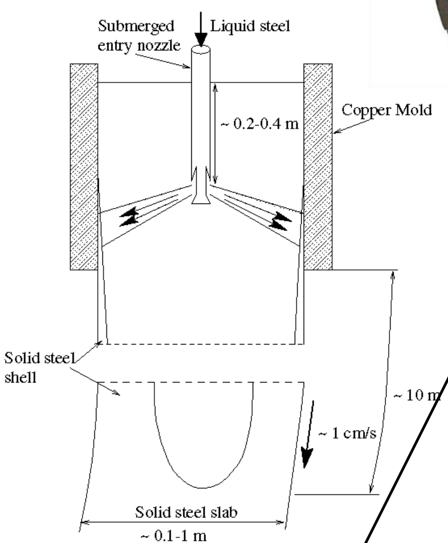
$\mathrm{Re}_m = UL/\eta = UL\sigma\mu_0$

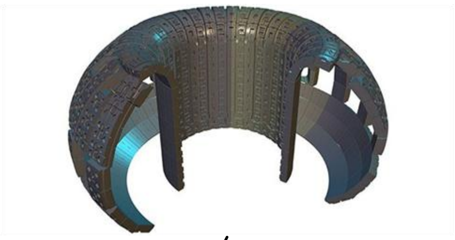
σ – Electrical conductivity

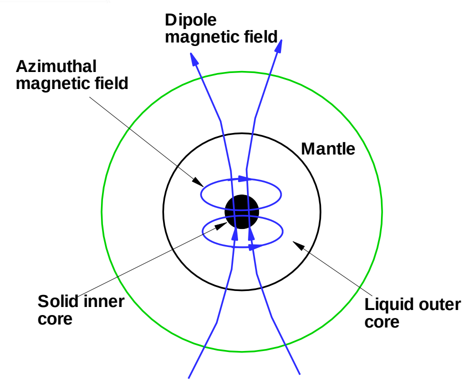
μ_0 – Magnetic permeability of vacuum

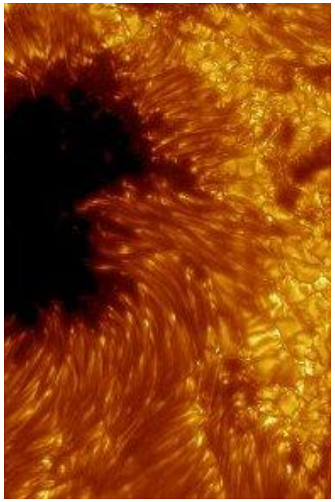
Liquid	σ [$\Omega^{-1}m^{-1}$]	η [m^2s^{-1}]
Sea water	5	6.3×10^6
Al (750 C)	4×10^6	0.2
Steel (1500 C)	0.7×10^6	1
Na (400 C)	6×10^6	0.13
Hg (20 C)	10^6	0.8
GaInSn (20 C)	3.3×10^6	0.25

Magnetic Reynolds number









0.01 - 1 $\qquad$ 10^2 $\qquad$ 10^8 $\qquad$ Re_m

MHD flows at low Re_m (quasi-static approximation)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} + \frac{\mathrm{Ha}^2}{\mathrm{Re}}\mathbf{j}\times\mathbf{e}_b$$

$$\nabla\cdot\mathbf{u} = \mathbf{0}$$

$$\mathbf{j} = -\nabla\phi + \mathbf{u}\times\mathbf{e}_b$$

$$\nabla^2\phi = (\nabla\times\mathbf{u})_b$$

$$\mathrm{Re} \equiv \frac{UL}{\nu}, \qquad \mathrm{Ha} \equiv BL\sqrt{\frac{\sigma}{\rho\nu}}$$

Effect of magnetic field on flow structures (far from walls)

Joule dissipation:

$$\frac{d}{dt}\int \frac{1}{2}\rho \mathbf{u}^2 dV = -\int \varepsilon dV - \int \mu dV$$

$$\mu = \frac{1}{\sigma}\int \mathbf{J}^2 dV$$

Effect of magnetic field on flow structures (far from walls)

Joule dissipation:

$$\frac{d}{dt}\int \frac{1}{2}\rho \mathbf{u}^2 dV = -\int \varepsilon dV - \int \mu dV$$

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MHD flows are found in laminar or transitional state more often than ordinary hydrodynamic flows

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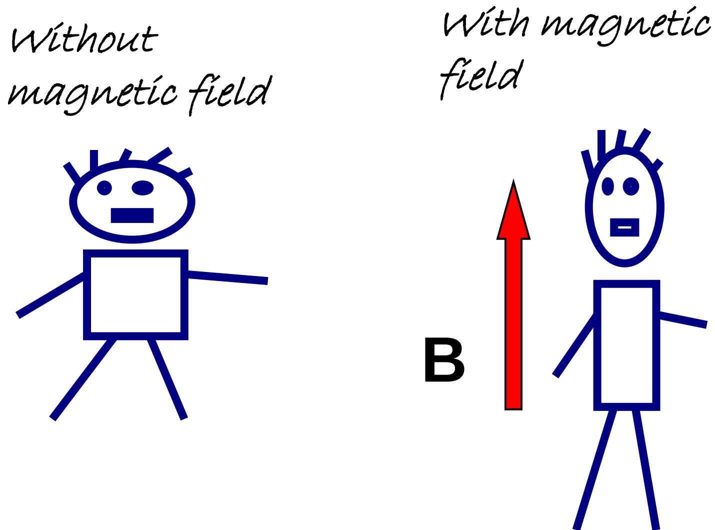
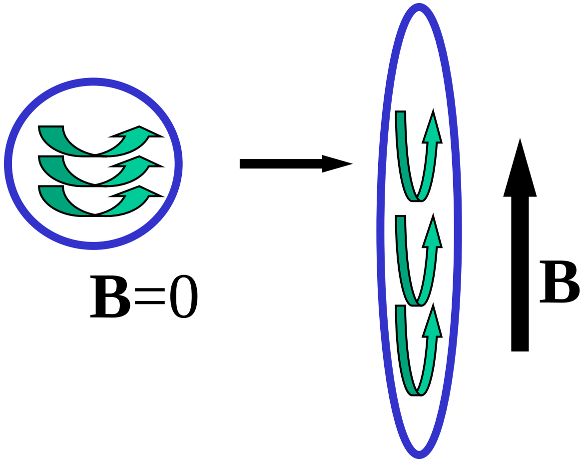
MHD flows are found in laminar or transitional state more often than ordinary hydrodynamic flows

Anisotropy of flow structures:



Anisotropy of gradients at low Re_m



$$\mathbf{F} \approx \mathbf{F}[\mathbf{u}] = \frac{\sigma B^2}{\rho} \Delta^{-1} \frac{\partial^2 \mathbf{u}}{\partial z^2}$$

$$\hat{\mathbf{F}} \approx \hat{\mathbf{F}}[\hat{\mathbf{u}}] = \frac{\sigma B^2}{\rho} \frac{k_z^2}{k^2} \hat{\mathbf{u}} = \frac{\sigma B^2}{\rho} \cos^2\theta \, \hat{\mathbf{u}}$$

$$\mu(\mathbf{k}) = \sigma B^2 \rho^{-1} \, |\, \hat{\mathbf{u}}(\mathbf{k}, t) \,|^2 \, \cos^2\theta$$

Magnetic Field ↓

3D Isotropic

↓

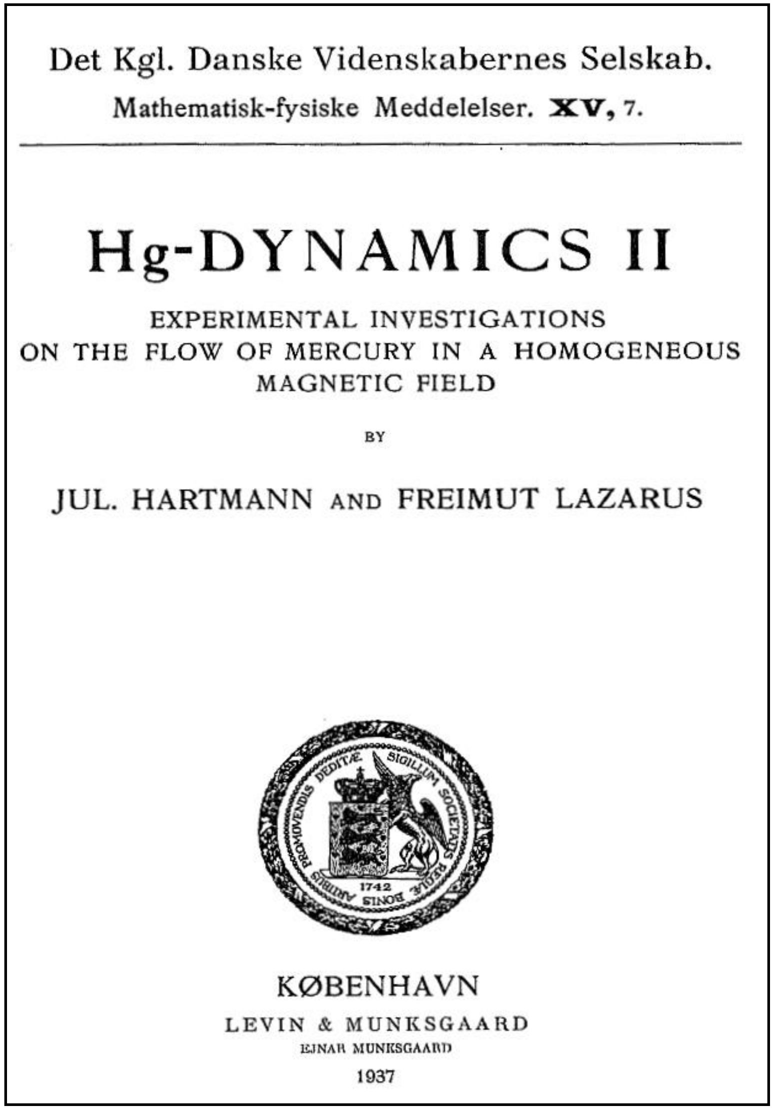
3D Anisotropic

↓

Quasi-2D

↑ Instability of 2D structures, Nonlinear Interaction

I. Archetypal MHD flow – duct with insulating walls in a uniform transverse magnetic field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Mathematisk-fysiske Meddelelser. **XV**, 7.

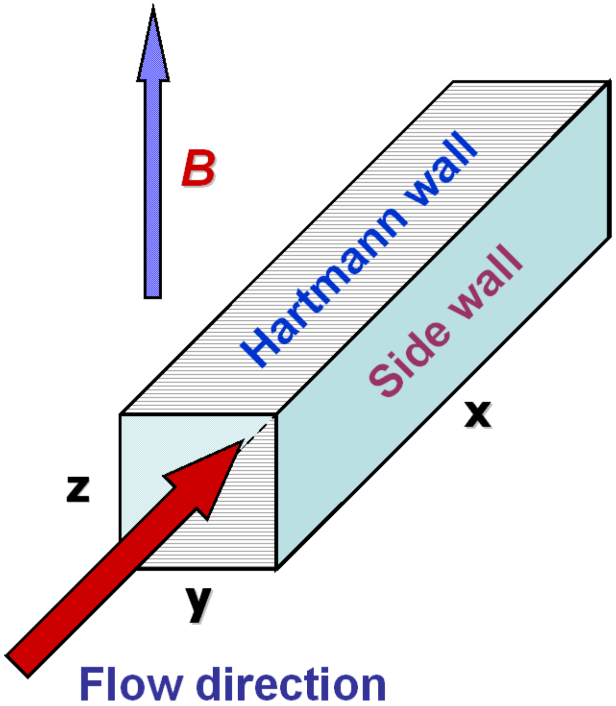
Hg-DYNAMICS II

EXPERIMENTAL INVESTIGATIONS ON THE FLOW OF MERCURY IN A HOMOGENEOUS MAGNETIC FIELD

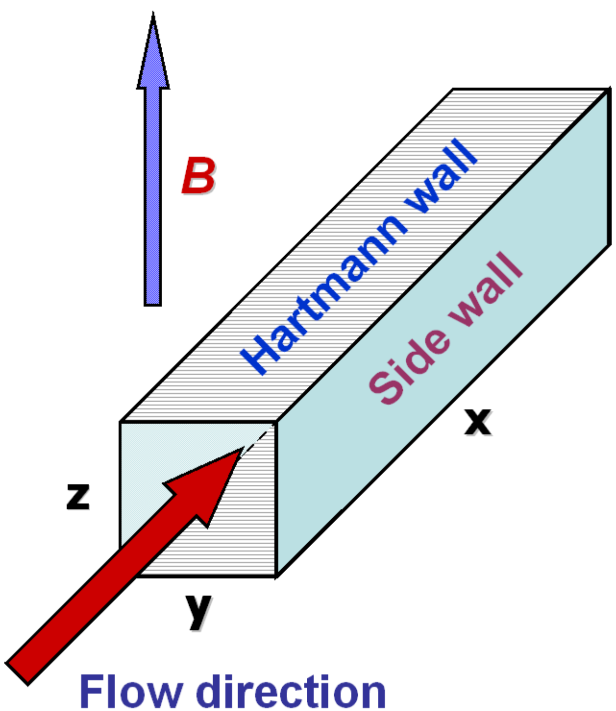
BY

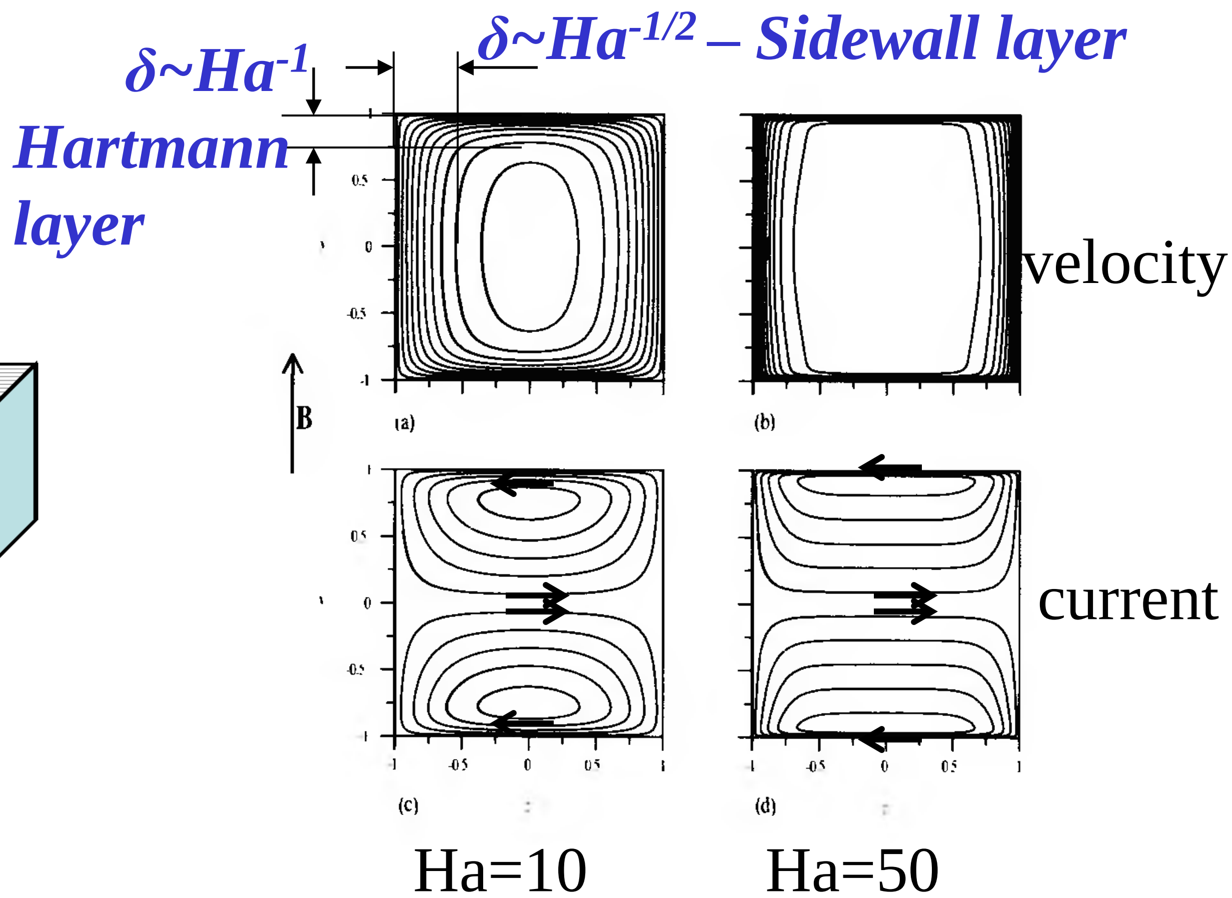
JUL. HARTMANN AND FREIMUT LAZARUS

KØBENHAVN
LEVIN & MUNKSGAARD
EJNAR MUNKSGAARD
1937



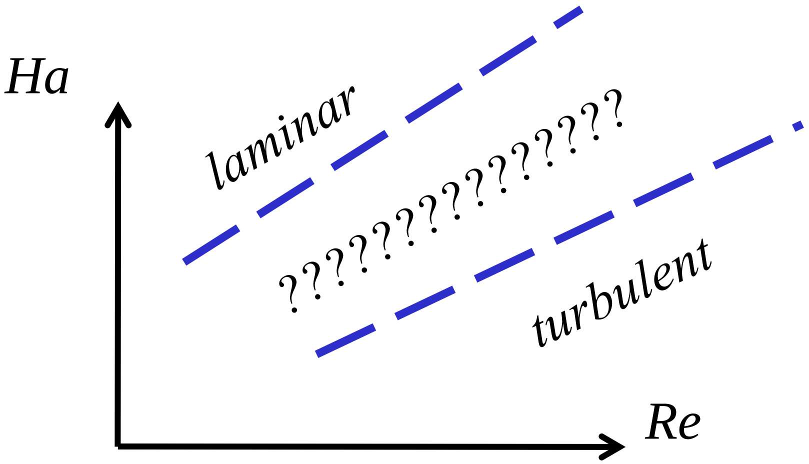
Flow structure: Flat core and MHD boundary layers





