On the Importance of Adaptivity for Higher-order Discretizations in Aerospace Applications

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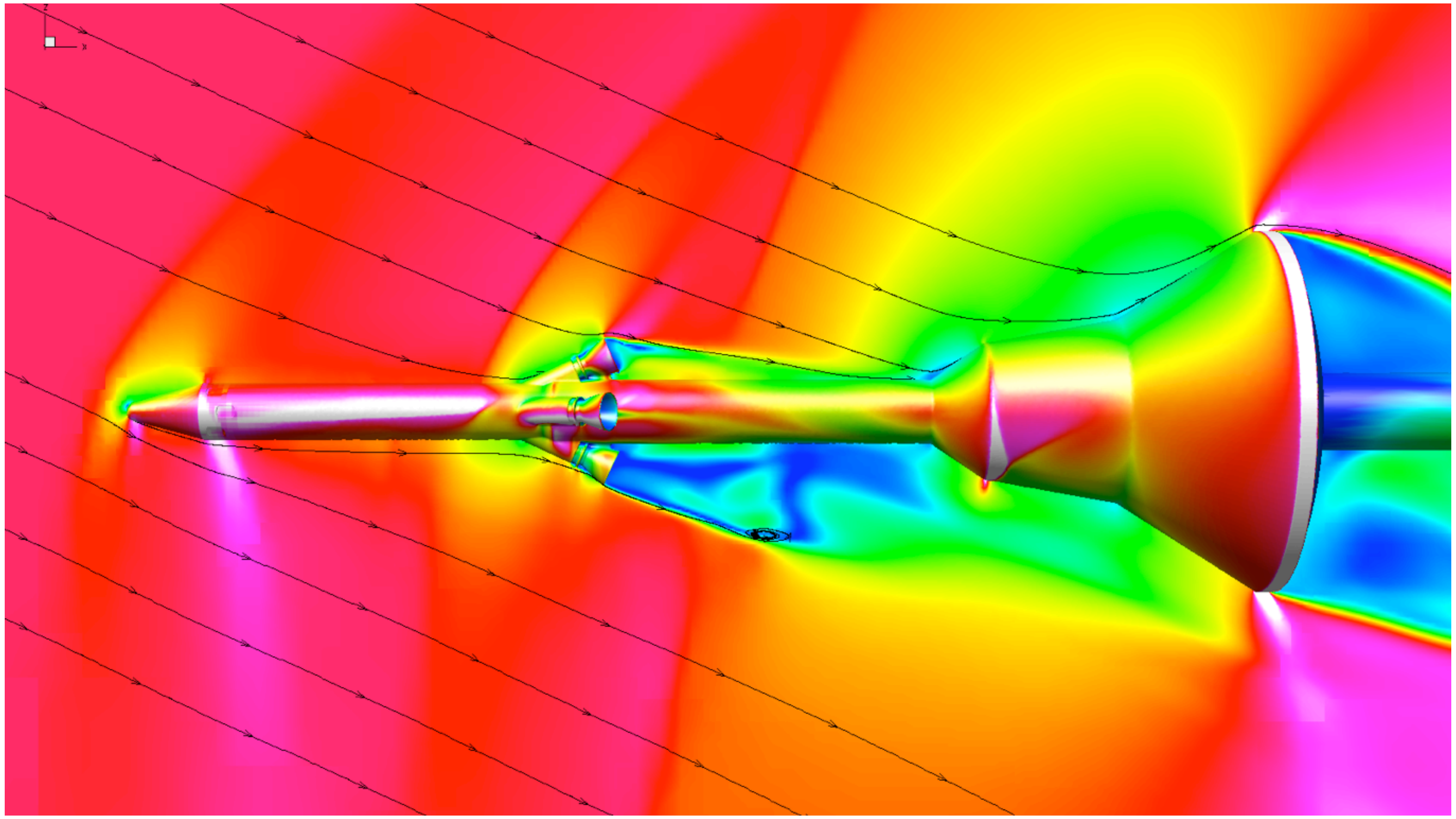
Motivation

Mach number distribution



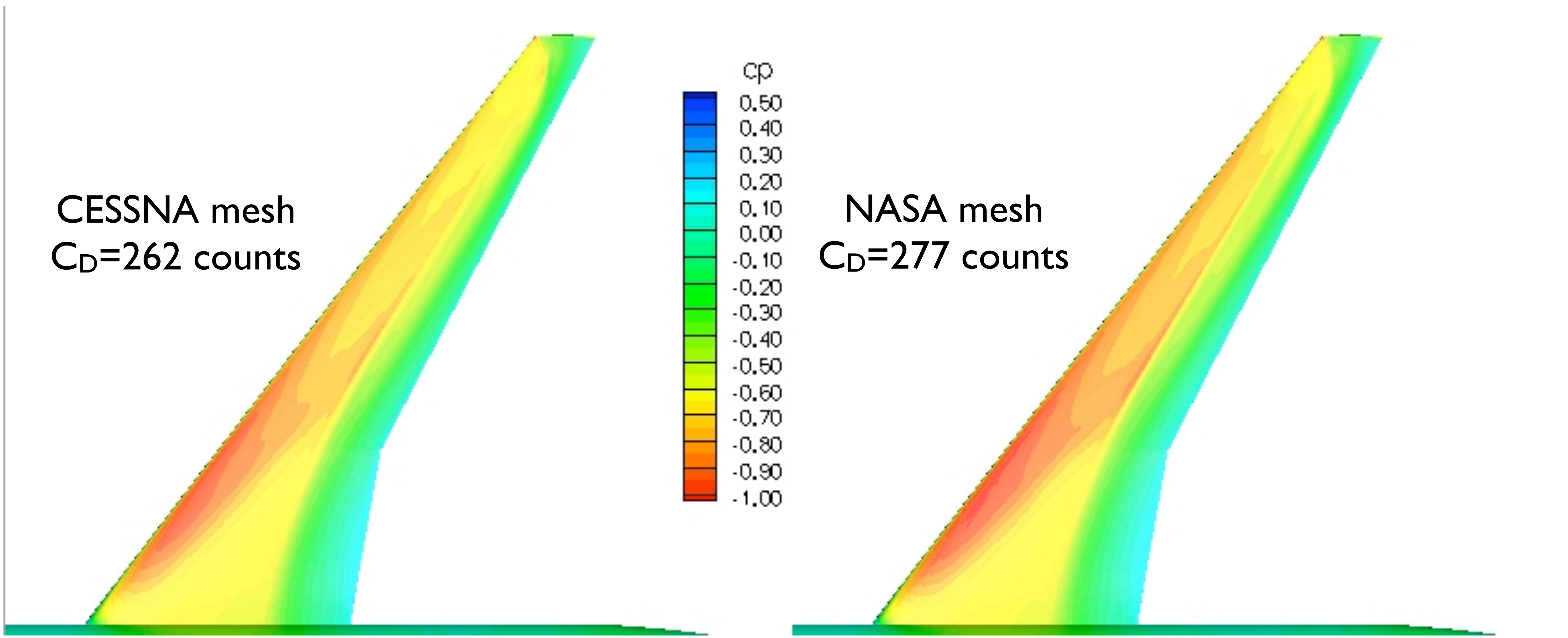
CFD solutions for complex problems have been made possible by increases in computational and algorithmic power

The Launch Abort Vehicle simulation above took 30 minutes to complete on 16 CPUs (Nemec et al, 2008, CART3D group)

Mesh “convergence” comparison

(Chaffin, 2009, DPW4 Presentation)

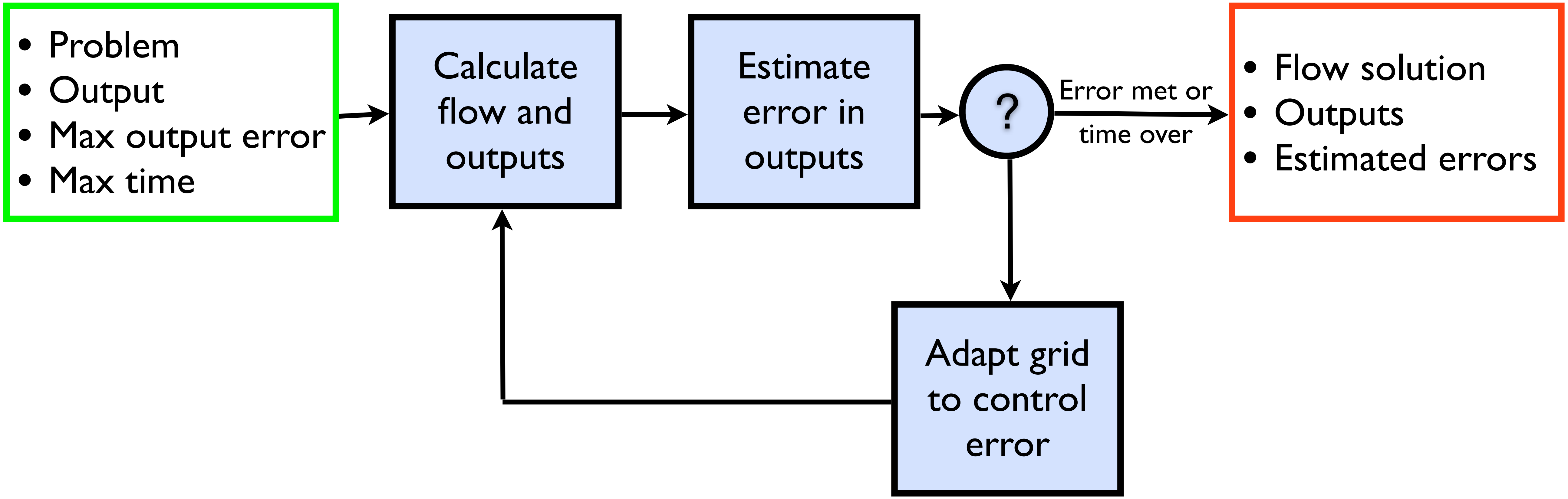
Same CFD code (NSU3D) run on two “best practice” meshes of about 40 million nodes



Which solution is most realistic?

How would this level of uncertainty be detected in practice?

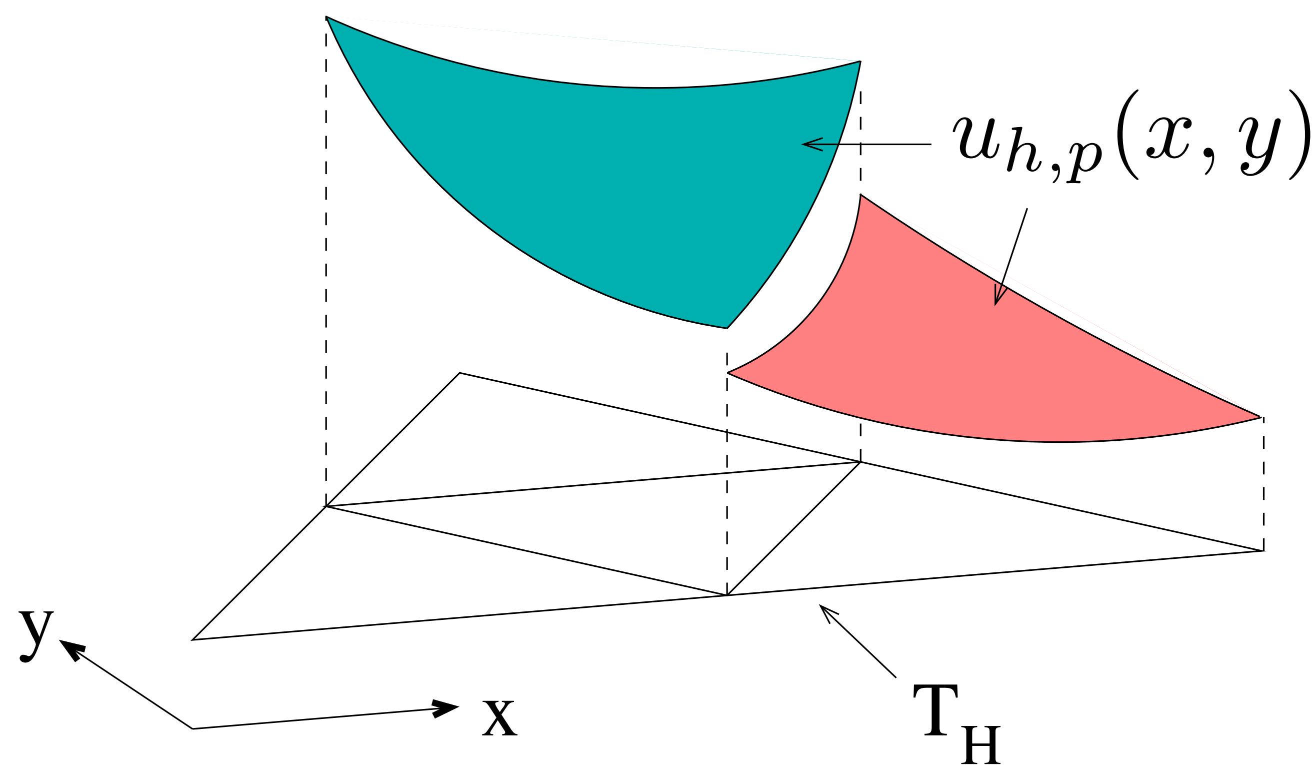
Output-based adaptation



Objective: Increase reliability of CFD by estimating and autonomously controlling error in outputs (e.g. drag or lift)

Higher-order Discontinuous Galerkin Finite Element Method (DGFEM)

- Approximations are degree p polynomials within elements but discontinuous between elements



DGFEM approximation: Find $u_{h,p} \in \mathcal{V}_{h,p}$ such that

$$\mathcal{R}_{h,p}(u_{h,p}, v_{h,p}) = 0, \qquad \forall v_{h,p} \in \mathcal{V}_{h,p}$$

Why higher order for CFD?

