Reflections on RANS* Modeling

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In collaboration with Strelets NTS group,
St. Petersburg, Russia

*Reynolds-Averaged Navier-Stokes

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Opinions on RANS* Modeling

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Outline

- **RANS is still in high demand, and will be for 50+ years**
  - Re-visit feasibility of Large-Eddy Simulation (LES) in real life
- **RANS and LES are not enemies, but partners**
  - Covering different regions in a Detached-Eddy Simulation (DES)
  - Direct Numerical Simulation and LES "educating" RANS models
- **Steady and Unsteady RANS, DES, for massive separation**
  - No simple answers, and many purposes
  - All simulation modes need to be understood
- **Progress in practical RANS models slight since 1990's**
  - Many impediments to decisive progress
  - The "Fundamental Paradox" of RANS modeling
  - New issue of multiple solutions
- **Comments on Reynolds-Stress-Transport Modeling**
  - Successes, but mostly away from aeronautical flows
- **Resilience of Logarithmic Law in pressure gradient: a DNS**
  - Example of "what we don't know" about turbulence
- **Summary and Grand Plan**

RANS Still in High Demand

- **In industrial steady/unsteady CFD**
- **For boundary layers in hybrid methods (1997 DES paper)**
  - LES is still unaffordable in leading-edge and nose regions
- **For wall region under an LES**
  - Work of Nikitin et al., Piomelli group, NTS, others
- **Also needed for:**
  - Small components next to large ones
  - Separation bubbles: this is up to the user
- **Trend towards initiating LES before separation in hybrid CFD**
  - RANS models will never be perfect, whereas LES improves with grid
  - Need unsteady quantities for noise and vibration
  - Challenge is generation of LES content
- **Is the hybrid method of the future zonal, or not?**
  - Zonal methods have successes in semi-complex situations
    - They give more control ("ZDES" work of S. Deck at ONERA)
  - Non-zonal methods are far more convenient

Feasibility of LES

- **The rationale for DES, in 1997, was:**
  - Pure LES for wings will not be feasible until 2045, assuming Moore's Law
    - I assumed "a factor of 5 every 5 years" but "a factor of 2 every 2 years" gives 2041 instead
  - This is even with full Wall Modeling inside the LES (unlimited Δx^+, etc.), and other favorable assumptions, such as perfect knowledge of δ and grid design
  - The LES needs 10^{11} grid points
  - Therefore, for now, ***the boundary layer needs RANS***
    - At least near the leading edge

Feasibility of LES

- **NASA-Ames/Stanford/CTR position on the cost of LES:**
  - Also for LES with Wall Modeling, as opposed to “wall-resolved” LES
  - **1979**, Chapman, AIAA J.: $N_{points} \sim Re^{2/5}$
    - Comes from averaging δ , the boundary-layer thickness (which is incorrect)
  - **2012**, Choi & Moin, Physics of Fluids: $N_{points} \sim Re$
    - Comes from averaging $1/\delta^2$ (which is correct)
  - Re is based on the lateral direction, and Re_z = O(500 million) for a wing
  - New rough estimate for grid points in full LES is much higher:

$$\frac{N_{2012}}{N_{1979}} \approx 165{,}000$$

- That is about 2^{17} or 34 years more to wait, if you apply Moore’s “2 in 2" law
- And do not forget the extra time steps needed