Question

Transition between laminar and turbulent flow regimes



R=Re/Ha=200 – 250 or 350 – 400 ?

Numerical method – Direct Numerical Simulations

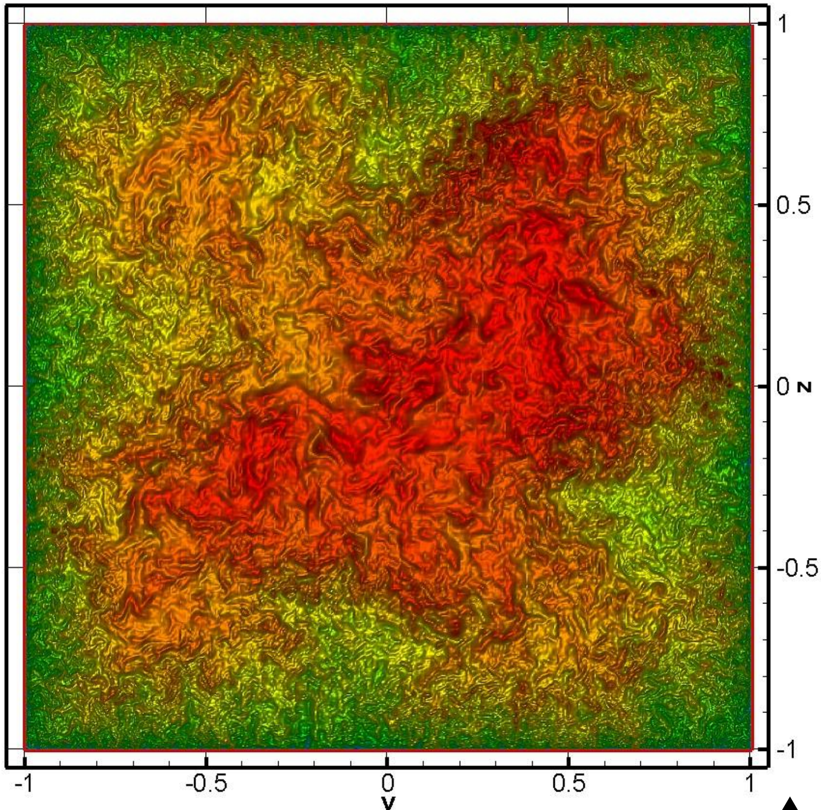
Finite difference solver (Krasnov et al, Comp. Fluids 2011)

Time advancing:	Adams-Bashforth/BWD 2nd order explicit with projection method for pressure/incompressibility
Grid arrangement:	Structured collocated grid with staggered fluxes
Viscous term:	2nd-order finite differences
Non-linear term:	2nd-order, divergent form, highly conservative operator (Morinishi et.al. 1998)
MHD term:	2nd-order, charge-conserving scheme for potential eq. and Lorentz force (Ni et.al. 2007)
Poisson solver:	Fourier expansion + 2D direct solver
Parallelization:	Hybrid parallelization: MPI + OpenMP

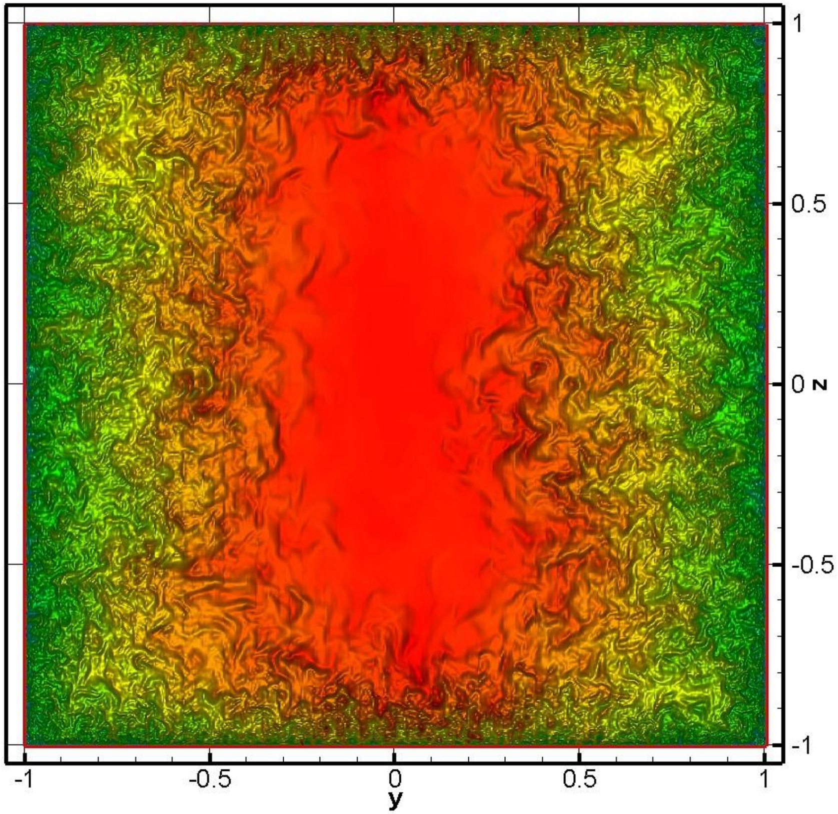
Parameters, Grid, Boundary conditions

- **$Re=10^5$** (in terms of mean velocity and half-width)
- **Ha=0, 100, 200, 300, 350, 400**
- Domain size: **4πx2x2**
- Numerical resolution: **2048x768x768**
- Nearly Chebyshev-Gauss-Lobatto wall clustering
- Electrically insulating walls
- Periodic inlet/exit