- Higher-order methods known to be more efficient than lower-order for problems with smooth flows
- Aerospace flows typically have limited smoothness
- Can higher-order methods be beneficial in aerospace applications?
- Adaptation key to realizing benefits of higher-order discretization on practical problems

Outputs & adjoints

- $\mathcal{J}_{h,p}(u_{h,p})$ is an output such as lift, drag, efficiency, etc.

- Consider a (infinitessimal) perturbation to the residual such that,

$$\mathcal{R}_{h,p}(u_{h,p} + \delta u_{h,p}, v_{h,p}) + (\delta r, v_{h,p}) = 0$$

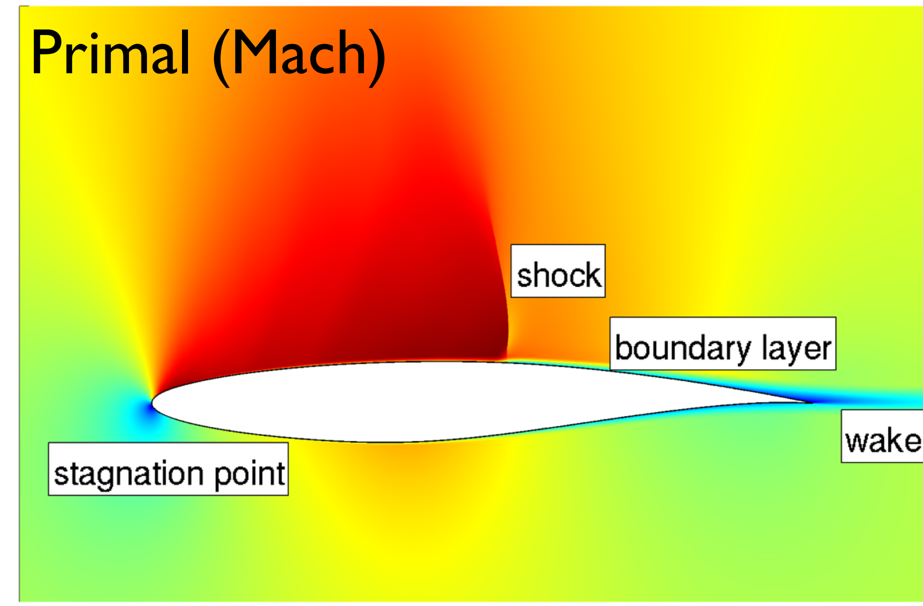
- The adjoint $\psi_{h,p} \in \mathcal{V}_{h,p}$ is the sensitivity of the output to a residual perturbation,

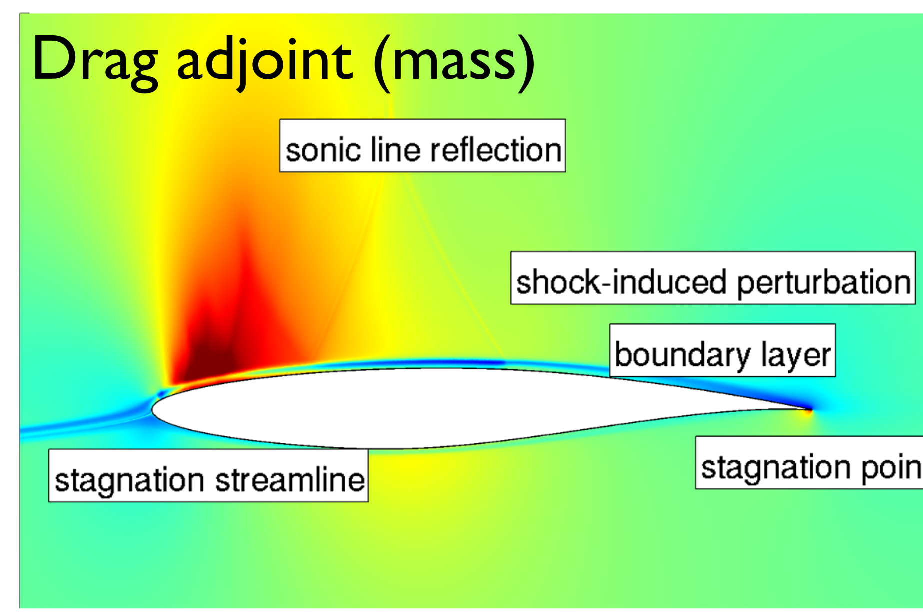
$$\delta \mathcal{J}_{h,p} \equiv (\delta r, \psi_{h,p})$$

- Interpretation: adjoint is transfer function between δr and $J_{h,p}$.

- In the infinite-dimensional case, the adjoint satisfies a linear PDE.

Transonic RANS example





Outputs & adjoints

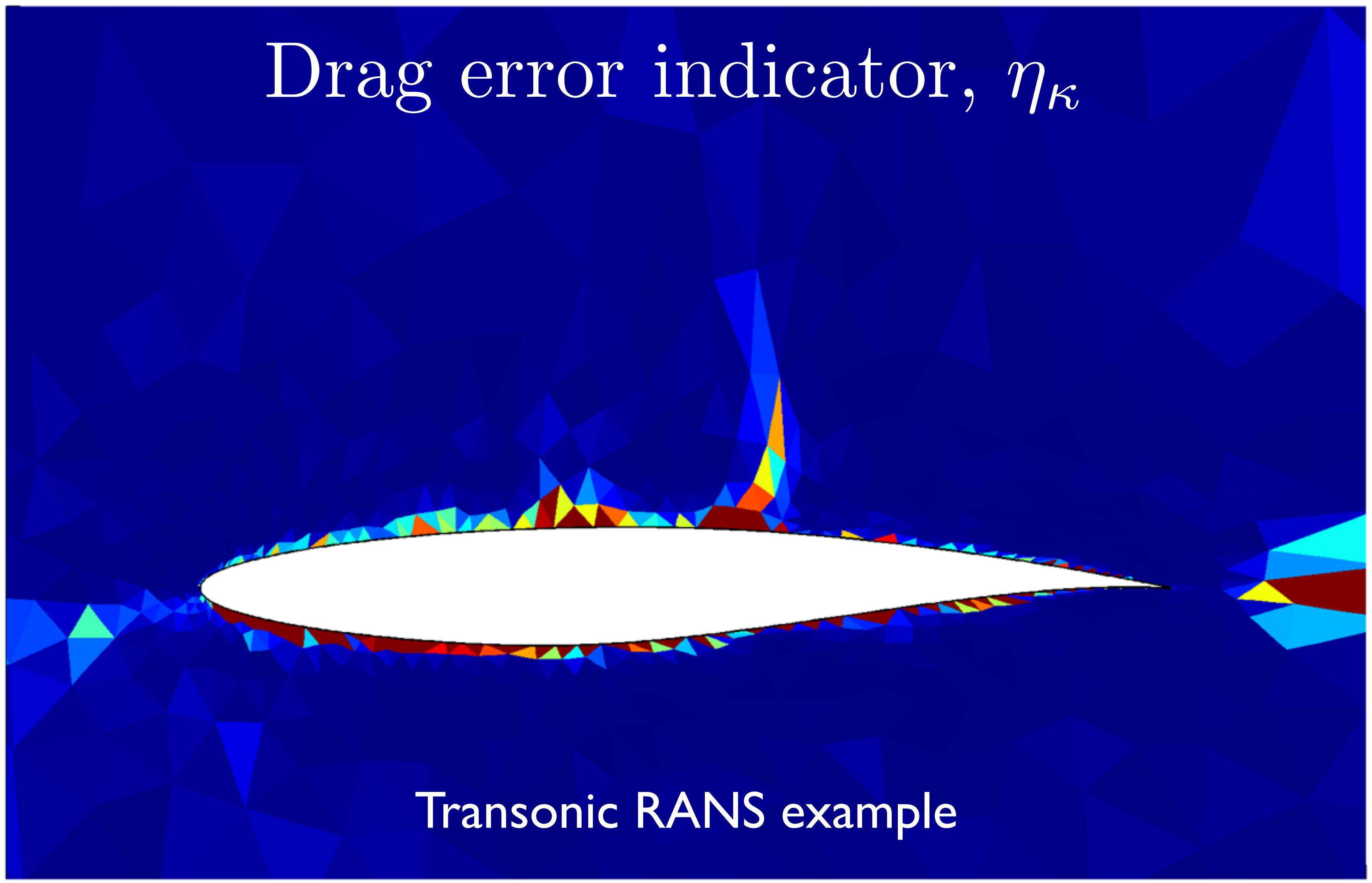
- $\mathcal{J}_{h,p}(u_{h,p})$ is an output such as lift, drag, efficiency, etc.