- Formula of Choi & Moin

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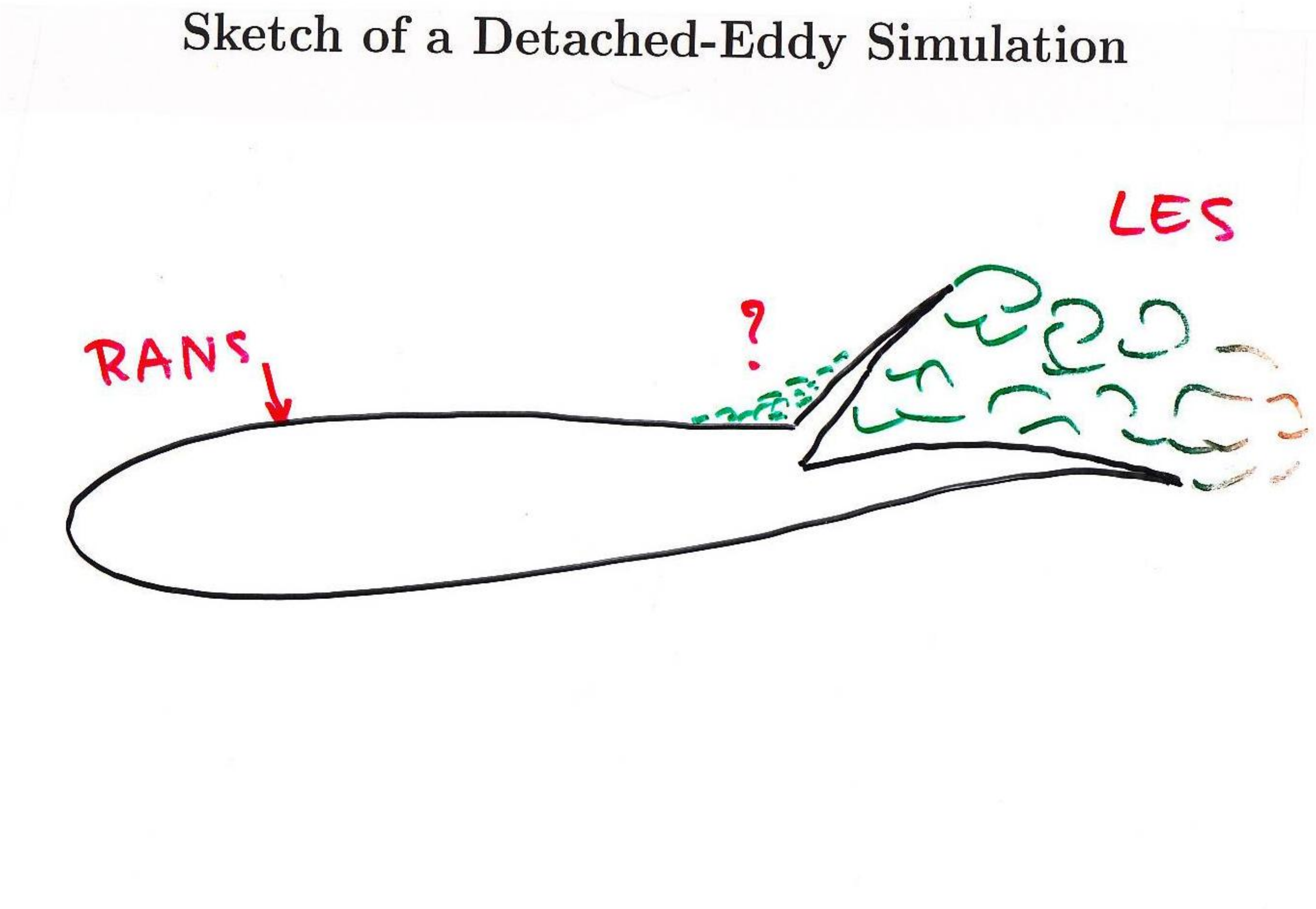
Original Sketch, 1997



"Natural" DES

- **Work of Chaderjian & Buning at NASA**
  - Lots of "worms!"
  - DES gives best Figure of Merit