Instantaneous streamwise velocity



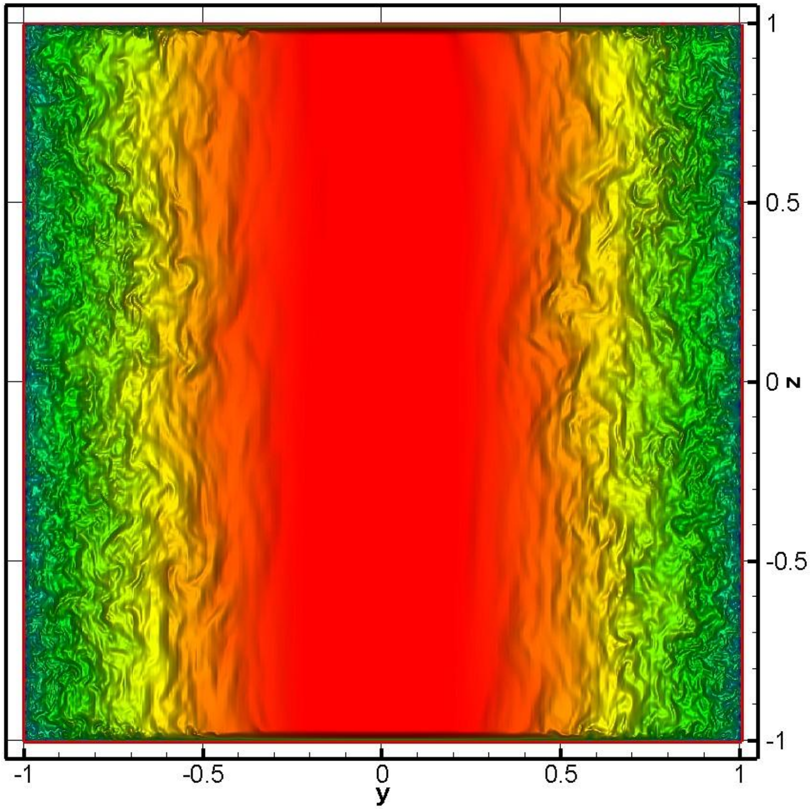
Ha=0



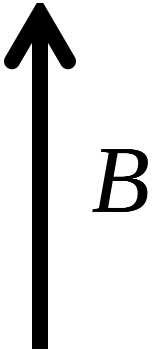
Ha=100

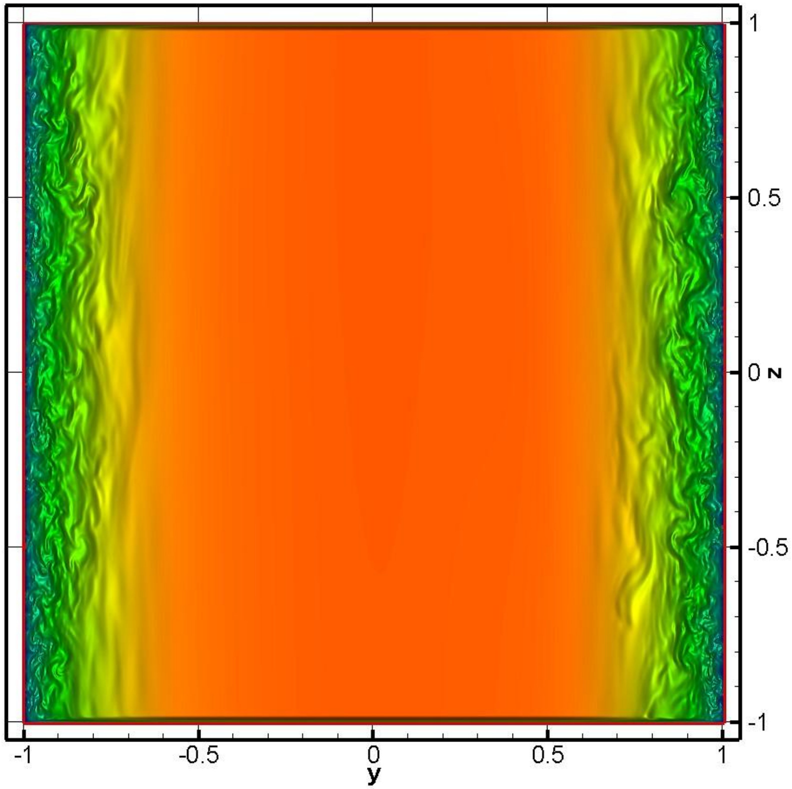
B

Instantaneous streamwise velocity



Ha=200

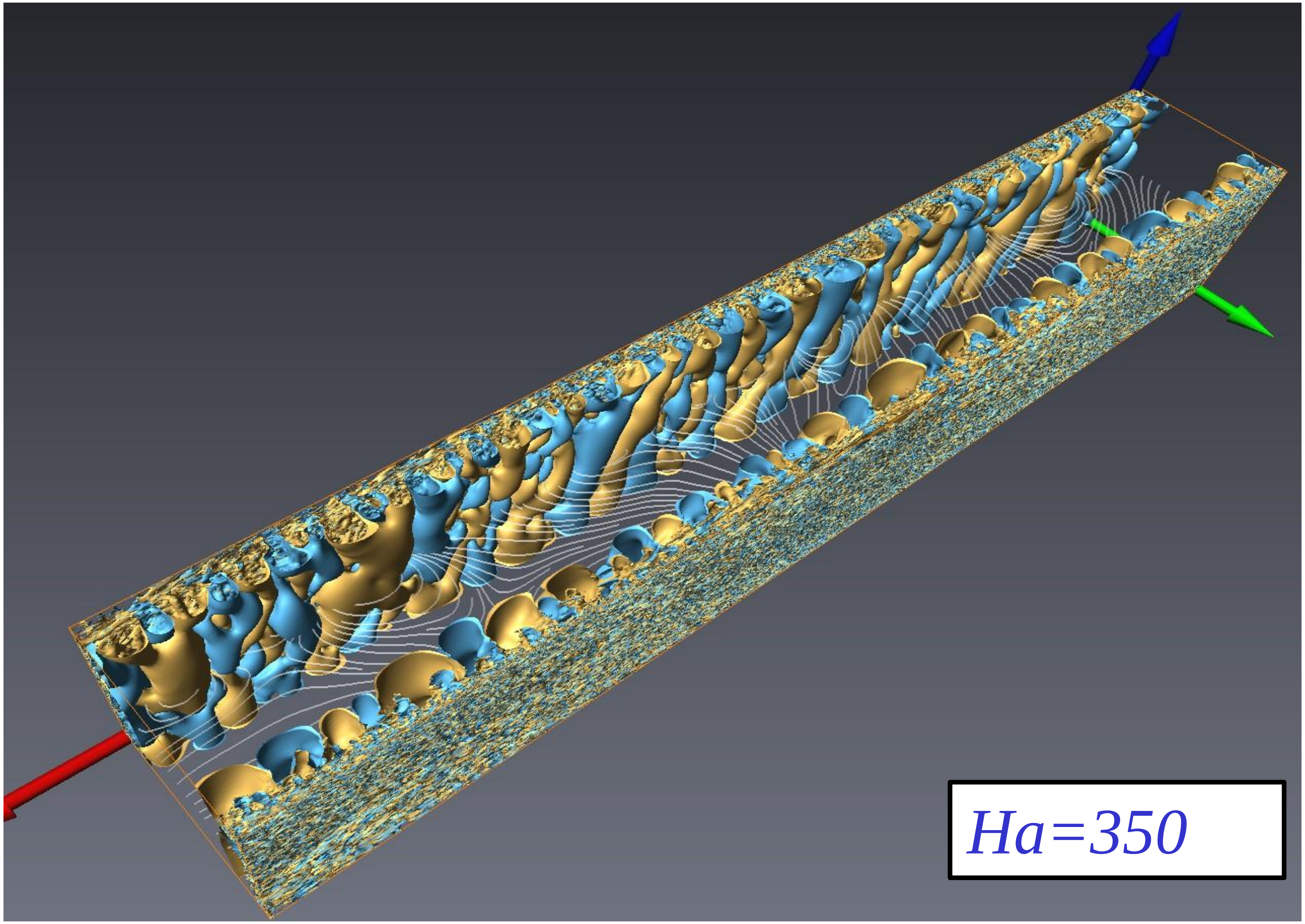




Ha=300

Laminar flow at Ha=400

Turbulence in sidewall layers



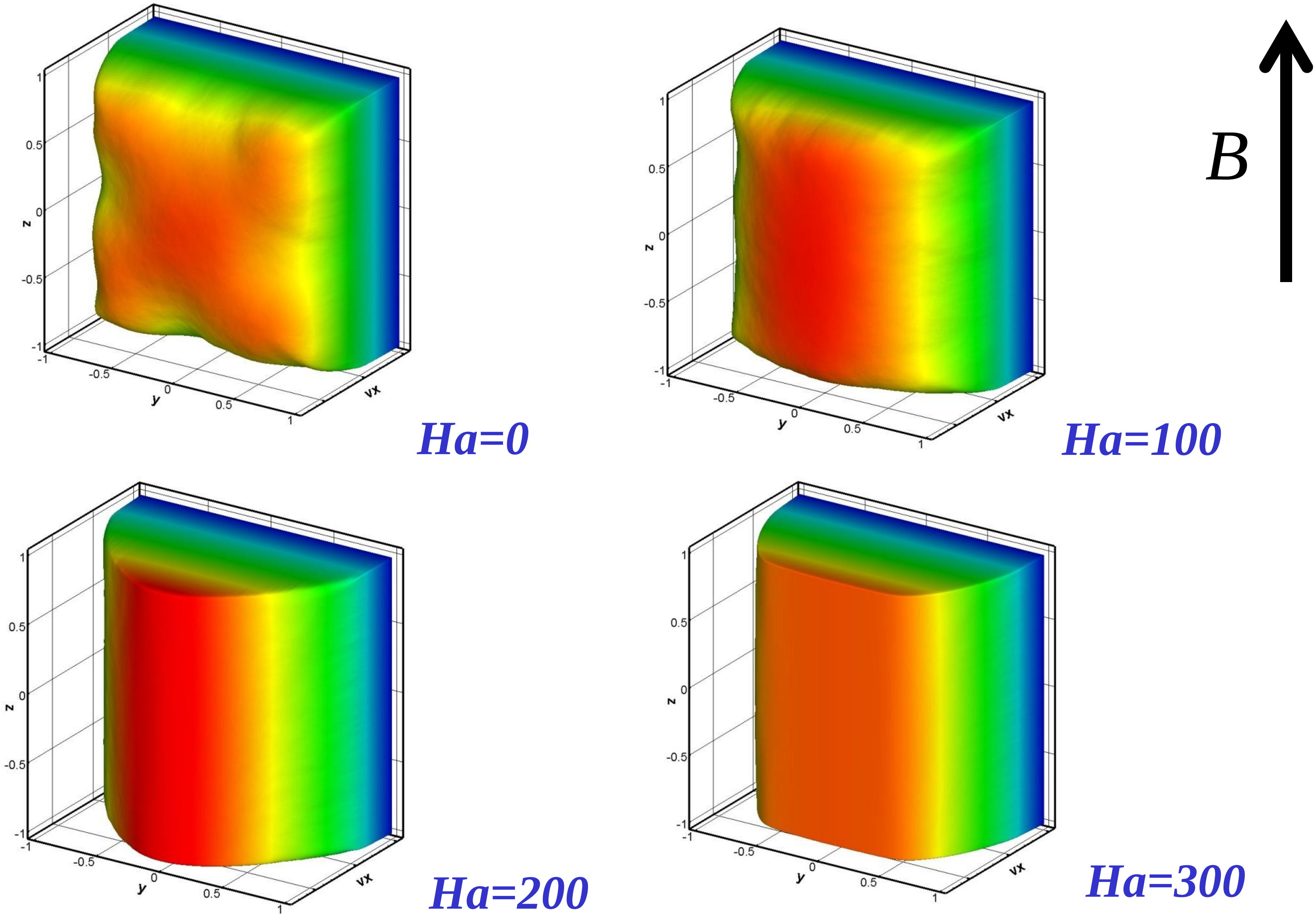
2^{nd} eigenvalue of $S_{ik}S_{kj}+\Omega_{ik}\Omega_{k}j$ (Jeong, Hussein, JFM 1995)

Summary of results

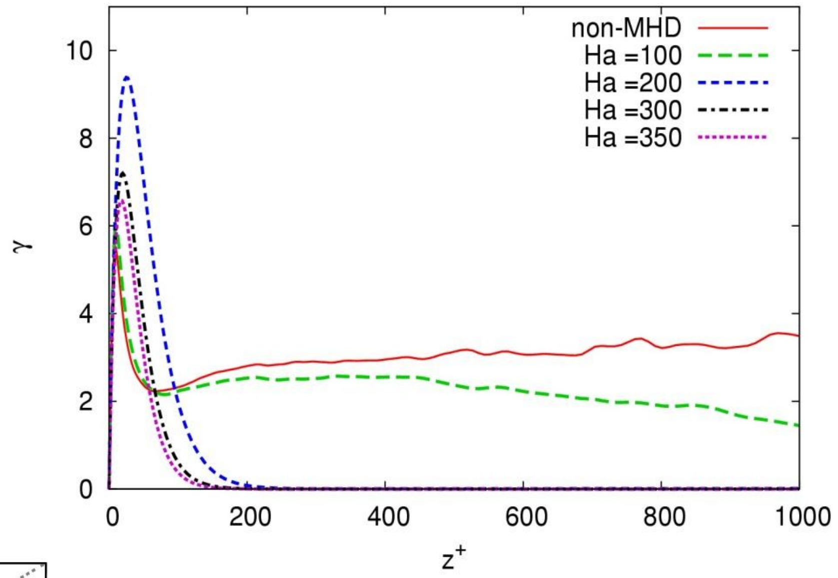
Ha	N	U_{cl}	$Re_{\tau,y}$	$Re_{\tau,z}$
0	0	1.1768	4253	4253
100	0.1	1.2304	3462	5269
200	0.4	1.3011	2487	5099
300	0.9	1.1466	1993	5865
350	1.225	1.1177	1901	6255
400 (laminar)	1.6	1.0465	1543	6512

Time-averaged in fully developed flow

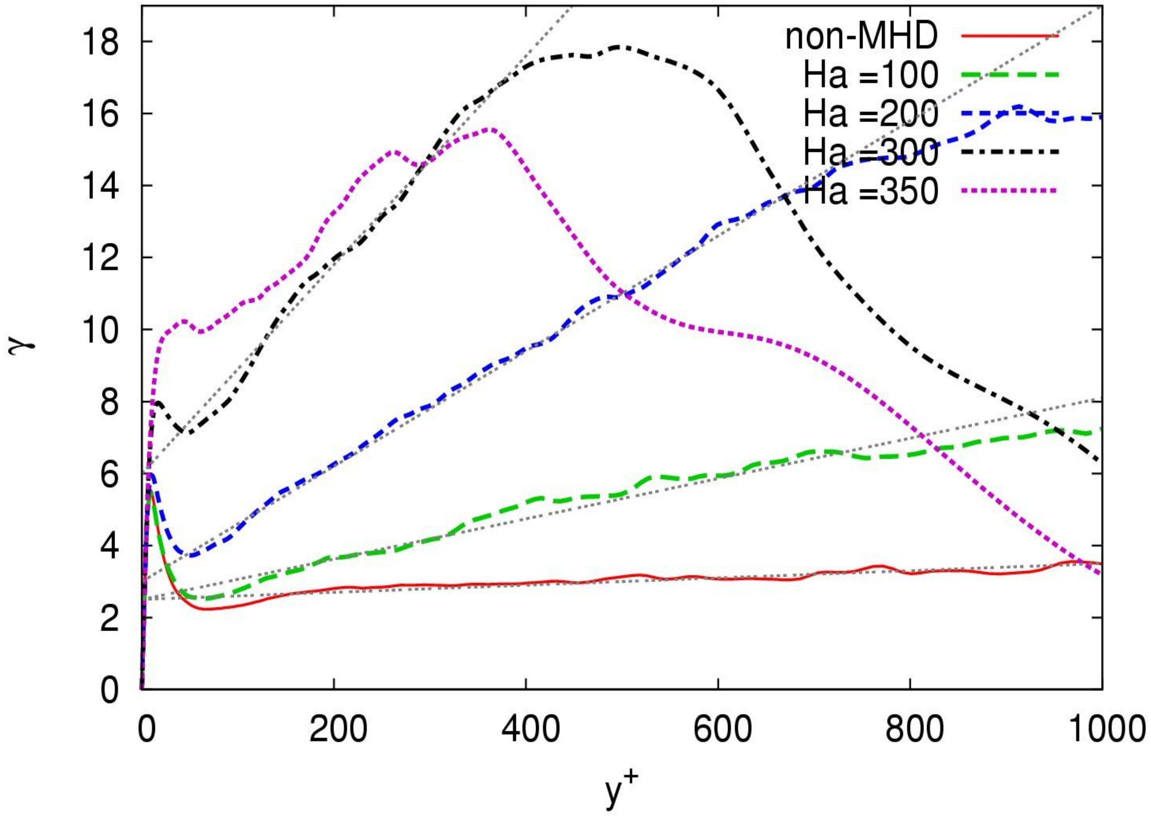
Mean streamwise velocity



Log-layer?



$\gamma = z^+ dU^+/dz^+$

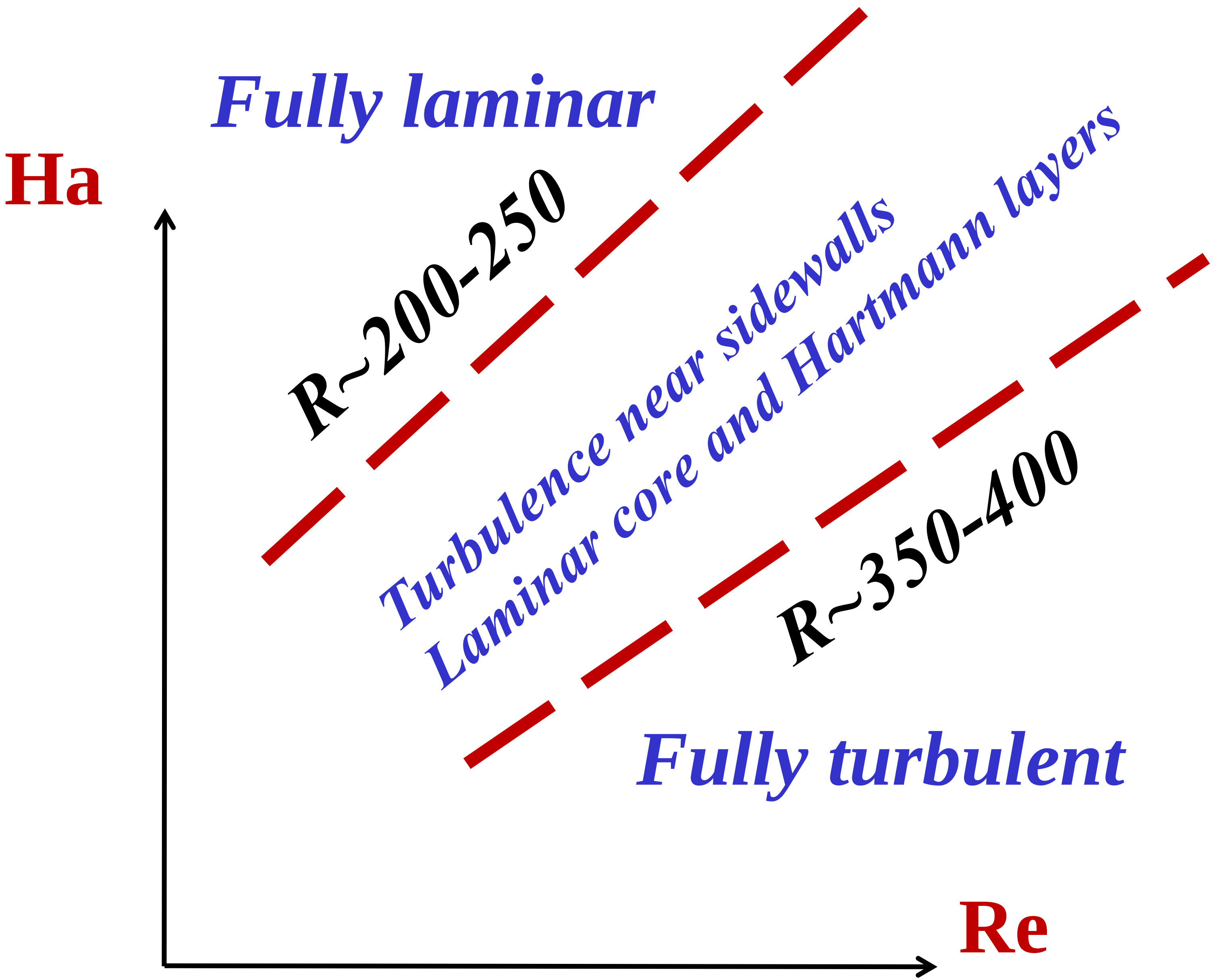


$\gamma = y^+ dU^+/dy^+$

Conclusions

• Transitional flow regimes with turbulence restricted to sidewall layers in a wide range of Ha
• Within sidewall layers, turbulence is small-scale and approaching isotropy near walls, but becomes large-scale, weak, and strongly anisotropic toward the center
• Non-trivial transformation of mean flow profile in the spanwise direction: lin-log

Krasnov et al, J. Fluid Mech. 2012, ***704****, 421-446*


Fully laminar
Ha
R~200-250
Turbulence near sidewalls
Laminar core and Hartmann layers
R~350-400
Fully turbulent
Re

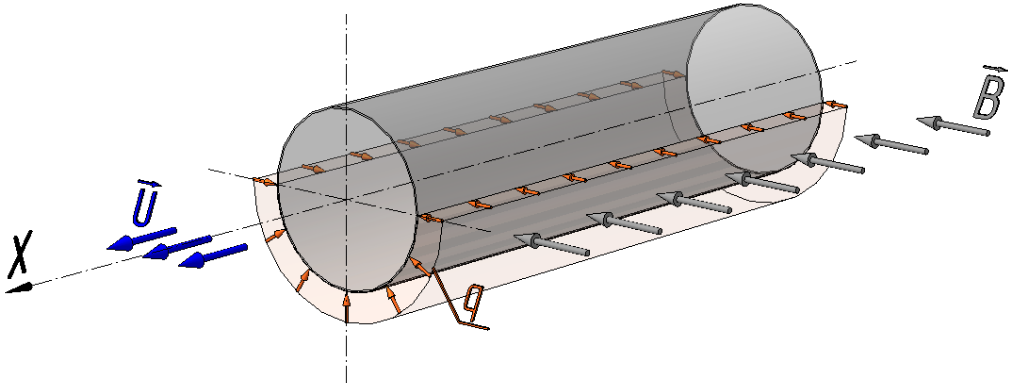
II. Mixed convection with strong transverse magnetic field

Flow of Hg in a horizontal pipe with transverse magnetic field: Institute of High Temperatures RAS

Pipe inner diameter: $d=19\ mm$
Walls: *stainless steel 0.5 mm*
Length of working segment: *2m*
Heated length: *0.812 m (43d)*
Uniform magn. field: *0.5m (26d)*
Max. heat flux: $q<55\ kW/m^2$
Max. magn. field: $B<1T$

Considered Case

- Horizontal pipe
- Perpendicular horizontal magnetic field
- Heated lower half
- Thermally insulated upper half