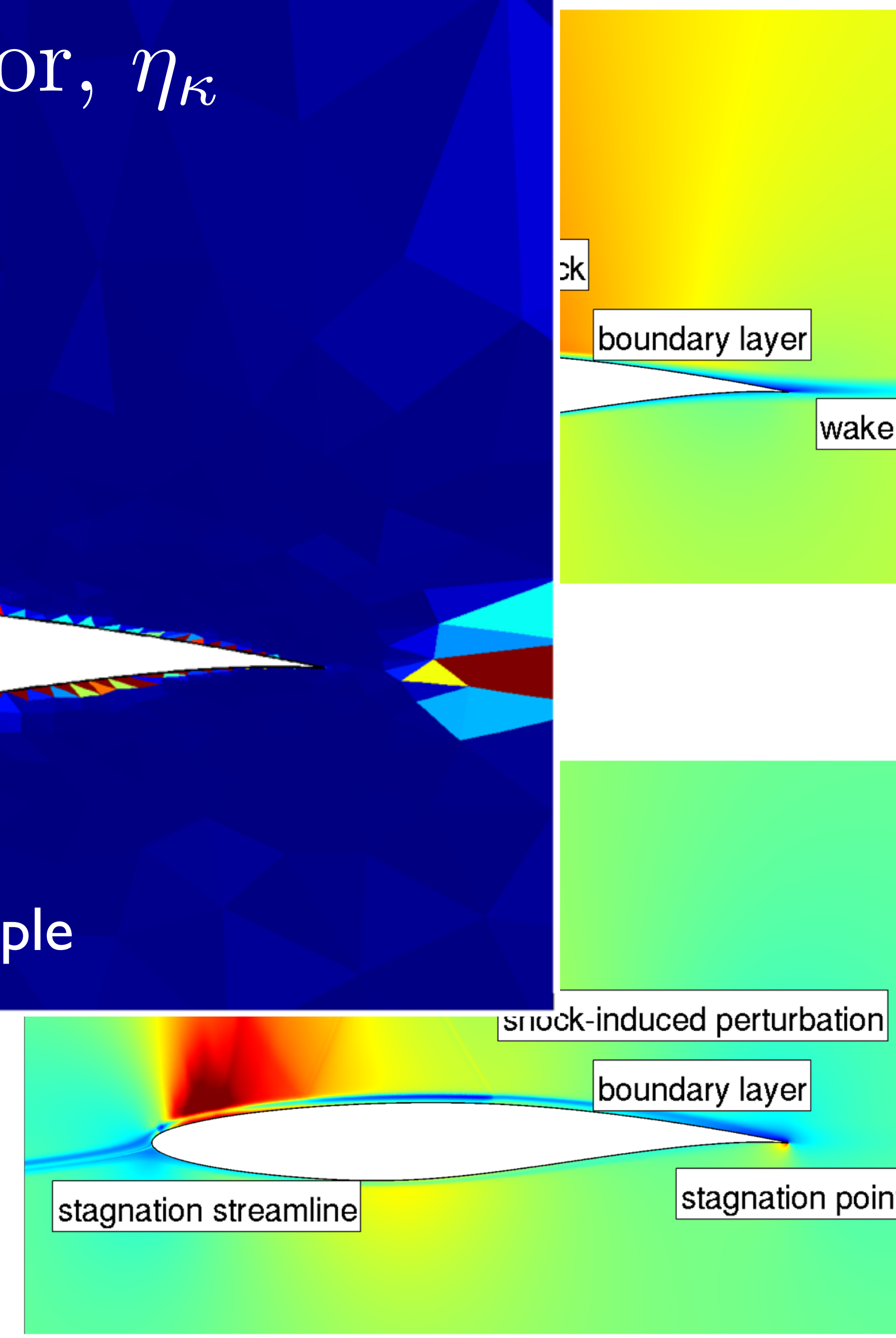
- Consider a
the residual

$\mathcal{R}_{h,p}(u_{h,p}$

- The adjoint
the output t

Transonic RANS example

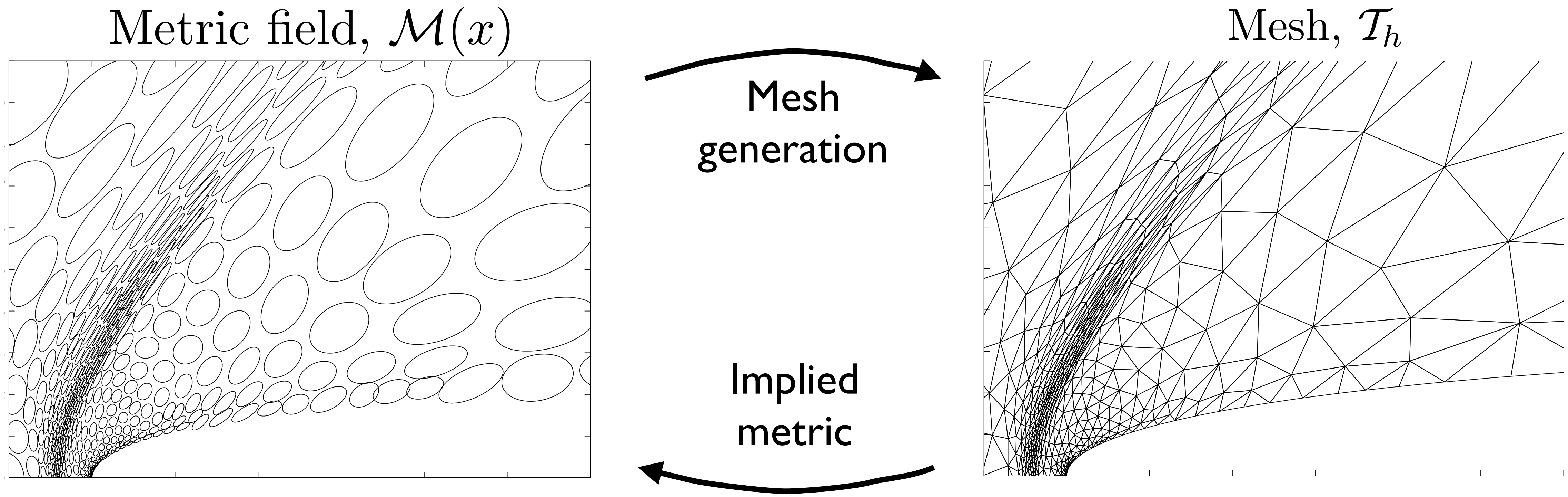




- Interpretation: adjoint is transfer function between δr and $J_{h,p}$.

- In the infinite-dimensional case, the adjoint satisfies a linear PDE.

Continuous Optimization: Mesh-metric Duality



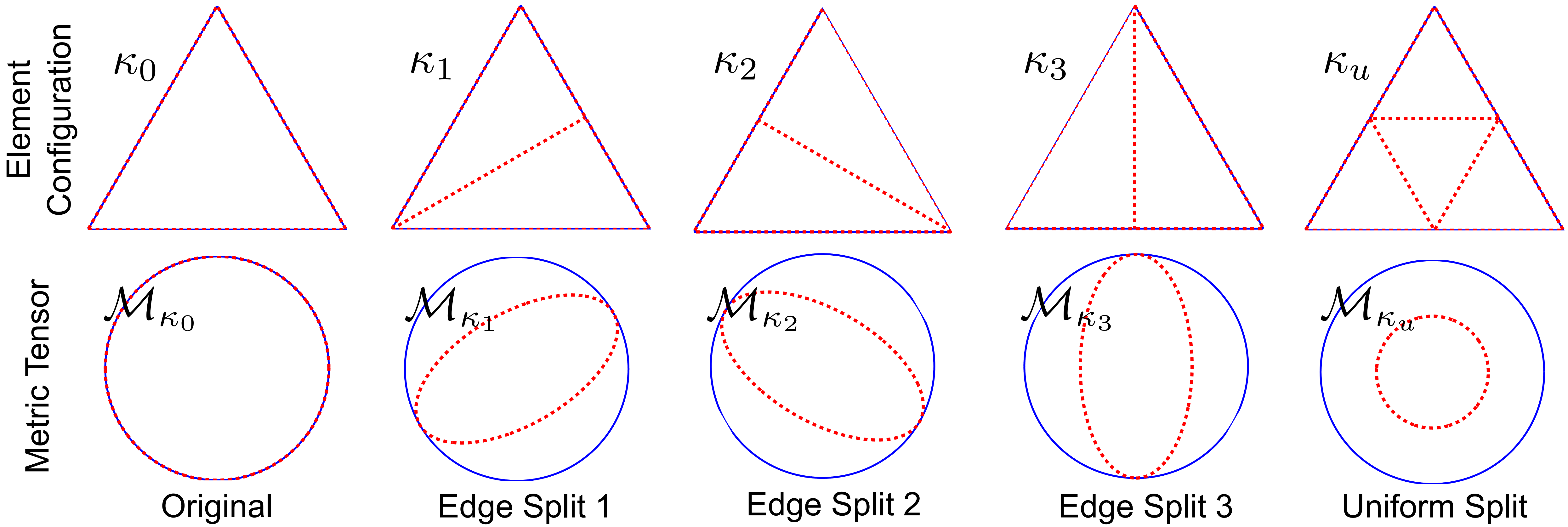
- (Intractable) discrete optimization problem

$$\mathcal{T}_h^* = \arg\inf_{\mathcal{T}_h} \mathcal{E}(\mathcal{T}_h) \quad \text{s.t.} \quad \mathcal{C}(\mathcal{T}_h) = \text{Cost}$$

- Continuous relaxation (Loiselle, 2009)

$$\mathcal{M}^* = \arg\inf_{\mathcal{M}} \mathcal{E}(\mathcal{M}) \quad \text{s.t.} \quad \mathcal{C}(\mathcal{M}) = \text{Cost}$$

Local sampling



- For each configuration, solve local problems keeping states outside of κ_0 fixed
- Determine error estimate $\eta_{\kappa_i} = R_{h,p}(u_{h,p}^{\kappa_i}, \psi_{h,p+1}|_{\kappa_0})$
- Produces a set of pairs, $\{\mathcal{M}_{\kappa_i}, \eta_{\kappa_i}\}$

MOESS Algorithm

(**M**esh **O**ptimization via **E**rror **S**ampling & **S**ynthesis)

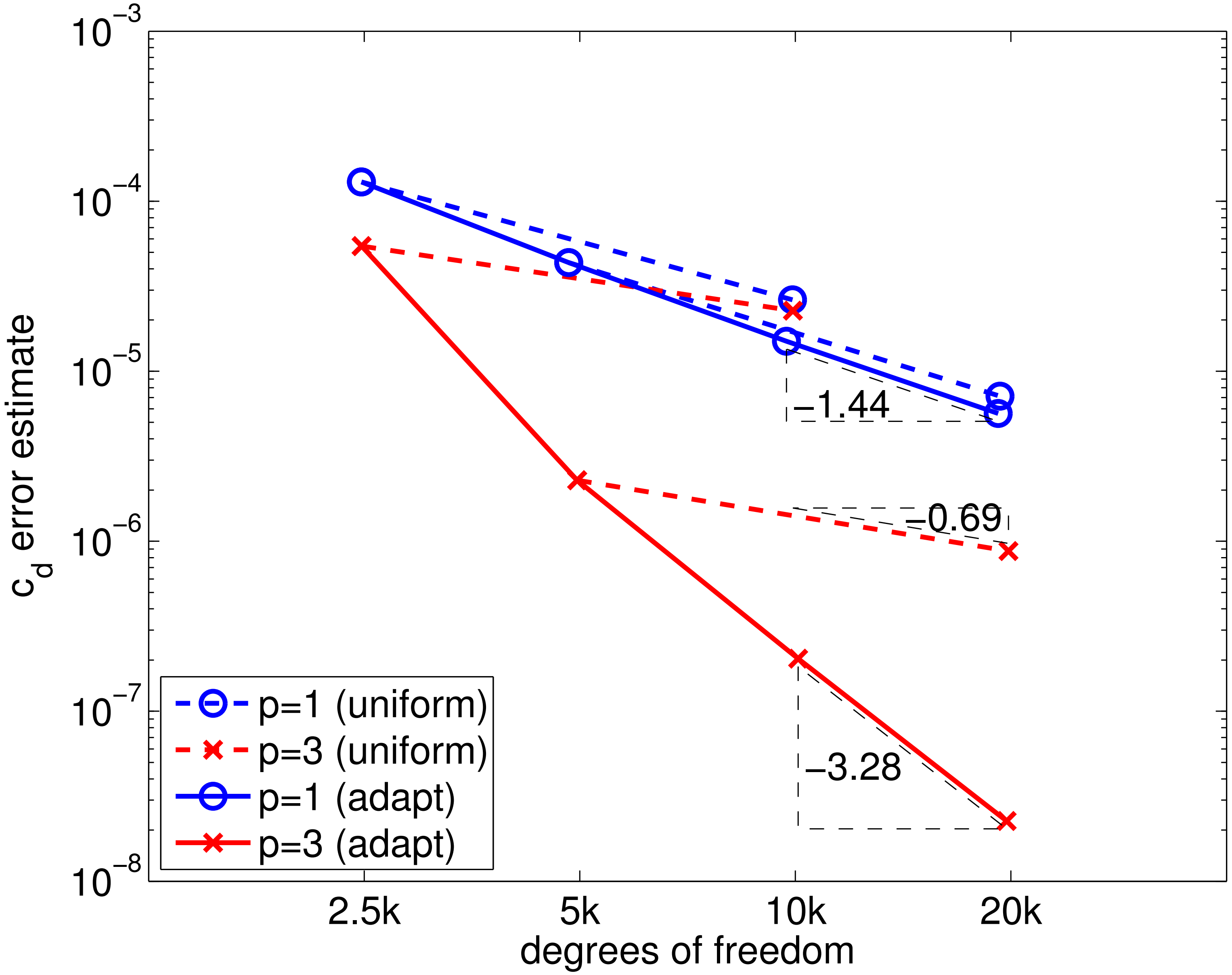
- Solve flow and adjoint on current grid
- Determine error-metric gradients via local sampling
- Utilize steepest descents algorithm to improve metric
- Remesh using improved metric

Yano & Darmofal, 2012

Impact of Adaptation on Higher-order Efficiency

Subsonic Euler

NACA 0012, $M = 0.5$, $\alpha = 2^\circ$



- Adaptive refinement is performed at $2,500$ and $5,000$ DOFs generating "optimal" meshes
- Uniform refinement (each element subdivided into four) is performed
- Uniform refinement compared to adaptive refinement at $10,000$ and $20,000$ DOFs

When singularities are present, adaptive refinement critical to realize benefits of higher order

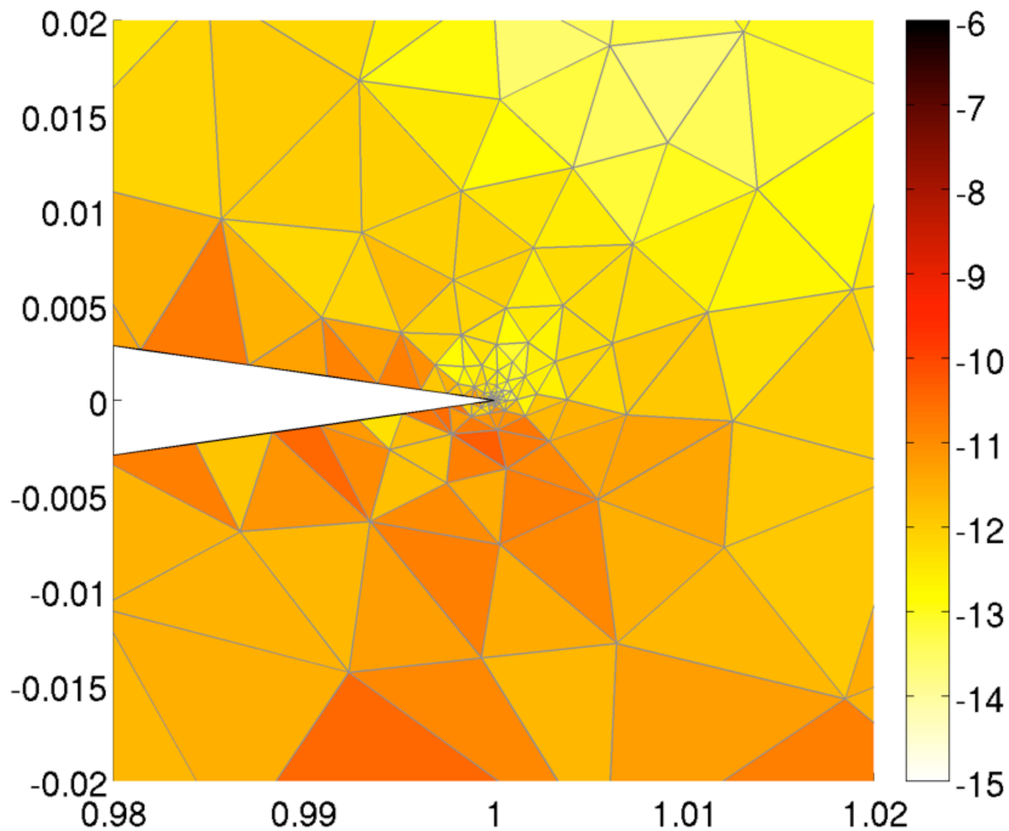
Impact of Adaptation on Higher-order Efficiency

Subsonic Euler

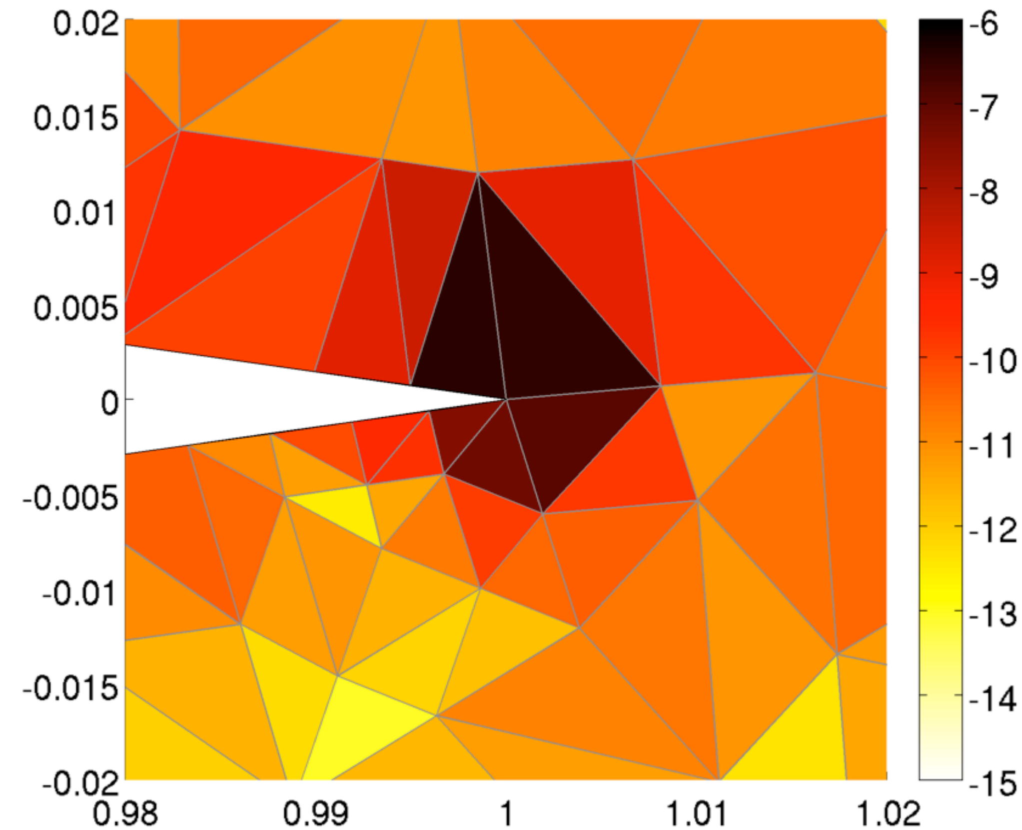
NACA 0012, $M = 0.5$, $\alpha = 2^\circ$

$p = 3$: Distribution of (log) elemental error ($\log_{10} \eta_\kappa$)

Optimized 20K DOF mesh



20K DOF mesh
uniformally refined from 5K DOF mesh

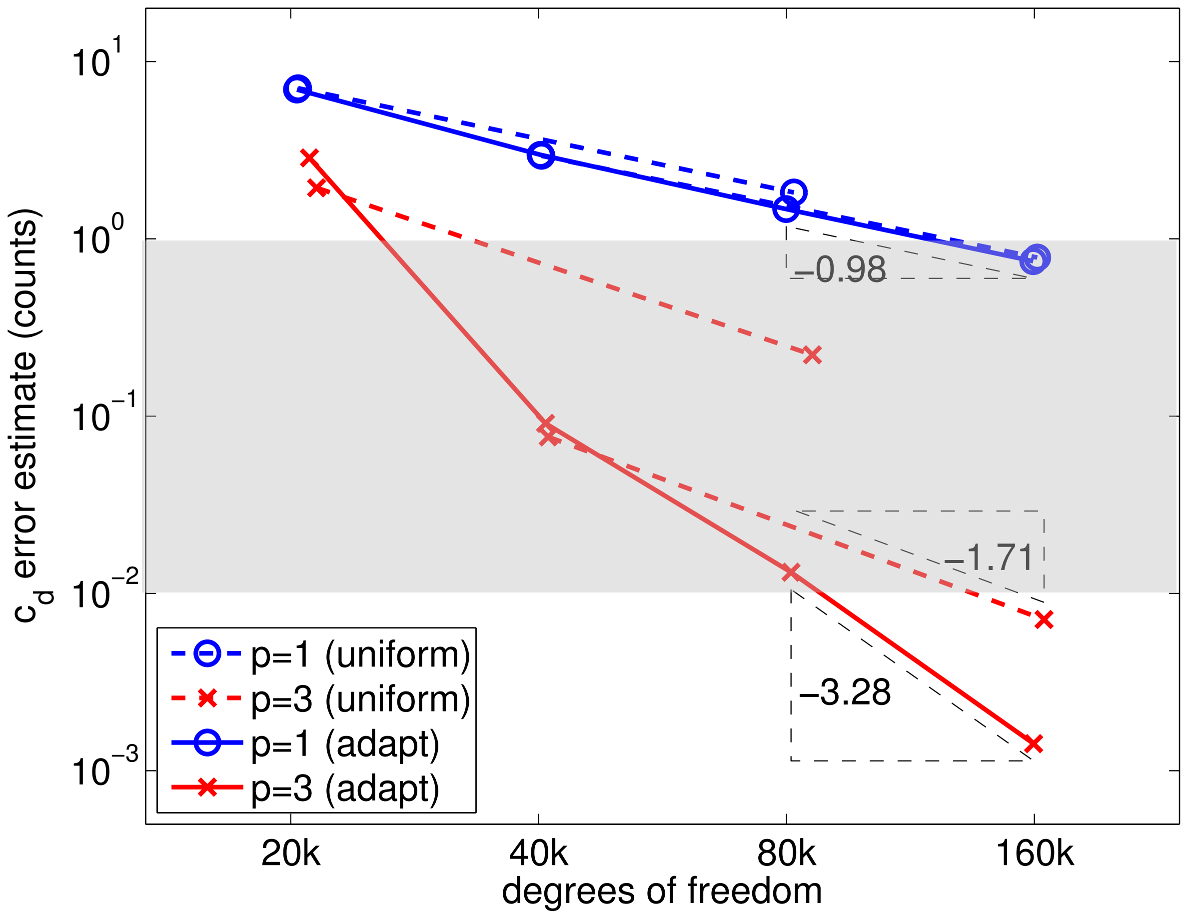


Adaptive mesh resolves trailing edge singularity more effectively

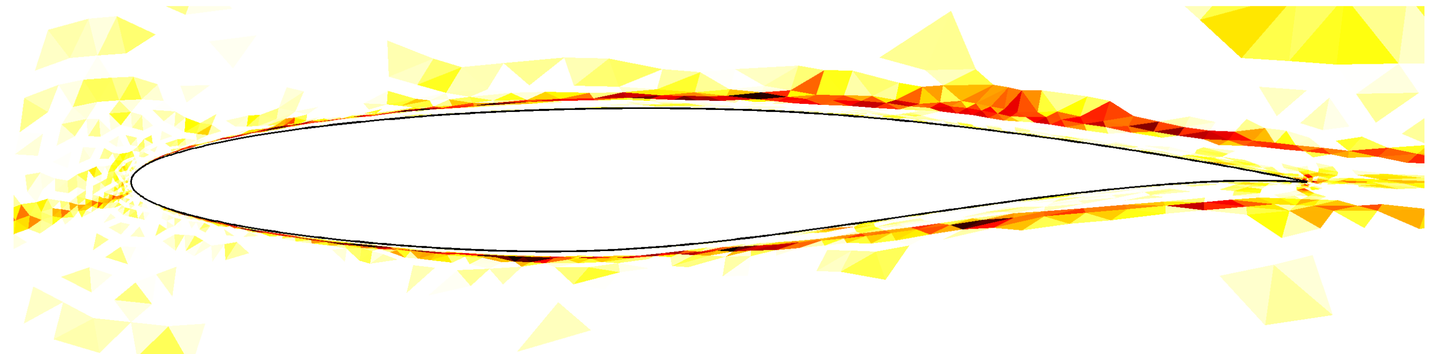
Adaptation, higher-order, and RANS

Subsonic RANS

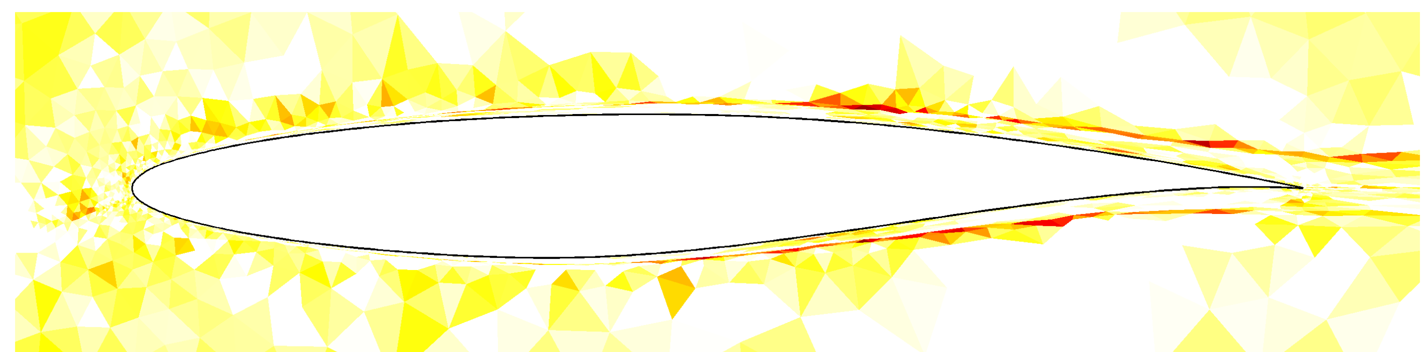
RAE2822, RANS, $M = 0.3$, $Re = 6.5 \times 10^6$, $\alpha = 2.31°$



Error indicator for160K DOF, p=3



Uniform refinement of 40K DOF to 160K DOF



Adaptive 160K DOF

To see full benefit of higher-order approximations, solution irregularities must be controlled: adaptation is critical