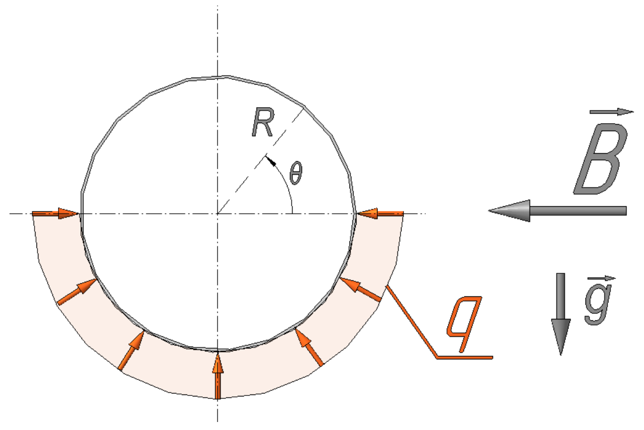


$Re_d=10^4$

$Ha_d=0, 100, 300, 500$

$Gr_d=8.3x10^7$ ($q=35$ kW/m^2)

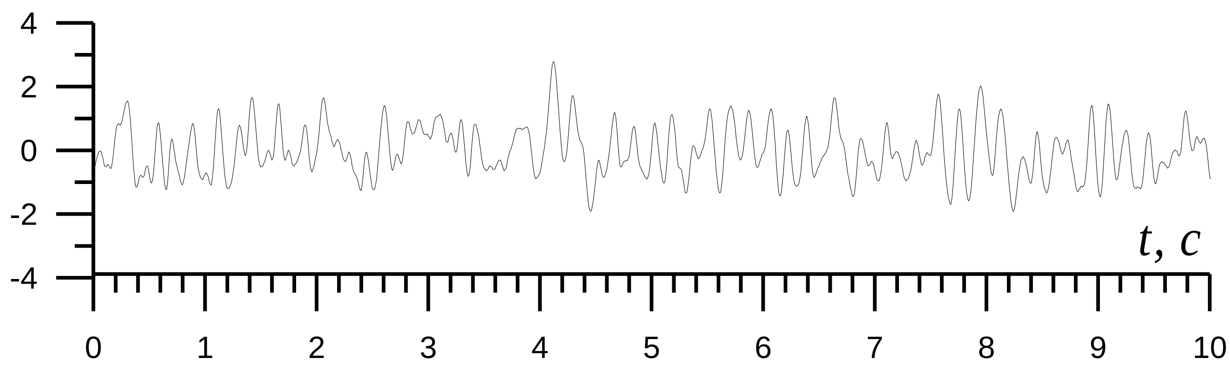
$Pr=0.022$



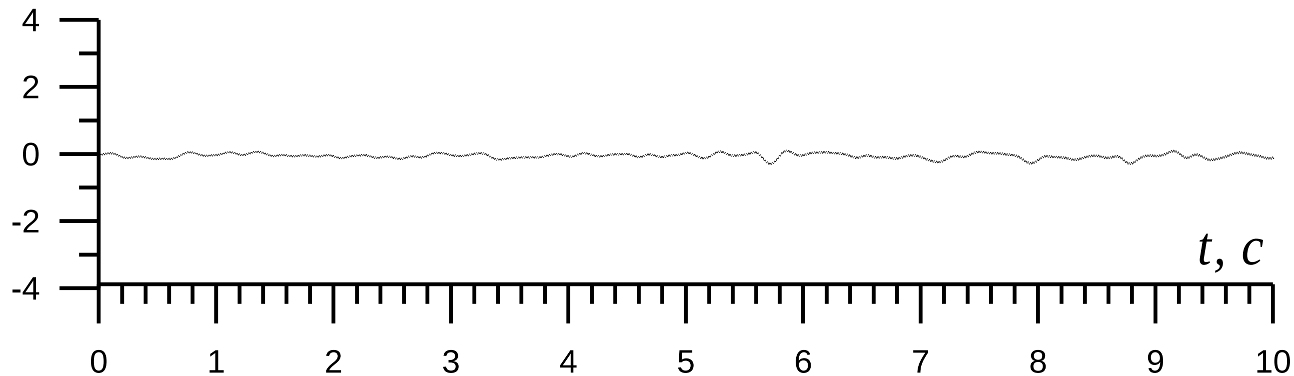
Experimental data

Temperature fluctuations: r=0.7R, bottom, x/d=40

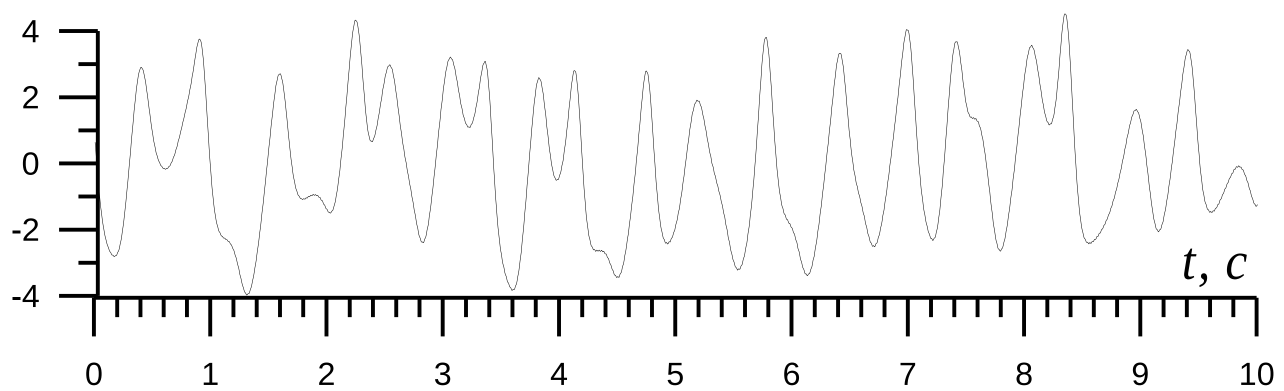
Ha=0



Ha=100

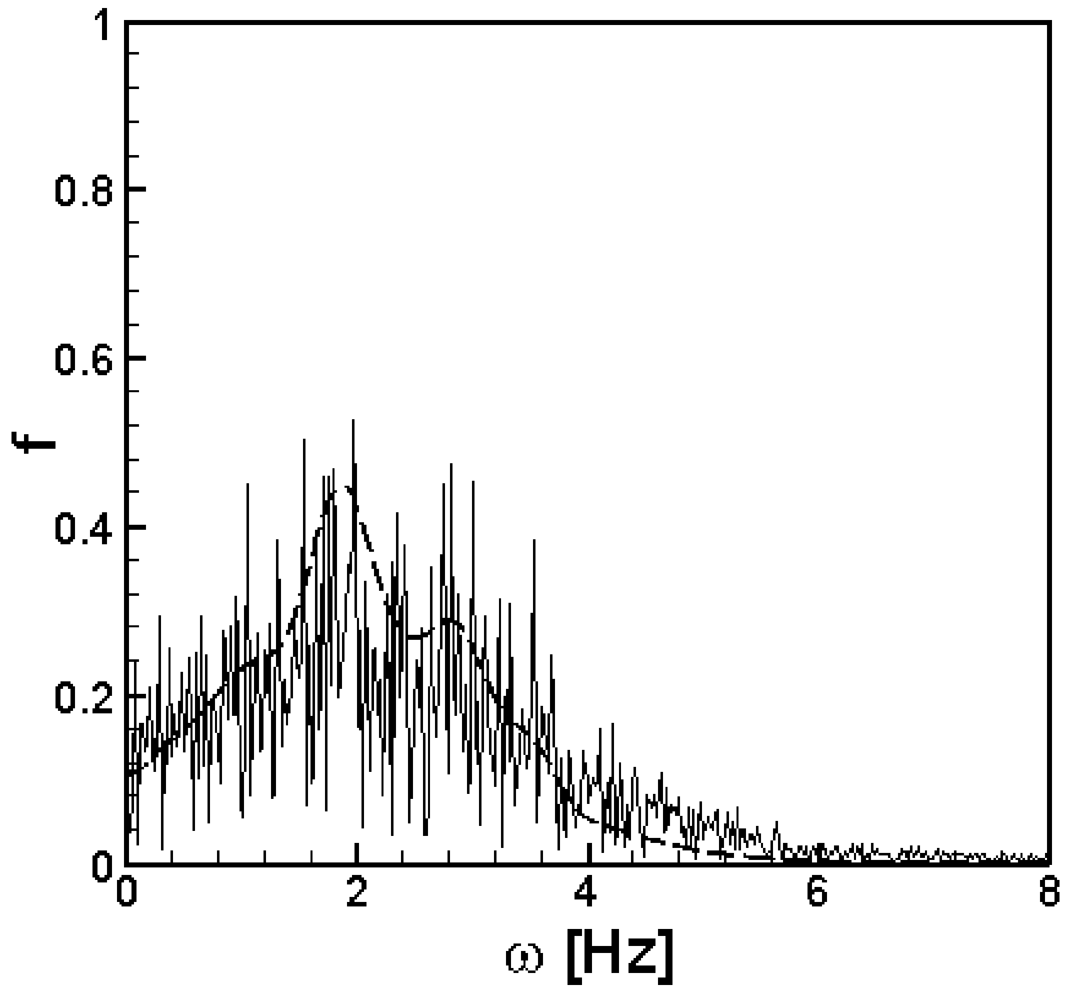


Ha=300



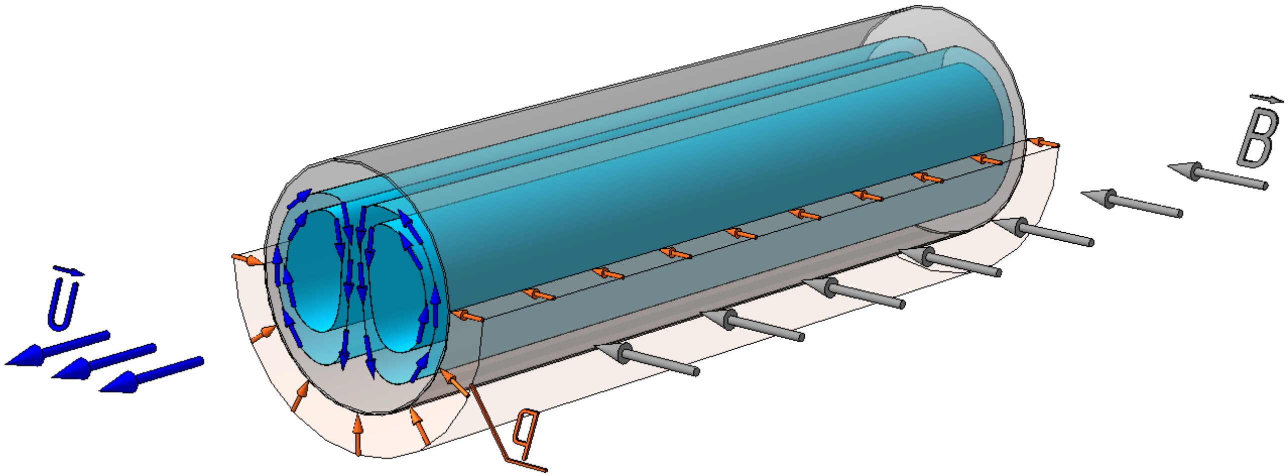
Experimental data

Ha=300

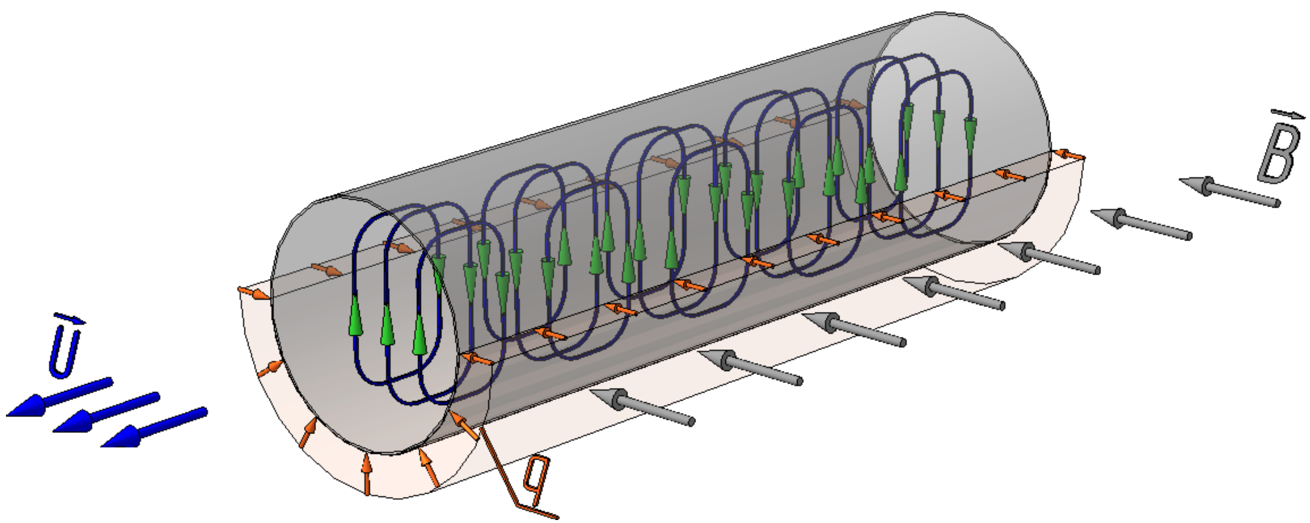


Hypothesis

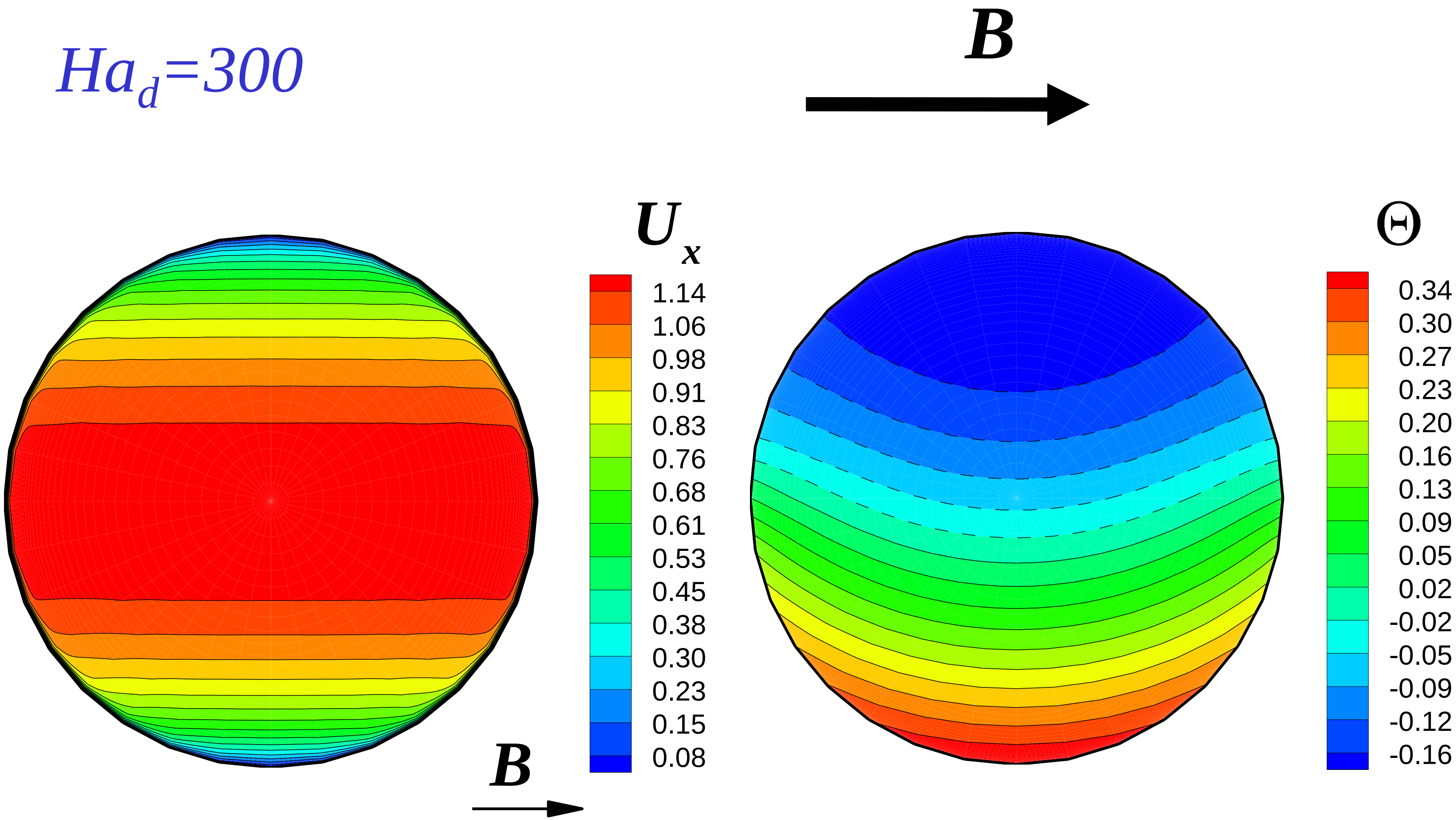
Ha=100



Ha=300