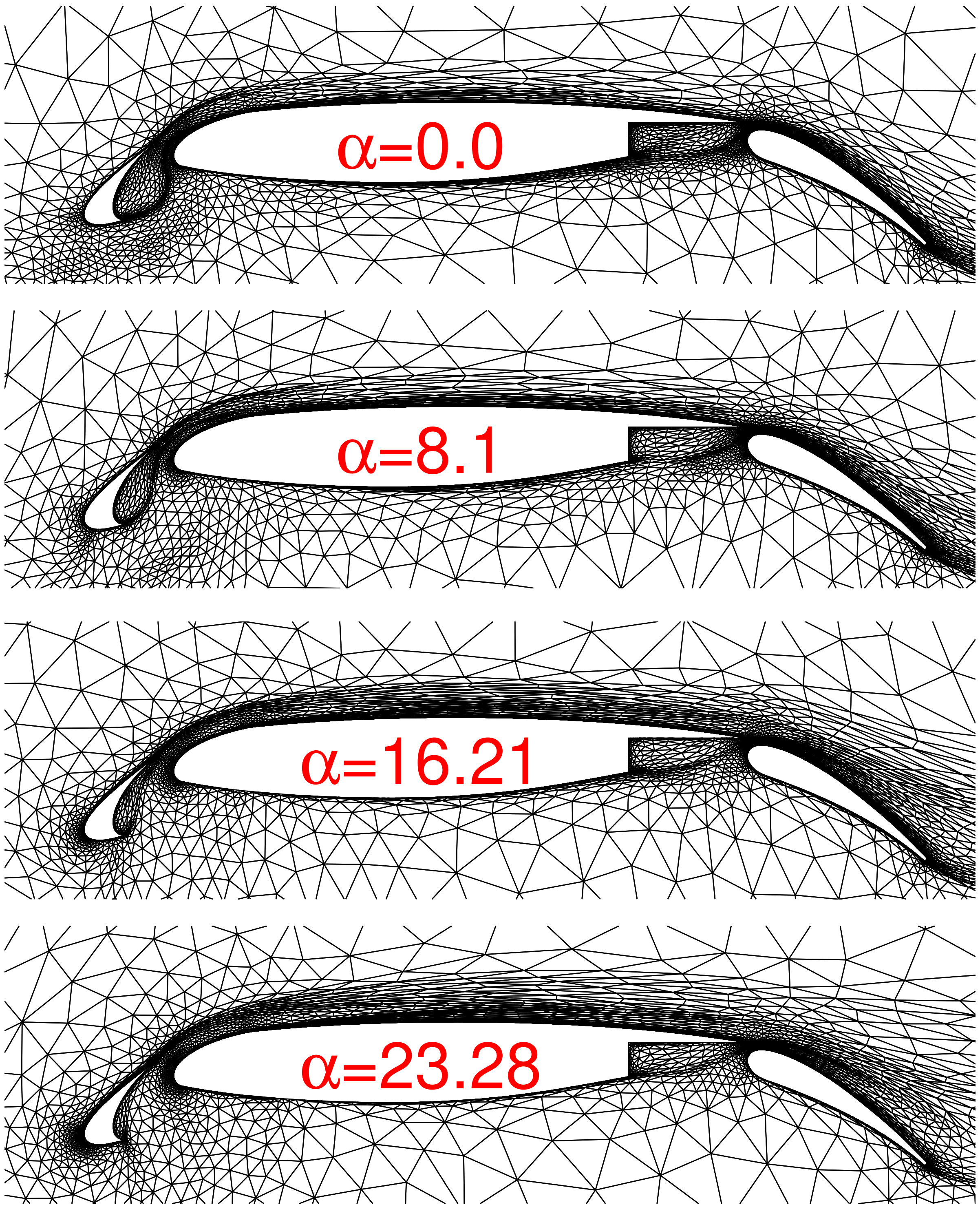
Yano, Modisette, Darmofal 2011

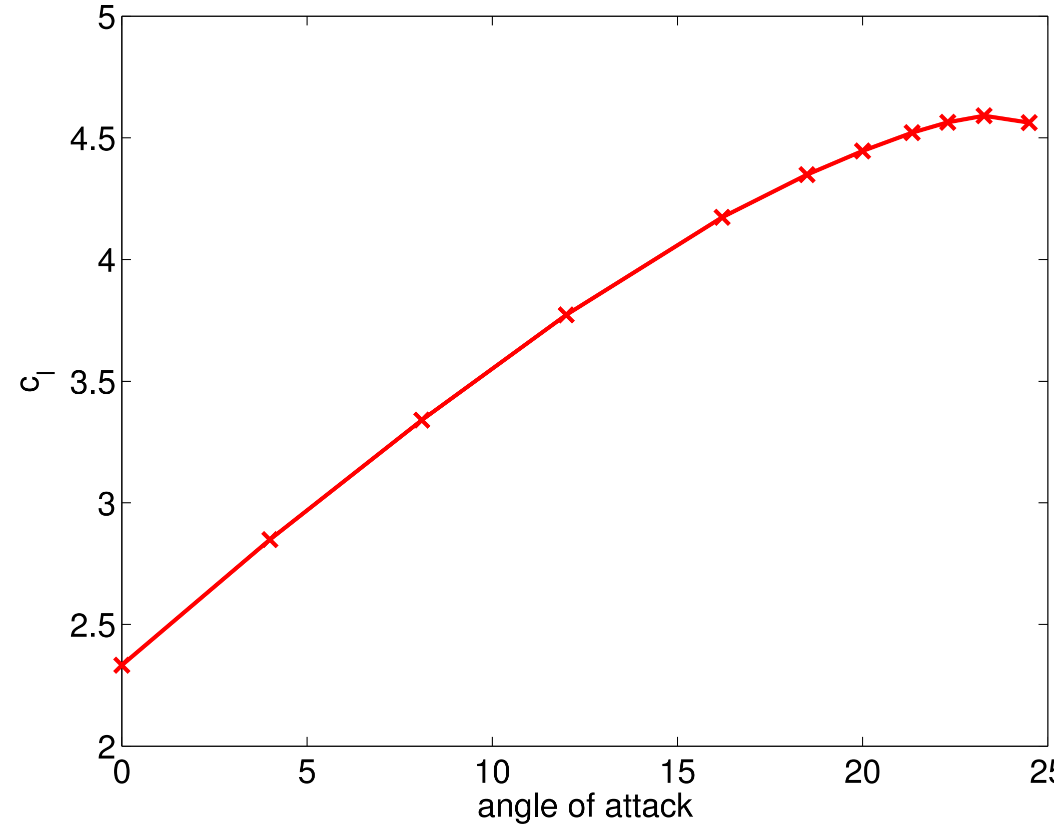
MDA-3 RANS

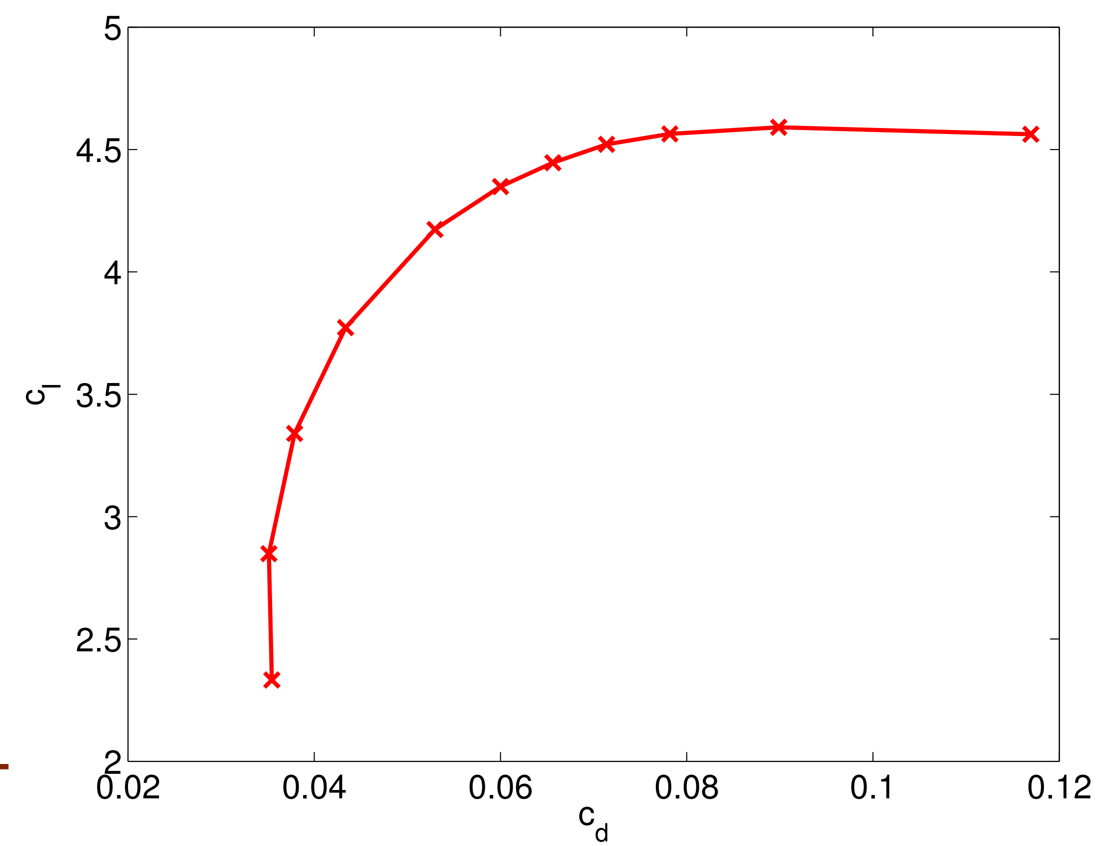
Lift adaptation,
boundary conforming

$M = 0.2,\ Re = 9 \times 10^6,$
$11\ \alpha$ values from 0 to $24.5°$

$p = 2$ results





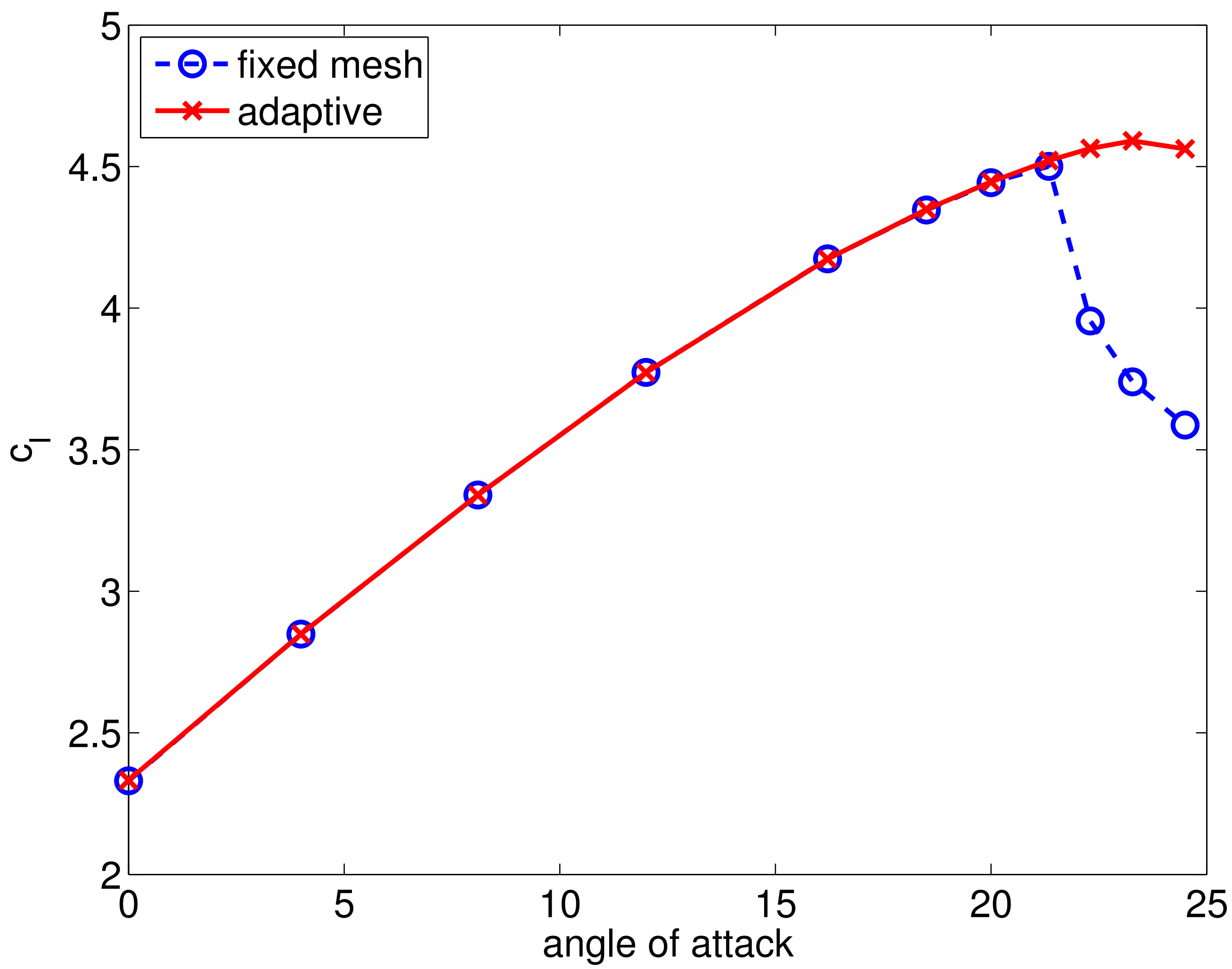


MDA-3 RANS

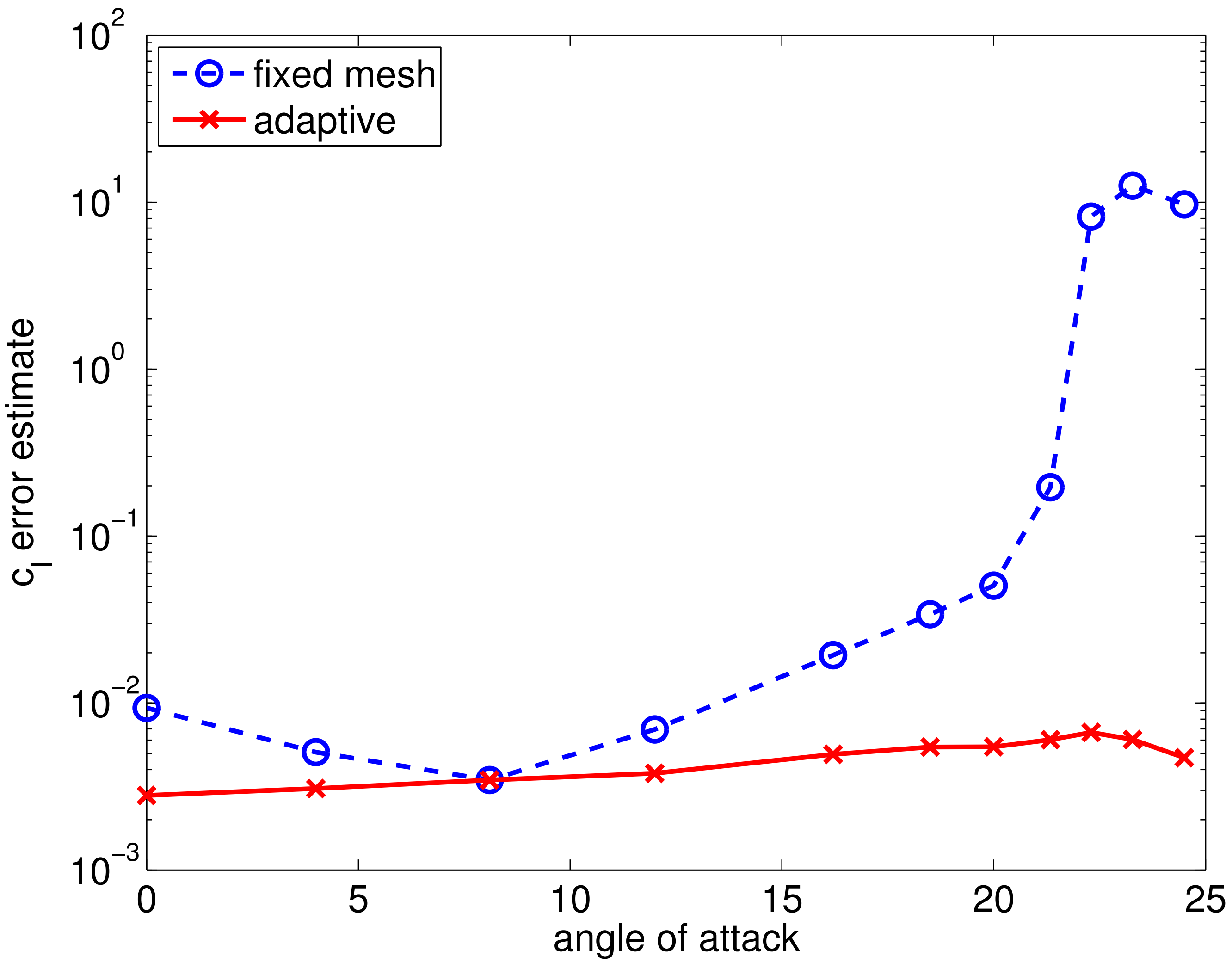
Lift adaptation,