Linear stability analysis: Base flow

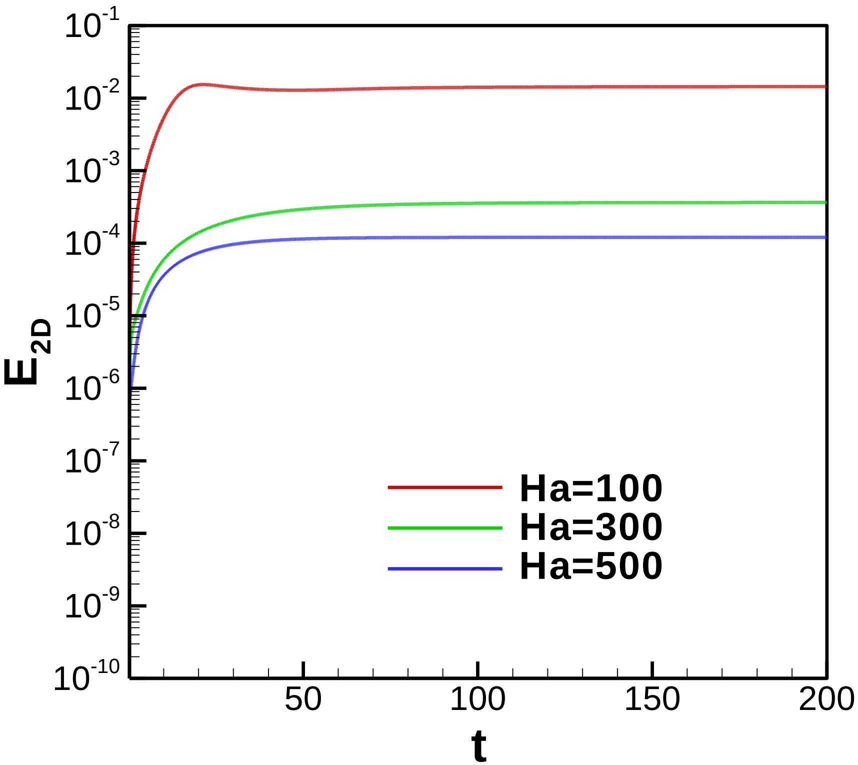


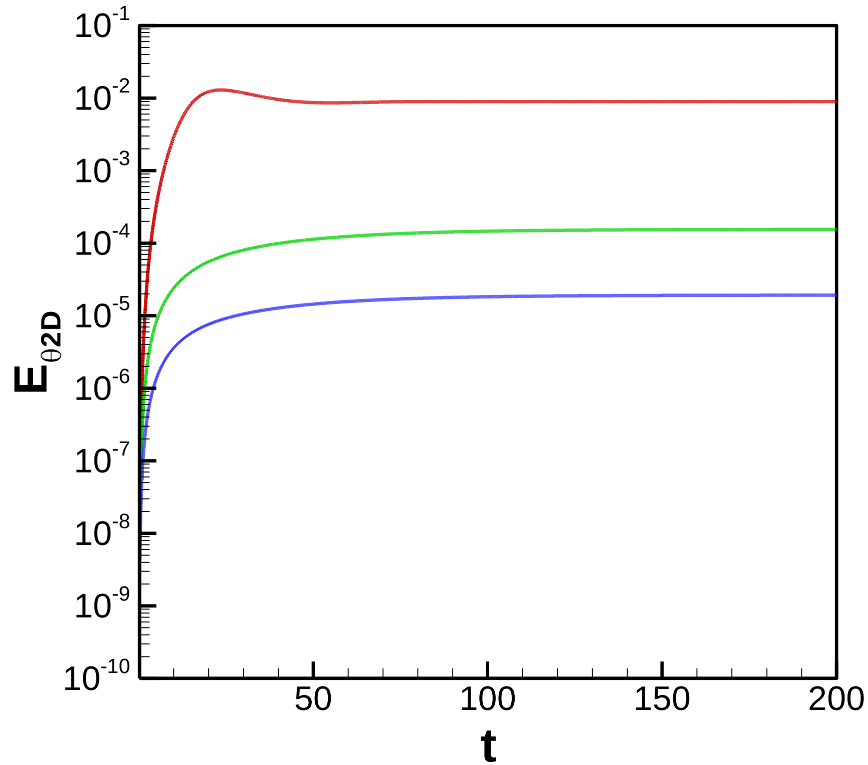
Linear Stability Analysis:
2D (streamwise-uniform) mode

Volume-averaged perturbations:

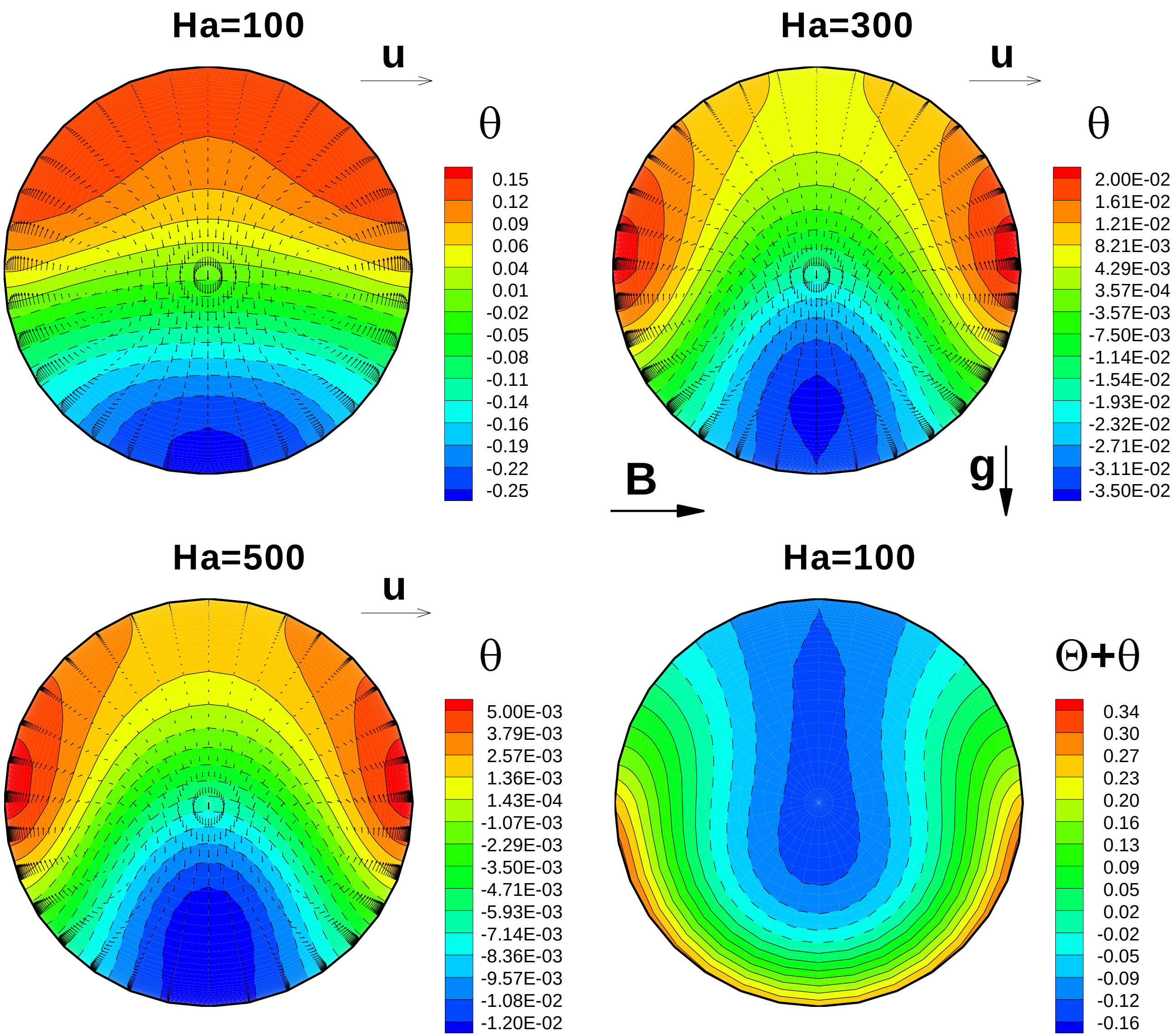
E2d, Eθ2d – x-independent mode

E3d, Eθ3d – mode of x-periodicity λ





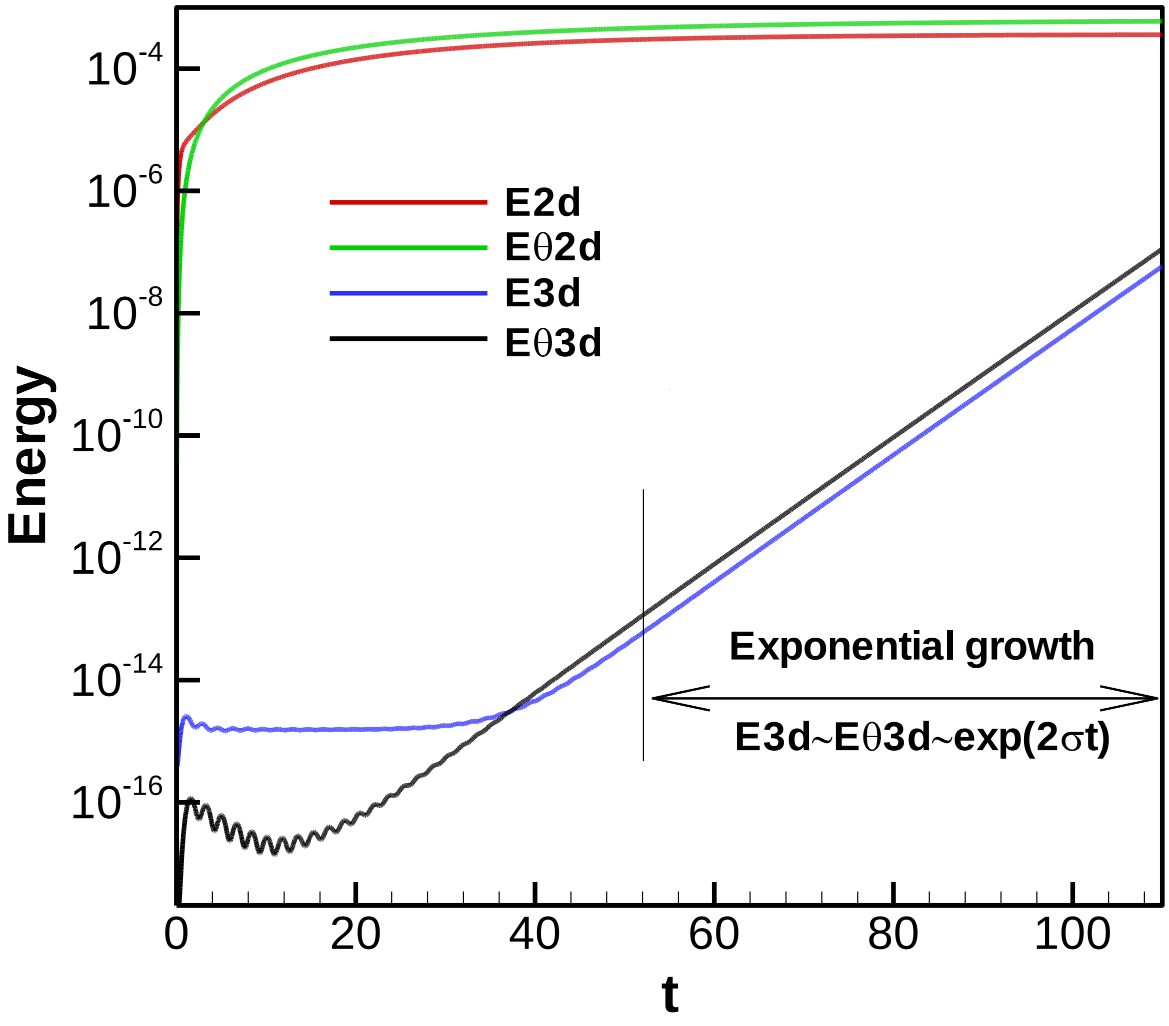
Linear Stability Analysis: 2D (streamwise-uniform) mode



Linear Stability Analysis: 2D + 3D modes

Example:

Ha_d=300, λ=1.0d

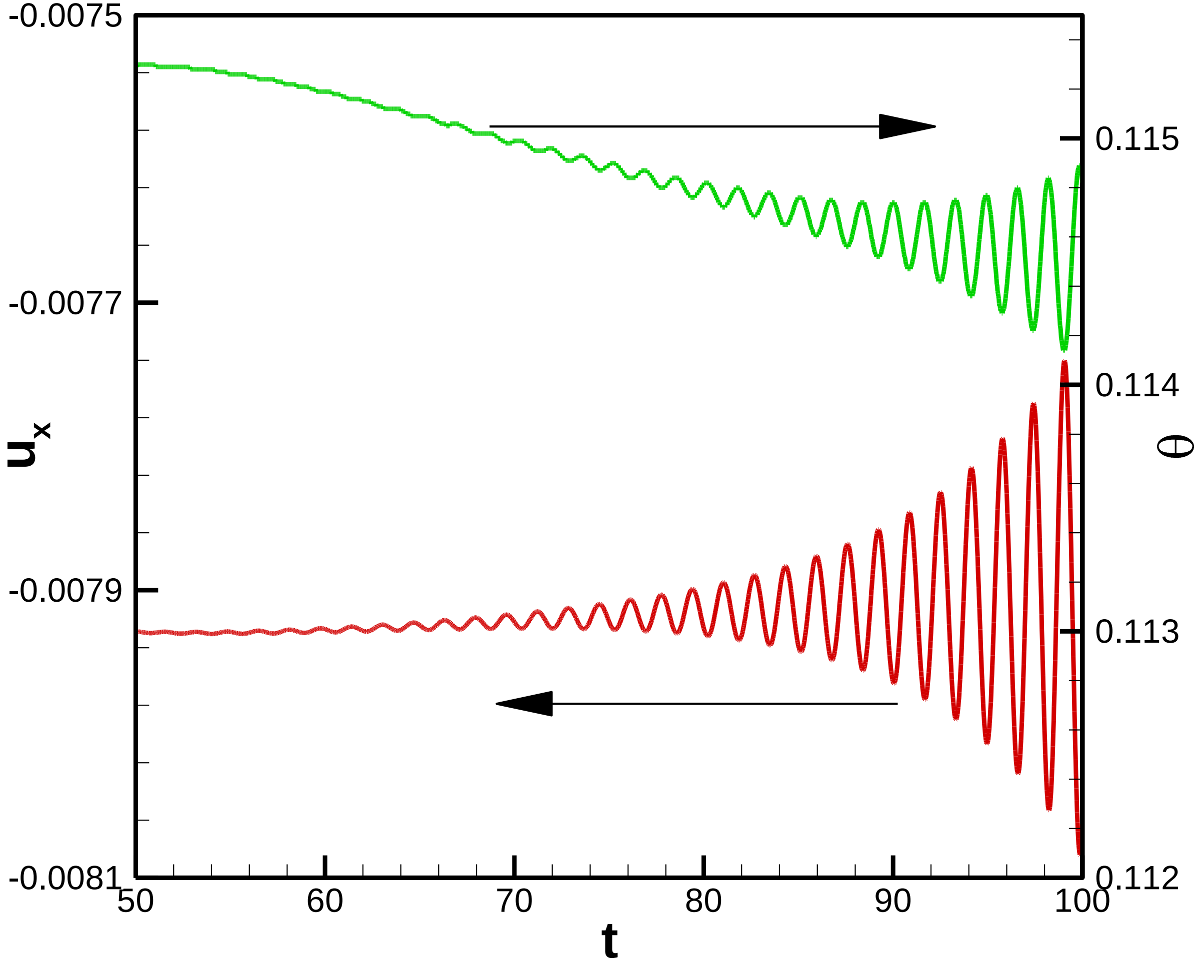


Linear Stability Analysis

Example:

Ha_d=300, λ=1.0d

Point signals of velocity and temperature during exponential growth

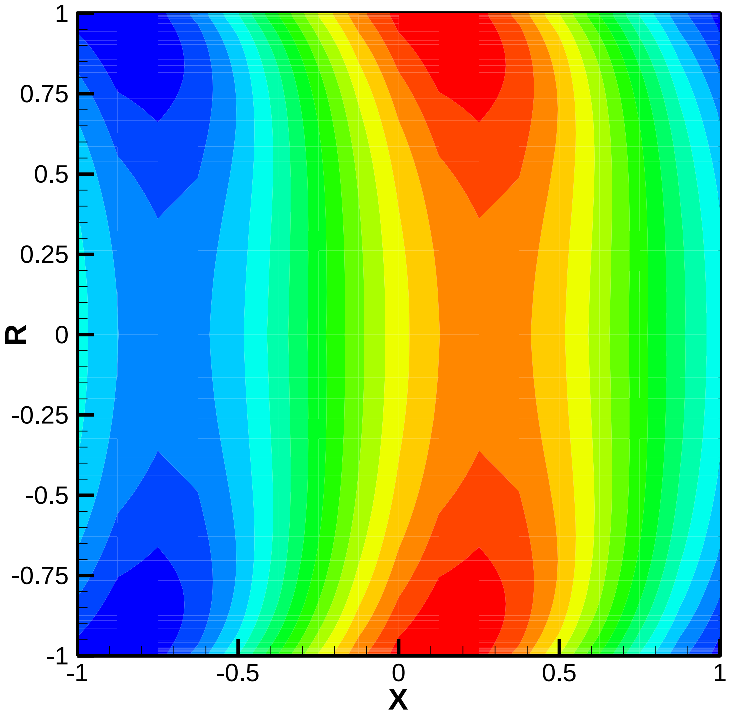


Linear Stability Analysis

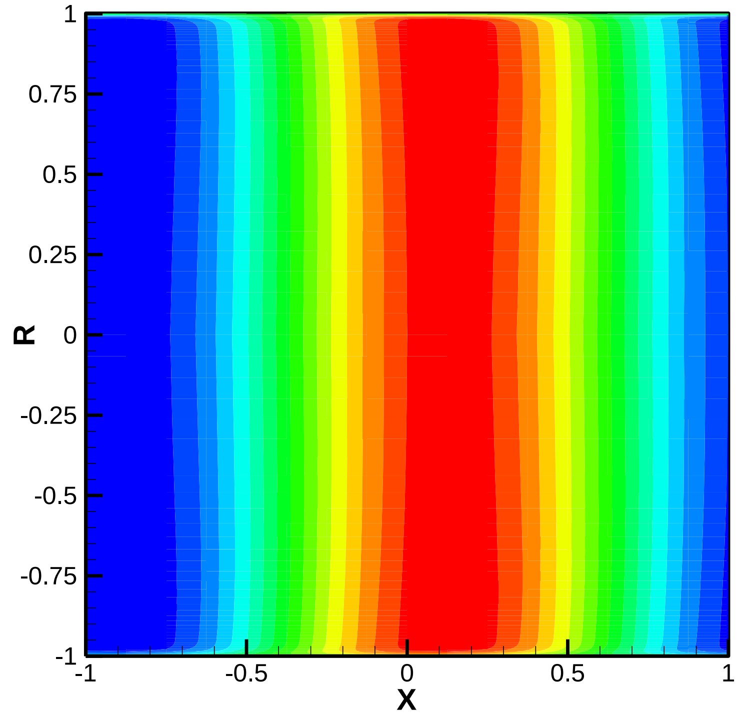
Example: Ha_d=300, λ=1.0d

t=100, horizontal cross-section through pipe axis

Temperature perturbations



Vertical velocity perturbations



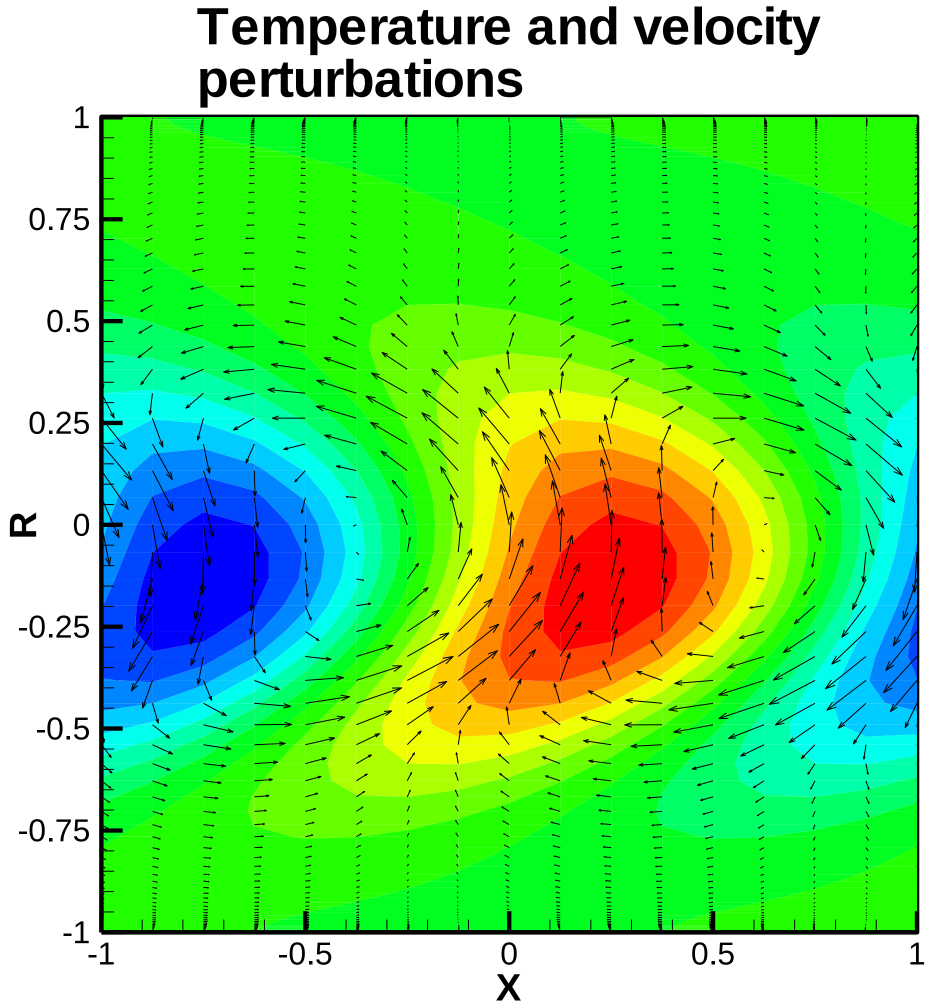
Magnetic field

Flow

Linear Stability Analysis

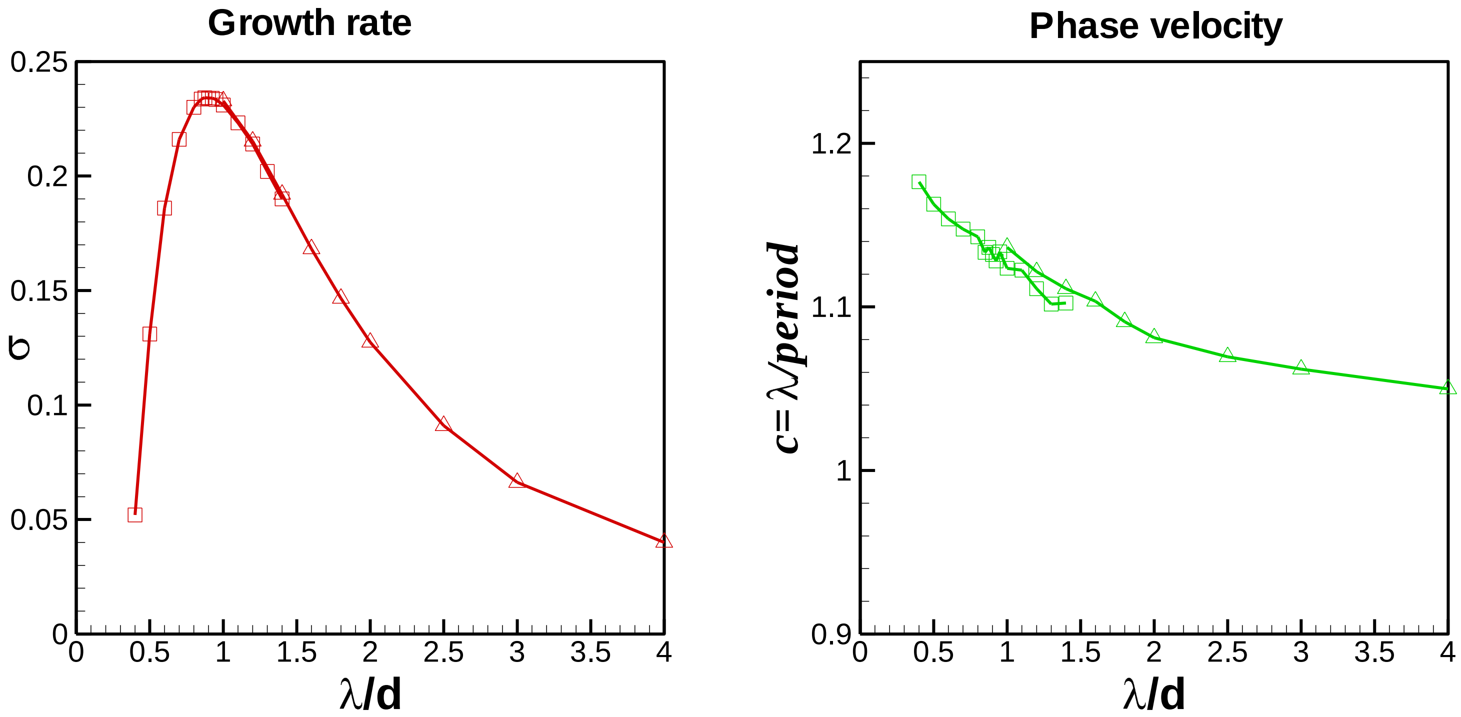
Example: Ha_d=300, λ=1.0d

t=100, vertical cross-section through pipe axis



Linear Stability Analysis

Ha_d=300

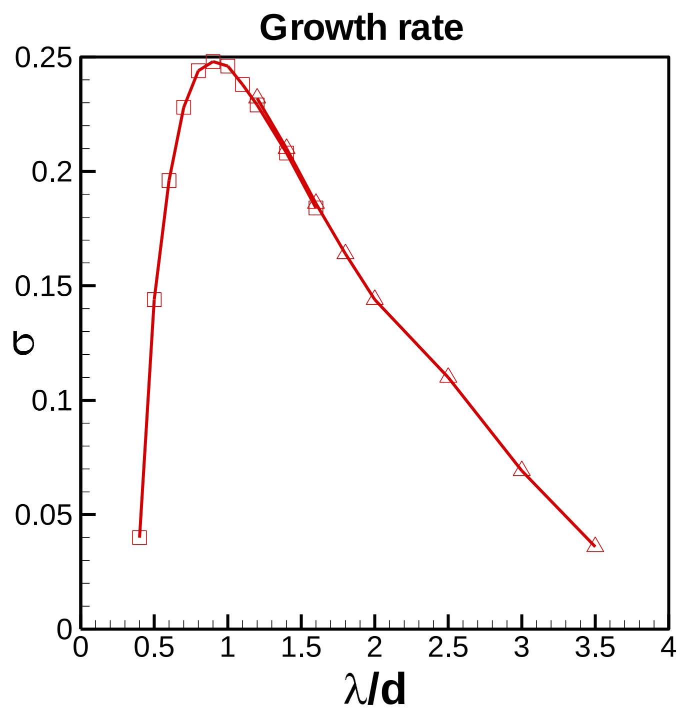


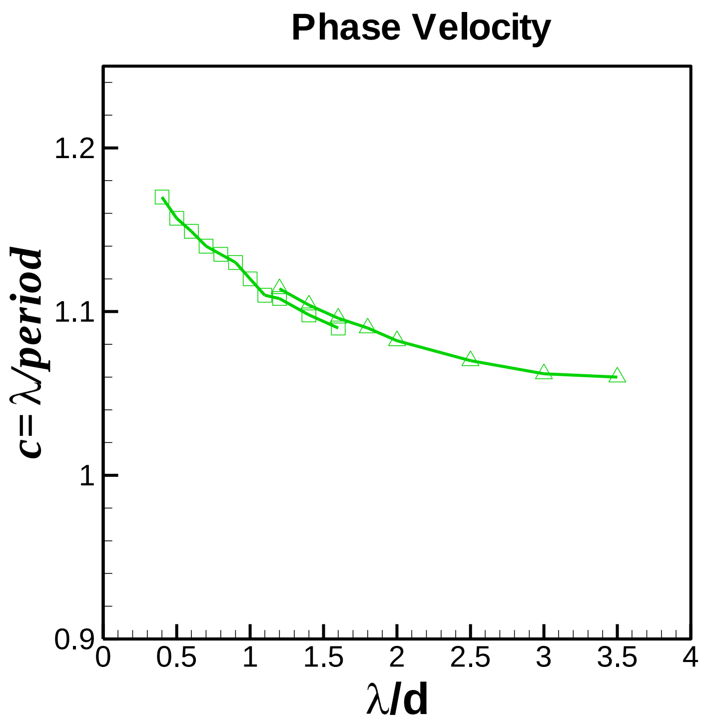
Fastest growing mode: ***λ=0.9d, period T=0.8***