boundary conforming

$M = 0.2,\ Re = 9 \times 10^6,$
$11\ \alpha$ values from 0 to 24.5°

- An appropriate mesh is critical (and subtle)
- Consider α sweep using $\alpha = 8.1^\circ$ optimized grid

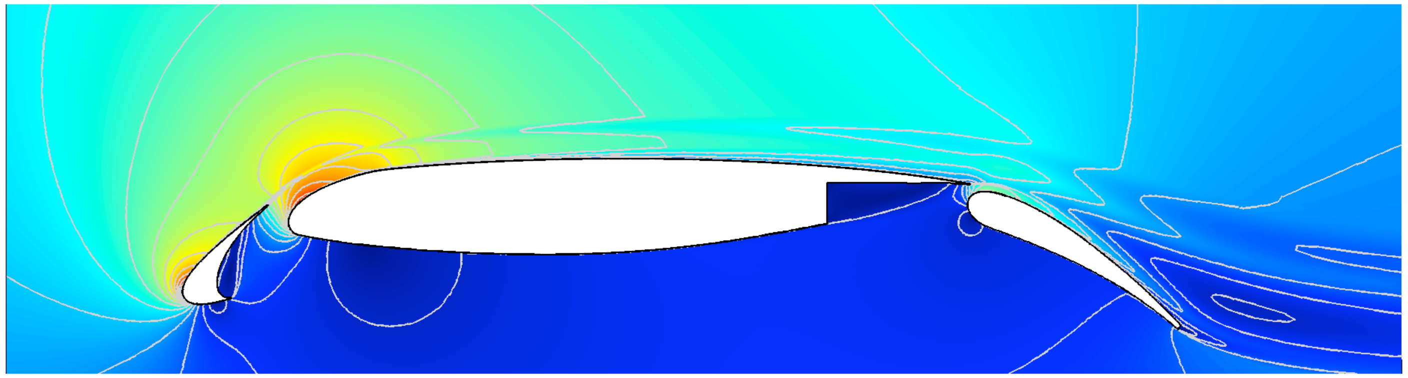


c_l

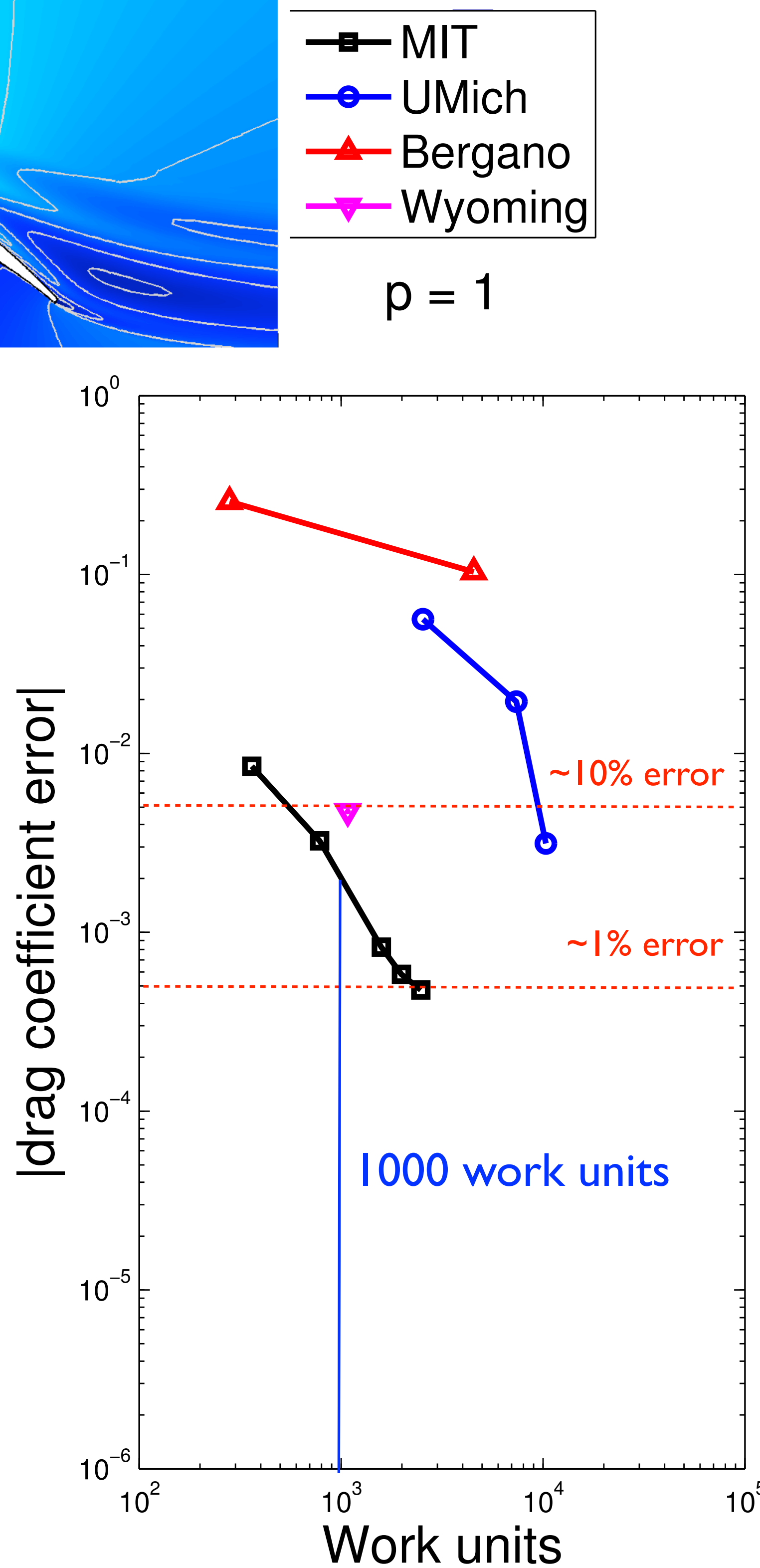


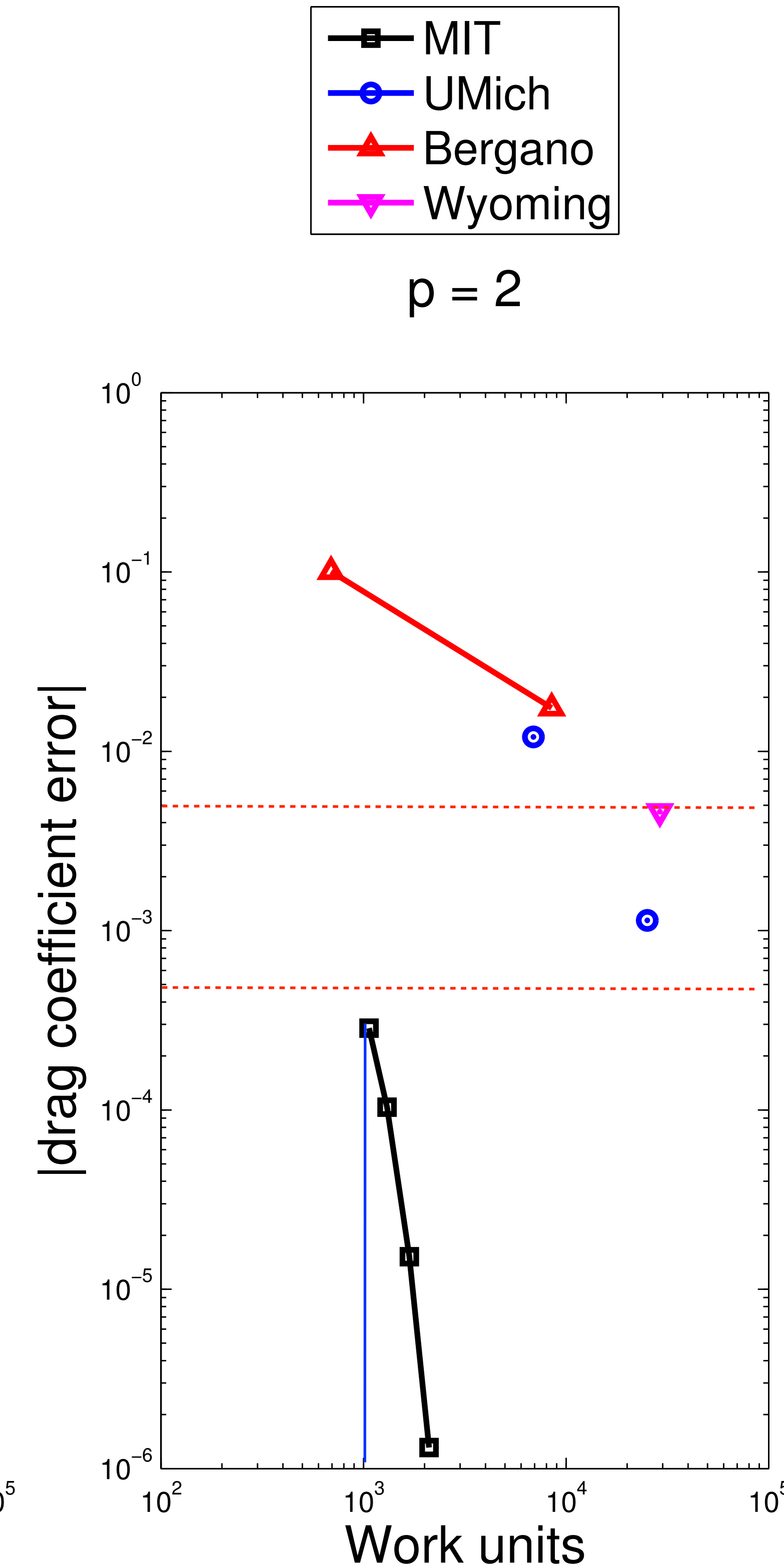
c_l error estimate

Multi-element Airfoil RANS Case: Higher-order Workshop (2012)