Dimensional frequency ~ 3.2 Hz **(compare with 2-3 Hz in experiment)**

Linear Stability Analysis

Ha_d=500





Fastest growing mode: ***λ=0.9d, period T=0.8***

Growth rate ~ 10% higher than at Ha=300

Linear Stability Analysis

Further results

- *Ha=100:*

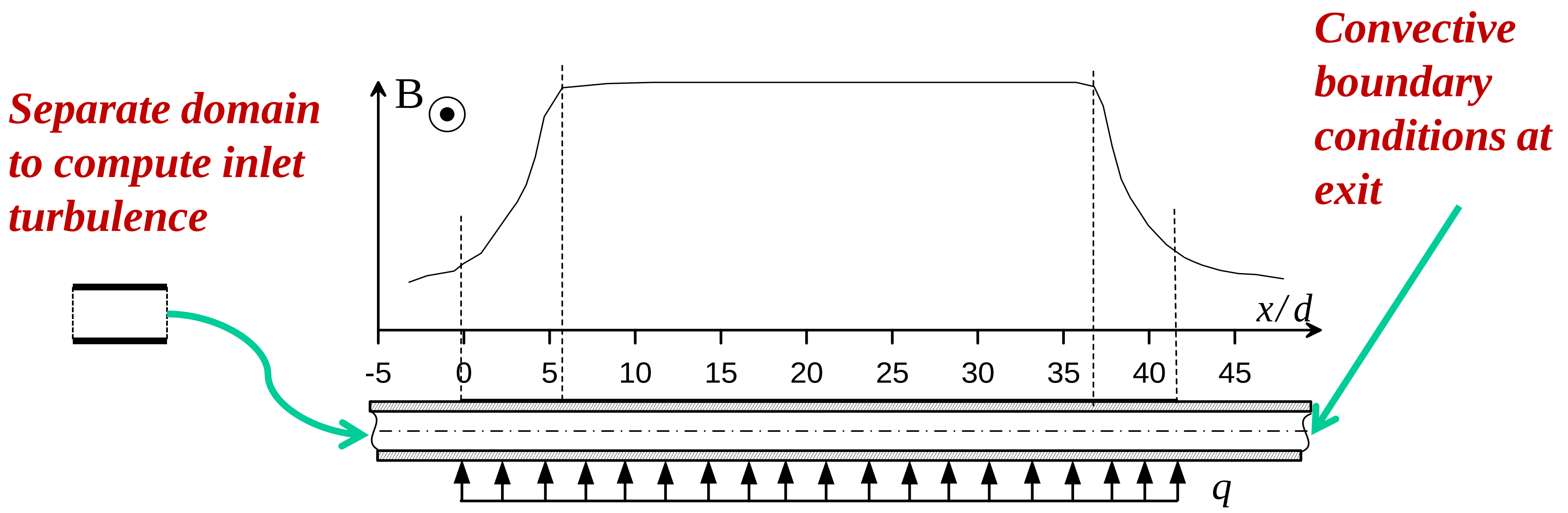
  No exponential growth of 3D modes found

- *Ha=300, but insufficient numerical resolution of boundary layers:*

  No exponential growth of 3D modes found

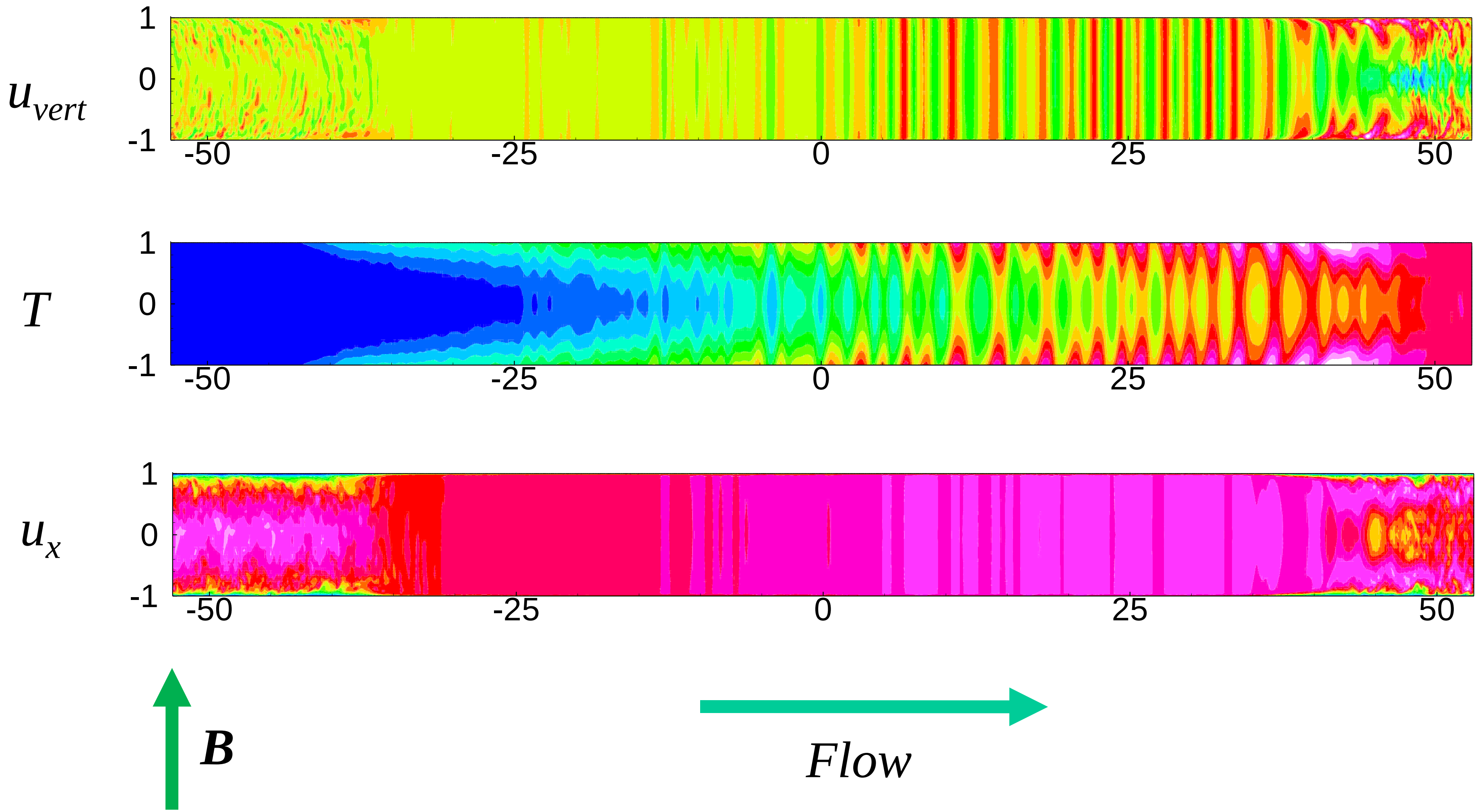
DNS of experiment's test section

- Realistic inlet/exit;
- div-free 2D distribution of magnetic field following experimental data
- x/d=53 – domain length;
- x/d=43 – heating area; x/d=31 – magnet;
- Resolution: N_r=90, N_θ=96, N_x=1696, A_r=3.0