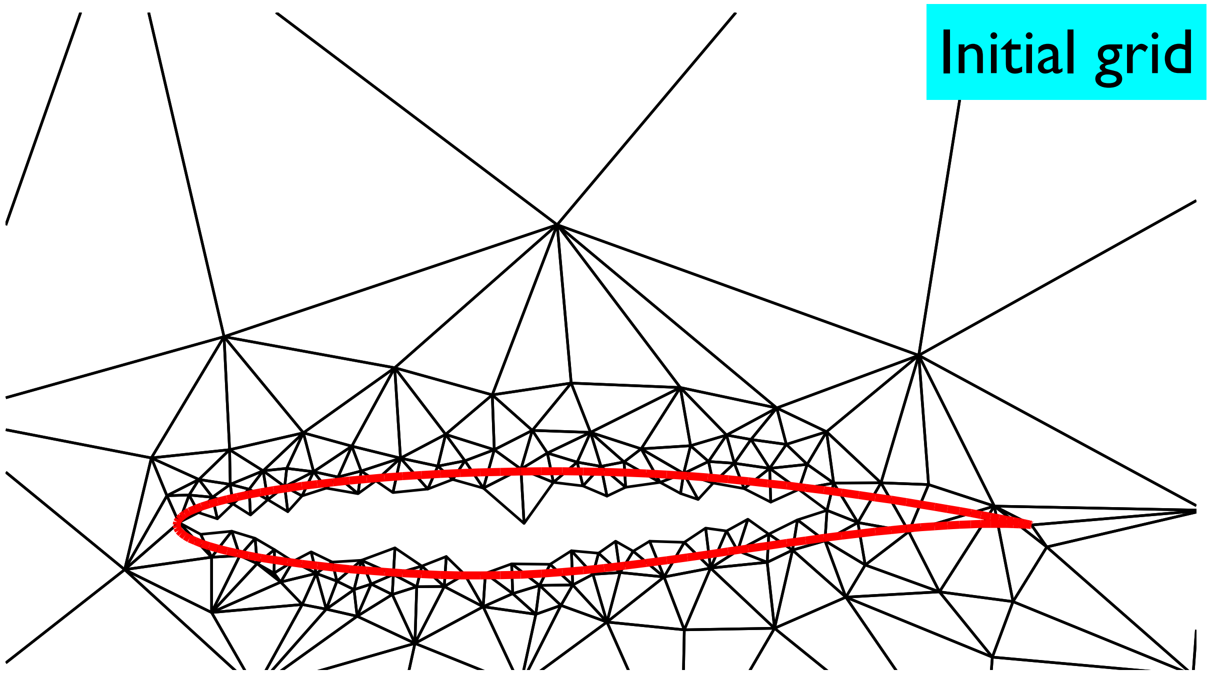
- UMich and Bergamo results use uniform mesh refinement
- Wyoming is an hp adaptive result
- MIT (h-adaptive) results 10-100 x more efficient
- MIT work includes adaptive time from a coarse initial mesh
- With adaptivity, p=2 error drops rapidly with small additional work

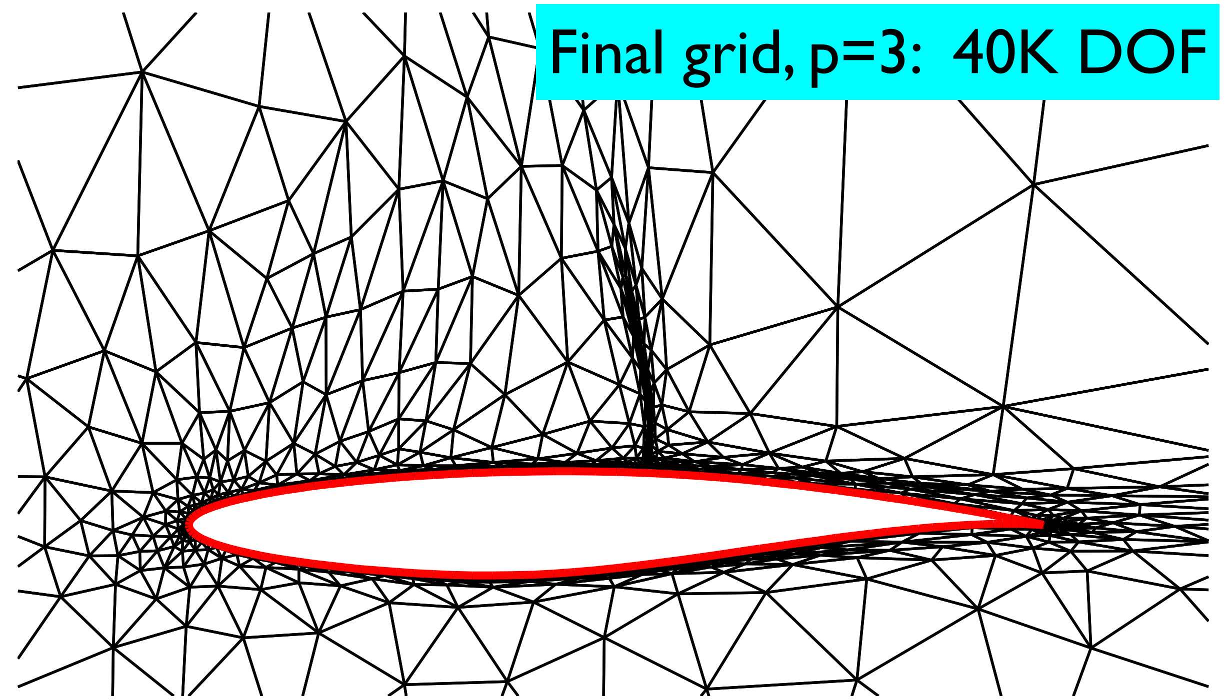


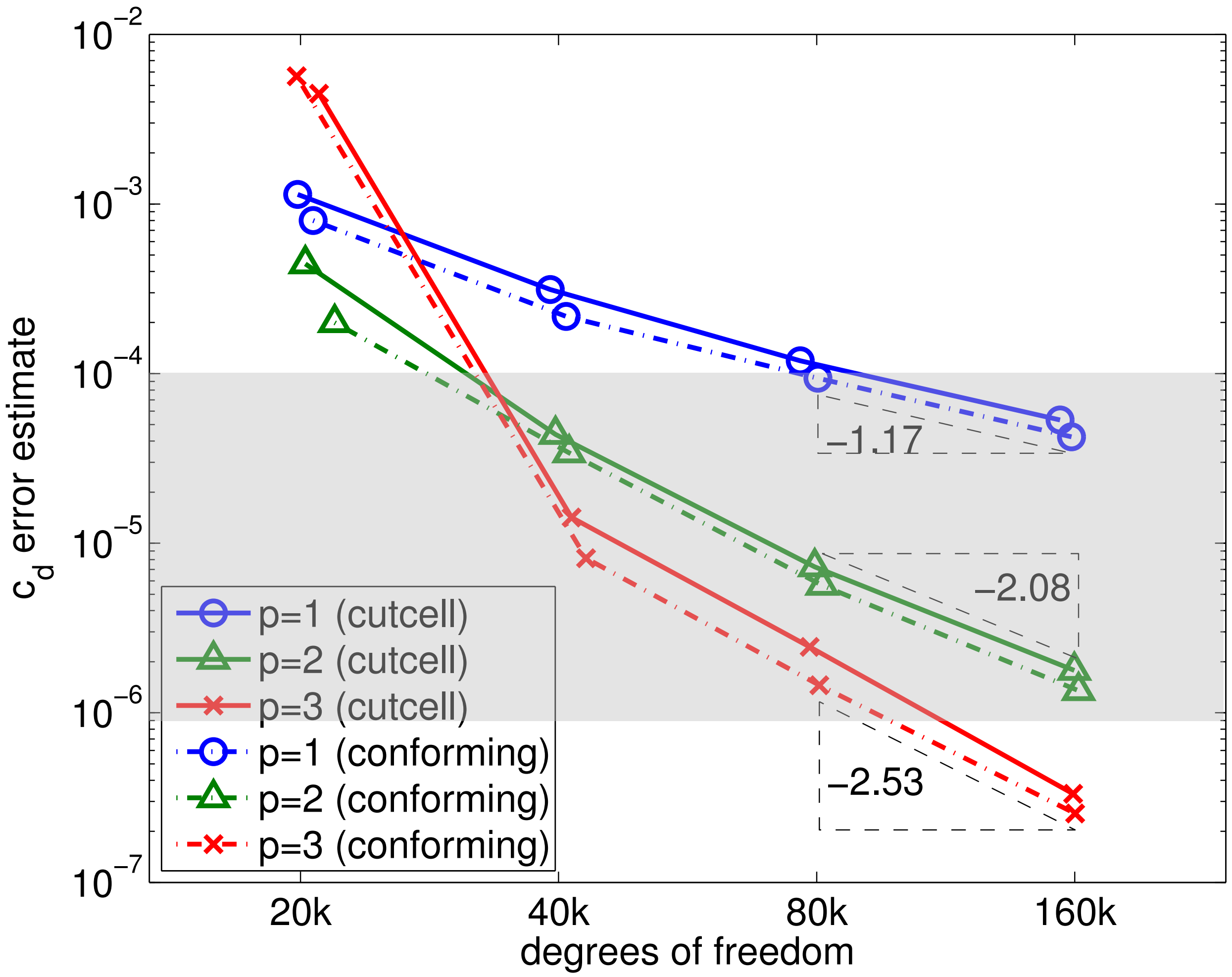


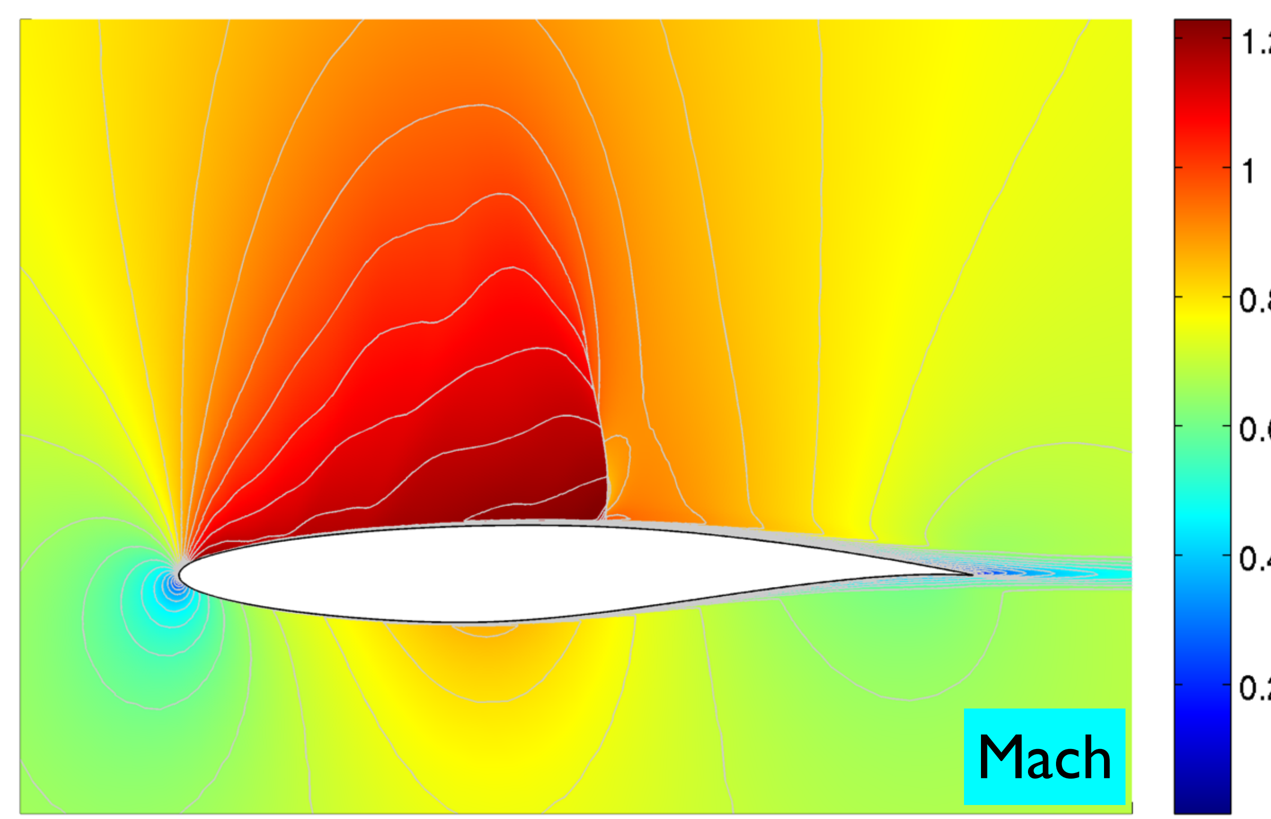
Cut-cell vs. boundary conforming

Transonic RANS   RAE2822, RANS, $M = 0.729$, $Re = 6.5 \times 10^6$, $\alpha = 2.31°$



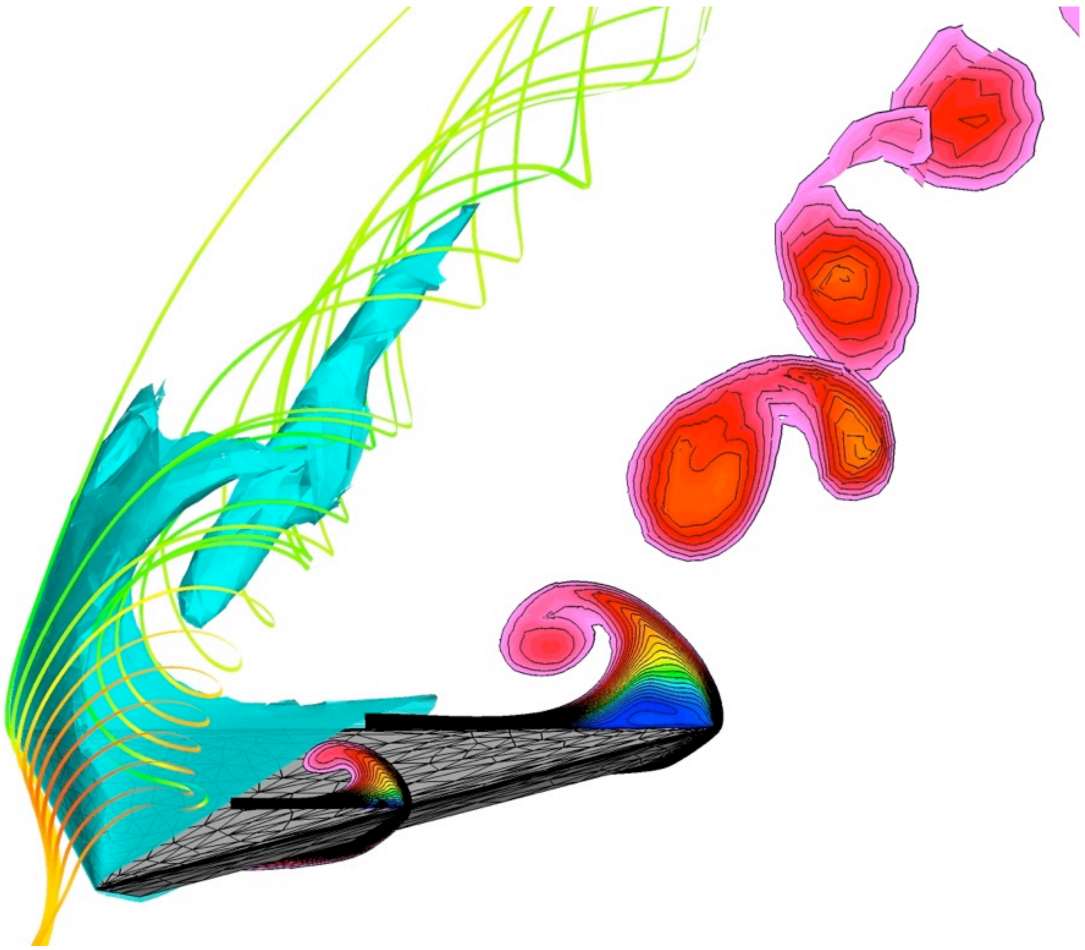


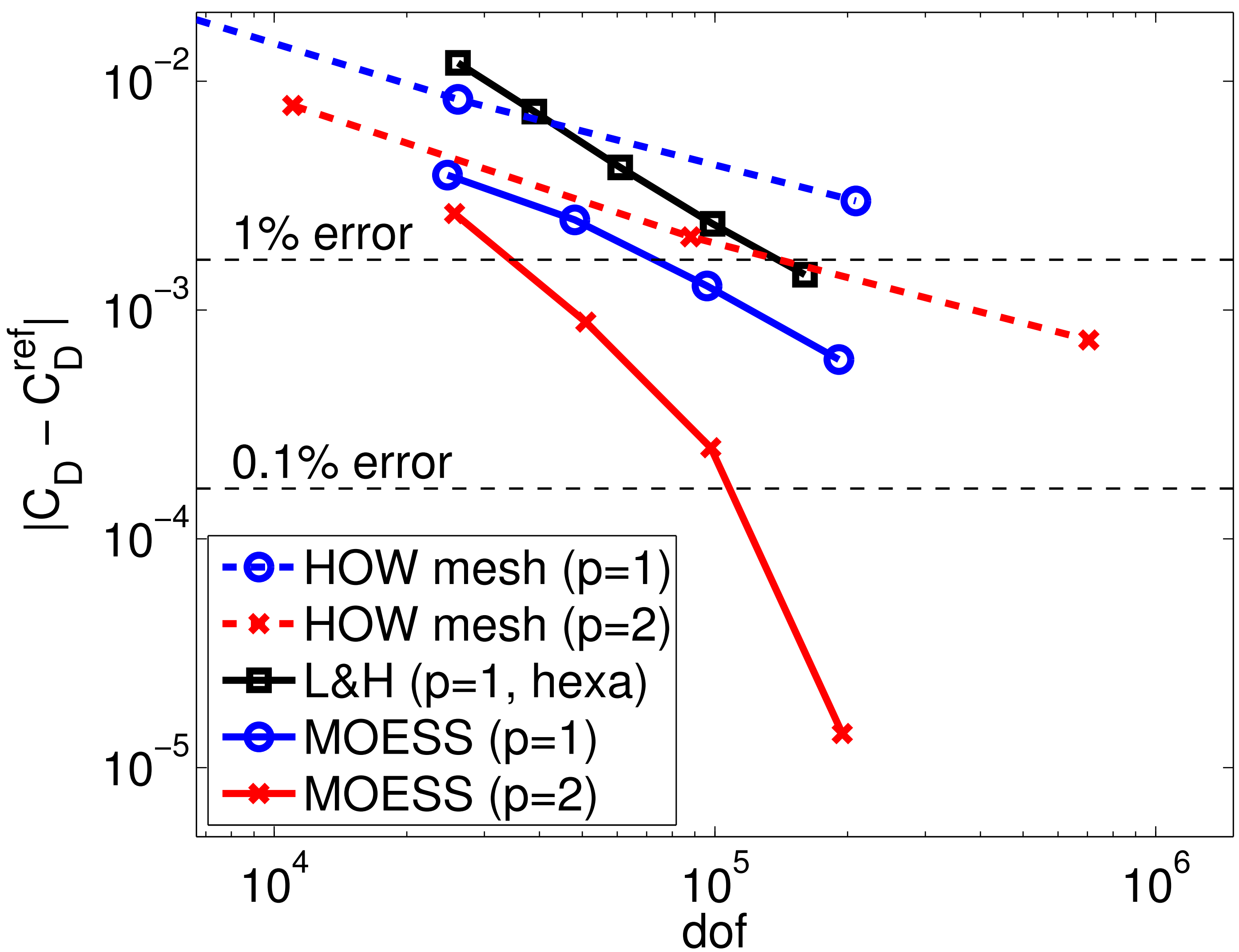




Laminar delta wing example

Higher Order Workshop Test Case 2012: $M = 0.3,\ Re = 4000,\ \alpha = 12.5°$





Adaptation critical for achieving higher-order convergence

(apriori meshes are uniform refinements)

Key take-away

Adaptation is critical to realize the performance benefits of higher-order discretizations on aerospace applications

Adaptive higher-order methods: Current Status & Challenges

- Higher-order 2D & 3D RANS (including shocks) demonstrated on *a priori* meshes (Bassi; Darmofal; Fidkowski; Hartmann; Mavriplis; Peraire, etc.)
- Robust anisotropic adaptation demonstrated for 2D steady RANS (Darmofal; Fidkowski; Hartmann)
- Proof of concept demonstrations for adaptive 3D RANS (Darmofal; Fidkowski; Hartmann)
- Challenges: higher-order adaptive meshing; robustness for under-resolved RANS; unsteadiness

Questions?

Affine-invariant metric framework

- Employ affine-invariant description of a metric space (Pennec et al, 2006)

$$S_\kappa = \log\left(\mathcal{M}_{\kappa_0}^{-1/2}\mathcal{M}_\kappa\mathcal{M}_{\kappa_0}^{-1/2}\right)$$

- S_κ (the step matrix) can be decomposed into $S_\kappa = s_\kappa I + \tilde{S}_\kappa$
- s_κ is isotropic and controls the area change
- $\tilde{S}_\kappa$ controls orientation and stretching changes
- First-order optimality conditions become

$$\begin{aligned}\frac{\partial \eta_\kappa}{\partial s_\kappa} - \lambda\frac{\partial \rho_\kappa}{\partial s_\kappa} &= 0\\ \frac{\partial \eta_\kappa}{\partial \tilde{S}_\kappa} &= 0\end{aligned}$$

Error model synthesis

- Define logarithmic error model $f_{\kappa_i} \equiv \log(\eta_{\kappa_i}/\eta_{\kappa_0})$

$$\{\mathcal{M}_{\kappa_i}, \eta_{\kappa_i}\} \to \{S_{\kappa_i}, f_{\kappa_i}\}$$

- Perform a least-squares fit to synthesis $f_\kappa(S_\kappa) = \text{tr}(R_\kappa S_\kappa)$:

$$R_\kappa = \arg\min_{Q \in Sym_d} \sum_{i=1}^{n_{\text{config}}} \left(f_{\kappa_i} - \text{tr}(Q S_{\kappa_i})\right)^2$$

- This gives $\eta_\kappa(S_\kappa) = \eta_{\kappa_0} \exp(r_\kappa s_\kappa d) \exp\left(\text{tr}\left(\tilde{R}_\kappa \tilde{S}_\kappa\right)\right)$
- For isotropic error and meshing this model reduces to,

$$\eta_\kappa^{\text{iso}}(h) = \eta_{\kappa_0} \left(\frac{h}{h_0}\right)^{r_\kappa^{\text{iso}}}$$

Continuous Metric Optimization

- **Problem Statement:** Seek optimal metric field

$$\mathcal{M}^* = \arg\min_{\mathcal{M}} \mathcal{E}(\mathcal{M}) \quad \text{s.t.} \quad \mathcal{C}(\mathcal{M}) = \text{Cost}$$

- Choose $\mathcal{E}(\mathcal{M}) = \sum_\kappa \eta_\kappa(\mathcal{M}_\kappa)$
- Choose $\mathcal{C}(\mathcal{M}) = \sum_\kappa \rho_\kappa(\mathcal{M}_\kappa)$
- ρ_κ are the DOF in region κ.