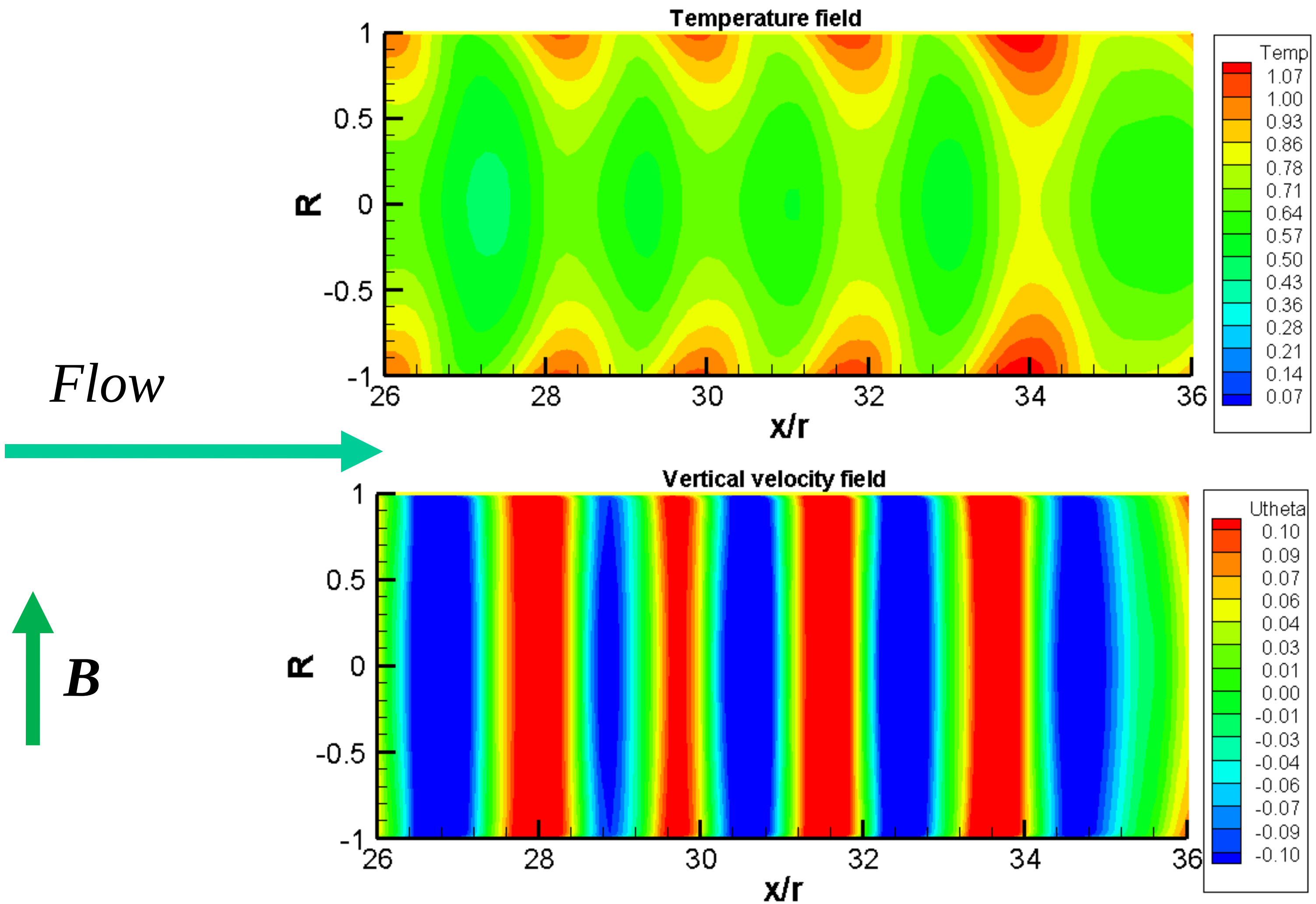
Fully developed flow, Ha=300

horizontal cross-section through pipe axis



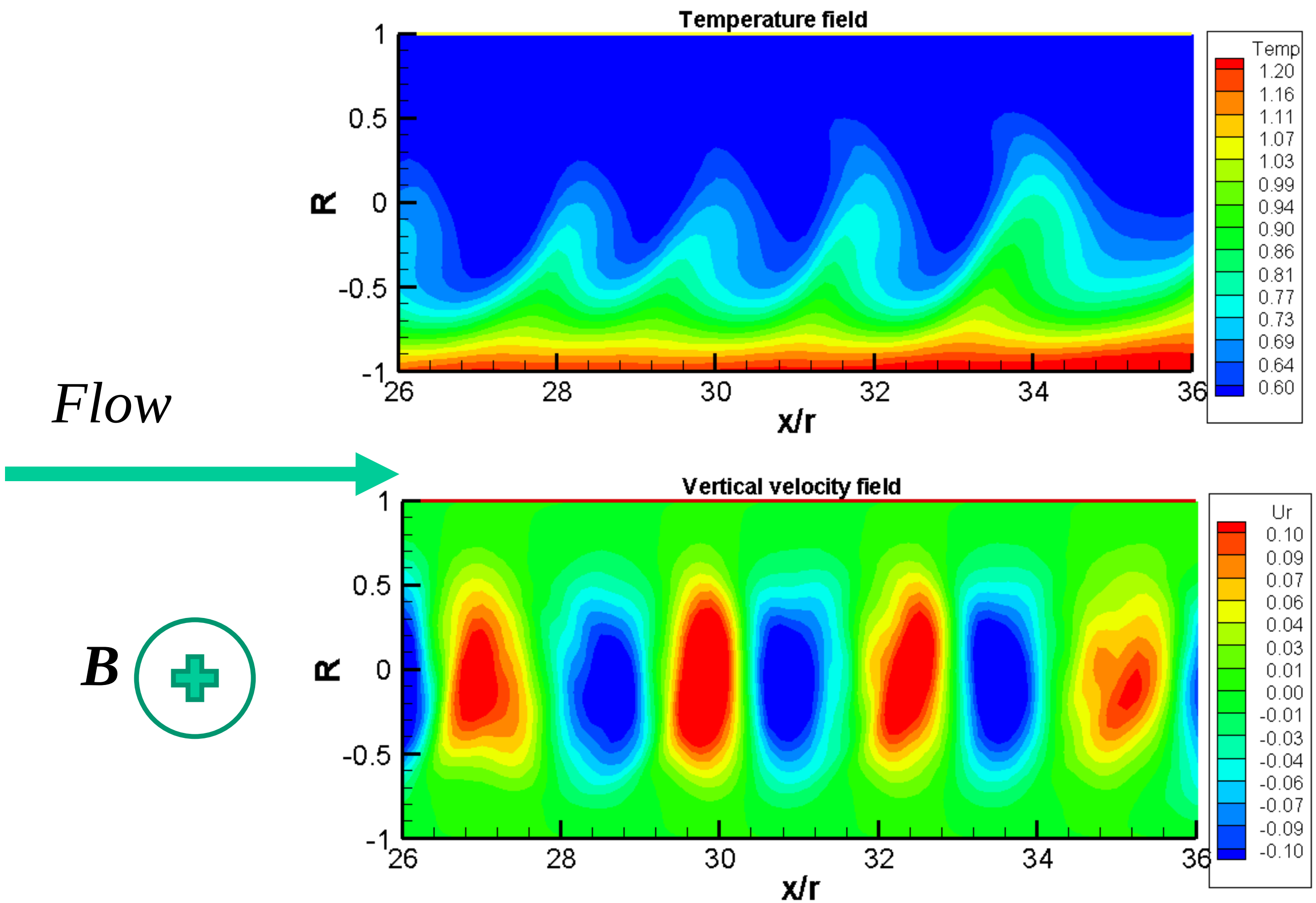
Fully developed flow, Ha=300

horizontal cross-section through pipe axis

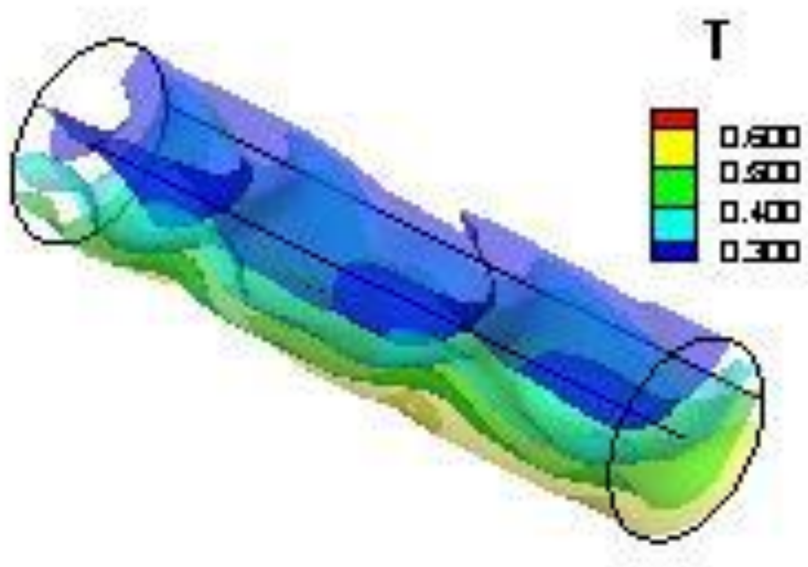


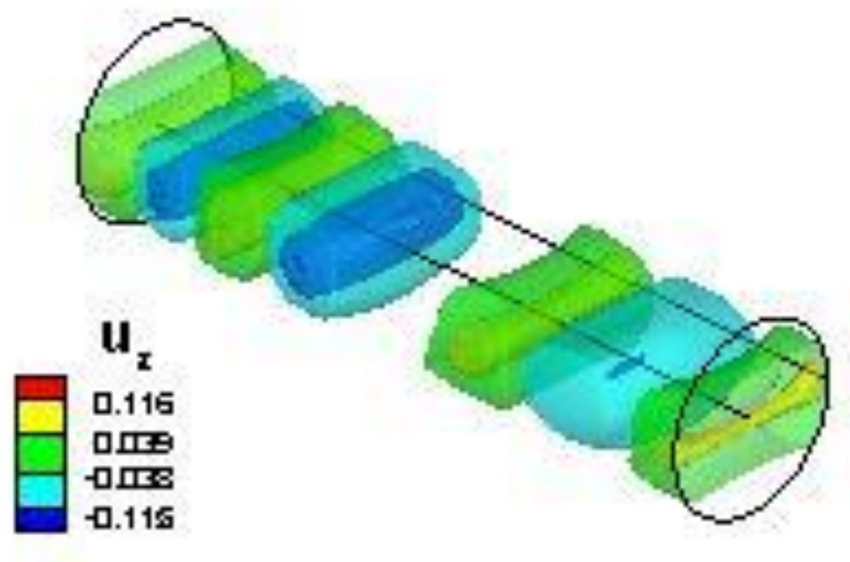
Fully developed flow, Ha=300

vertical cross-section through pipe axis



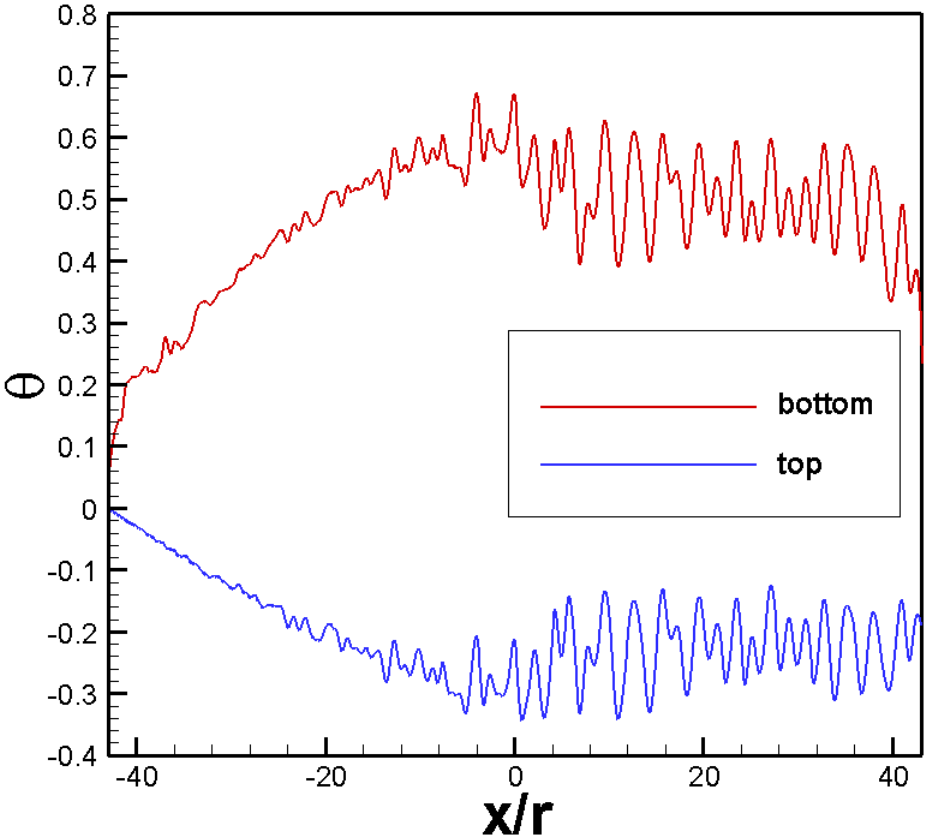
Fully developed flow, Ha=300



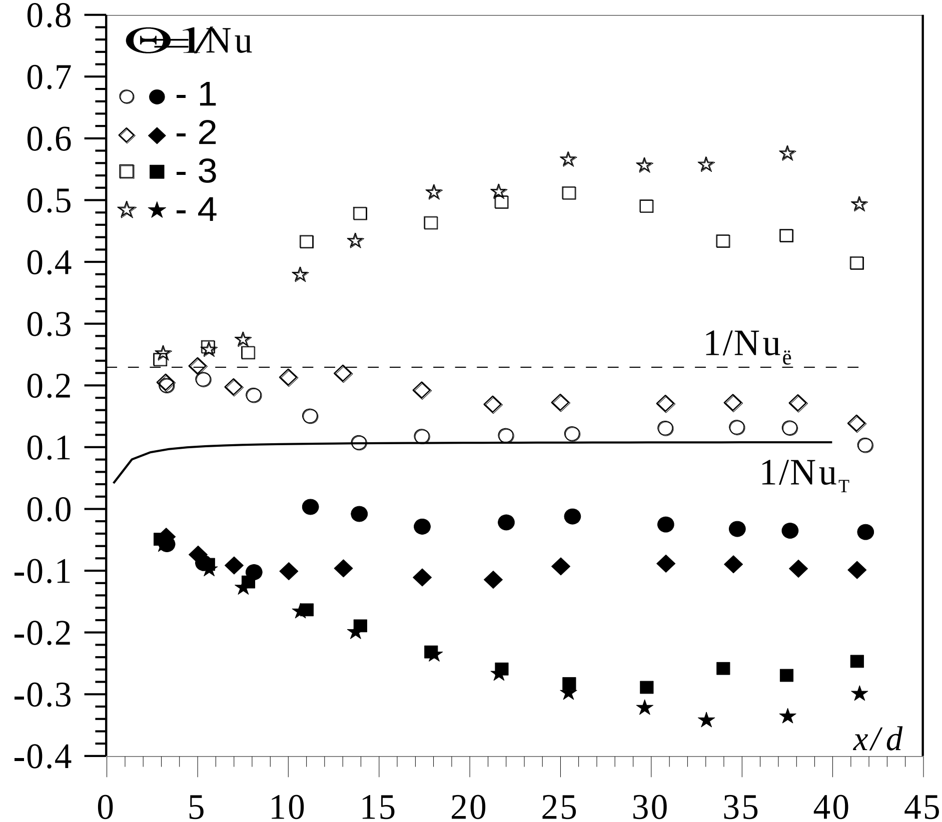


Comparison between DNS and experiment, Ha=300

Non-dimensional temperature top and bottom wall



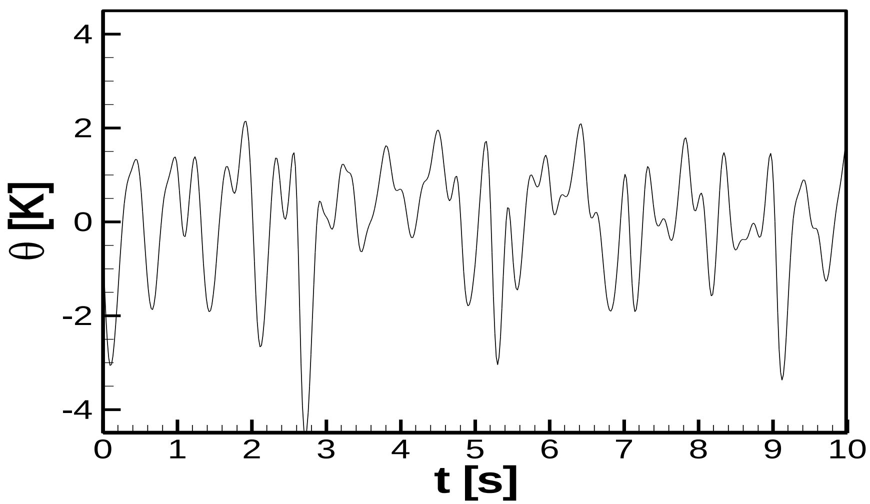
DNS



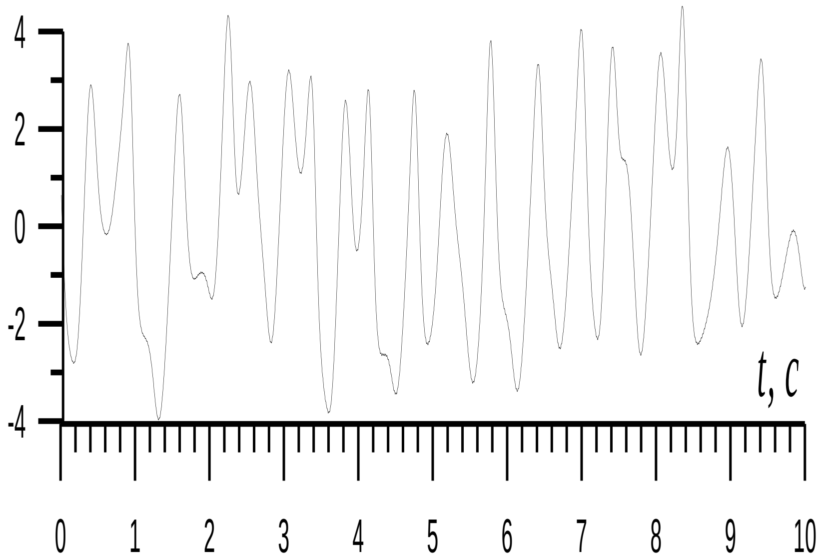
Experimental data (MPEI, JIHT RAS)
1 – Ha=0, 2 – Ha=100, 3 – Ha=300, 4 – Ha=500

Comparison between DNS and experiment, Ha=300

Temperature fluctuations: r=0.7R, bottom, x/d=37



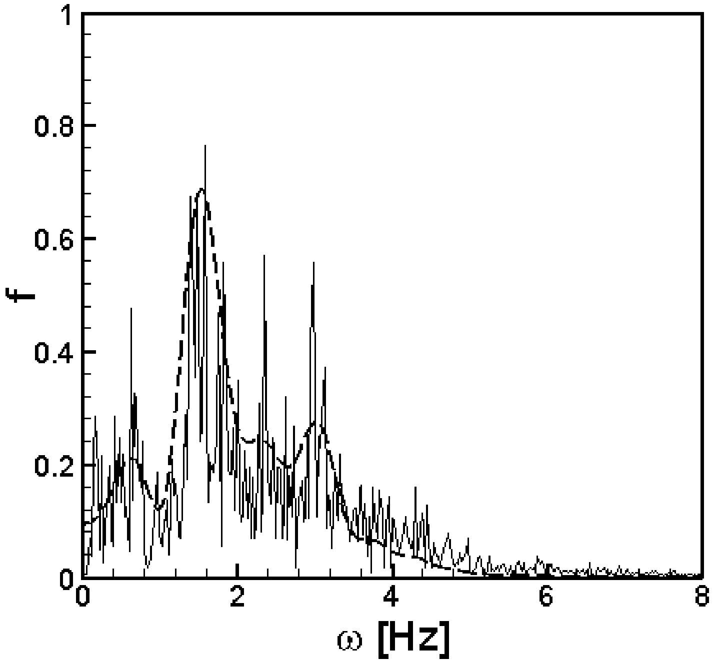
DNS



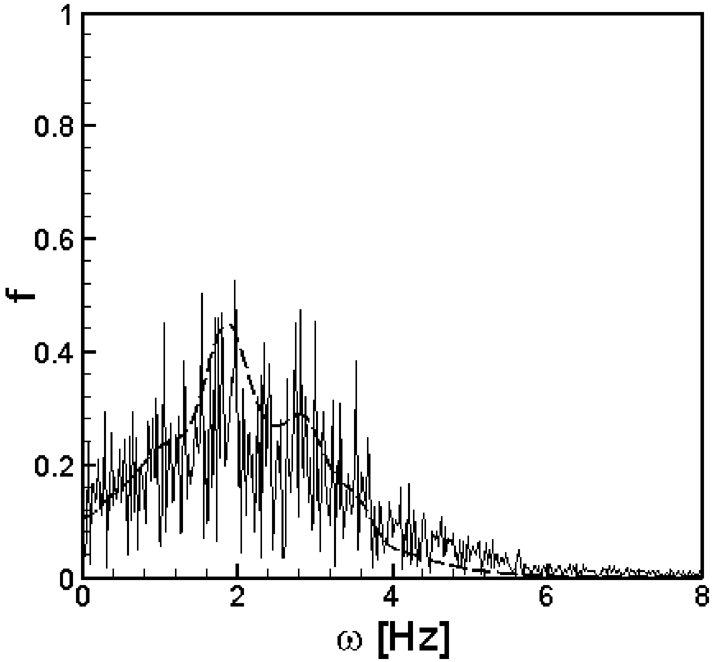
experiment

Comparison between DNS and experiment, Ha=300

Spectrum of temperature fluctuations: r=0.7R, bottom, x/d=37



DNS



experiment

Conclusions:

Temperature fluctuations observed in experiments at Ha=300 (but not at Ha=100) are explained by reorientation of thermal convection rolls so that their axes are parallel to the magnetic field

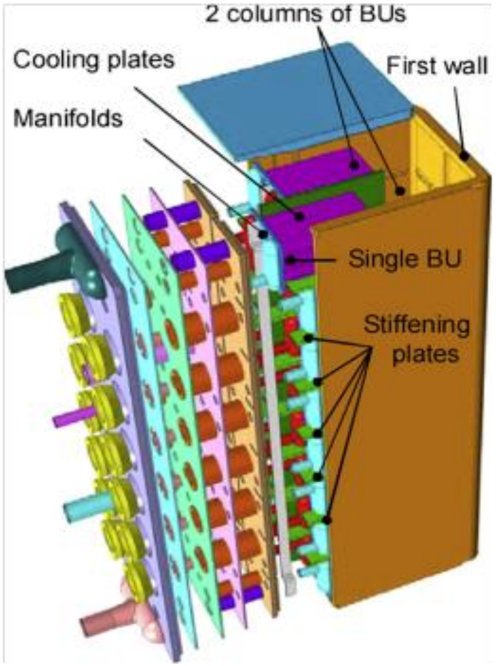
This type of convection is detected in linear stability analysis and confirmed in a large-scale DNS

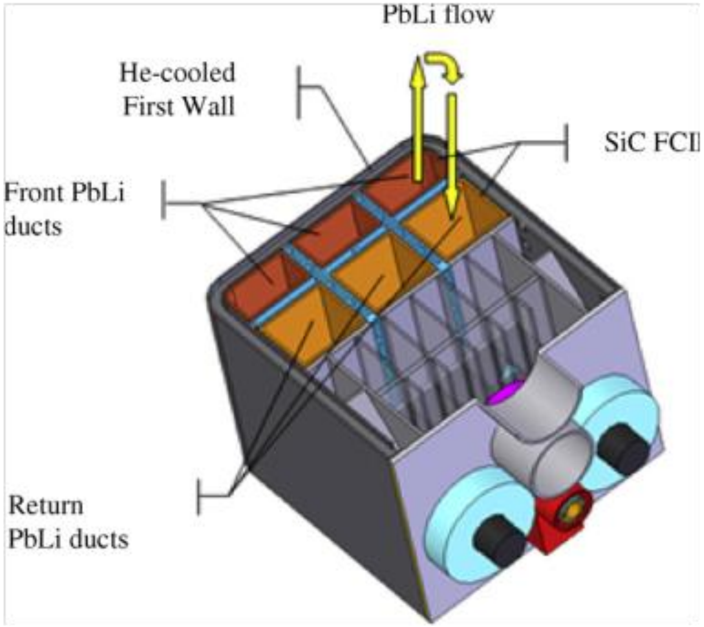
The results of numerical model are in good quantitative agreement with experimental results

Good numerical resolution of Hartmann layers is critical for capturing the flow behavior

Possibility of strong temperature fluctuations caused by convection has to be considered in design of MHD liquid metal heat exchangers

Implications for LM blanket design





HCLL

DCLL

$$Gr \sim 10^{10} - 10^{12}$$

$$Ha \sim 10^{4}$$