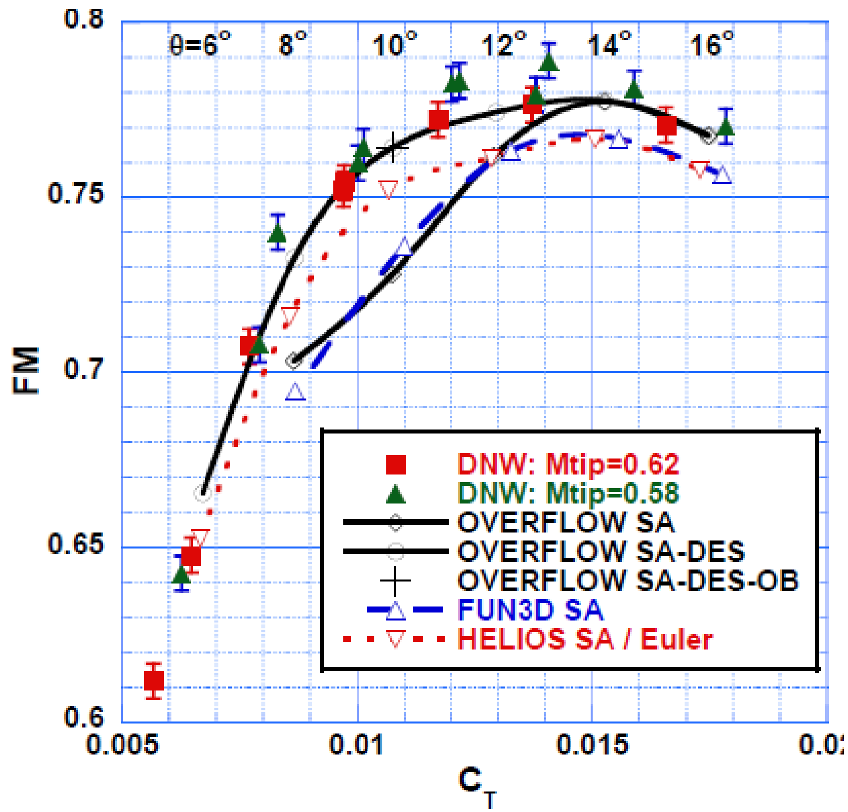
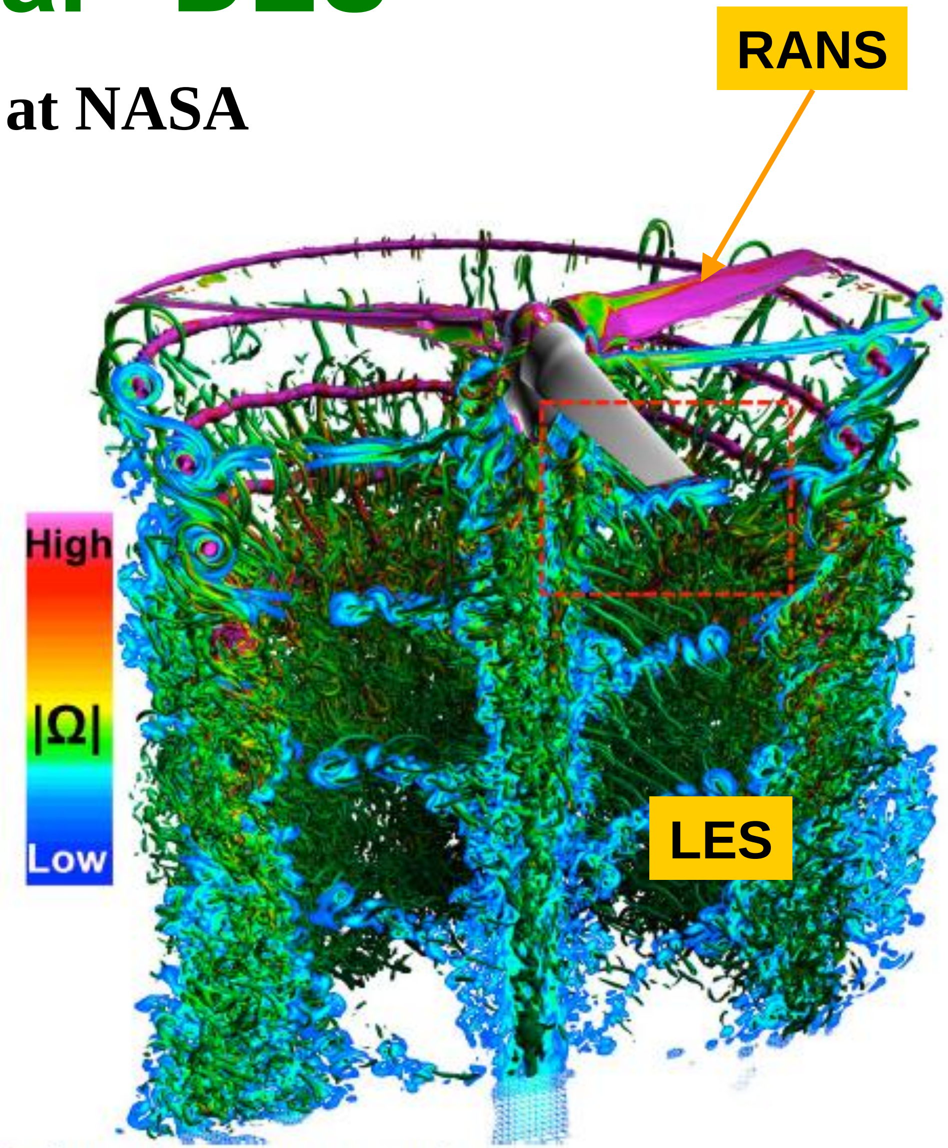


Figure 13 Figure of merit variation with C_T for the TRAM rotor in hover. M_{tip} =0.625, Re=2.1 million.

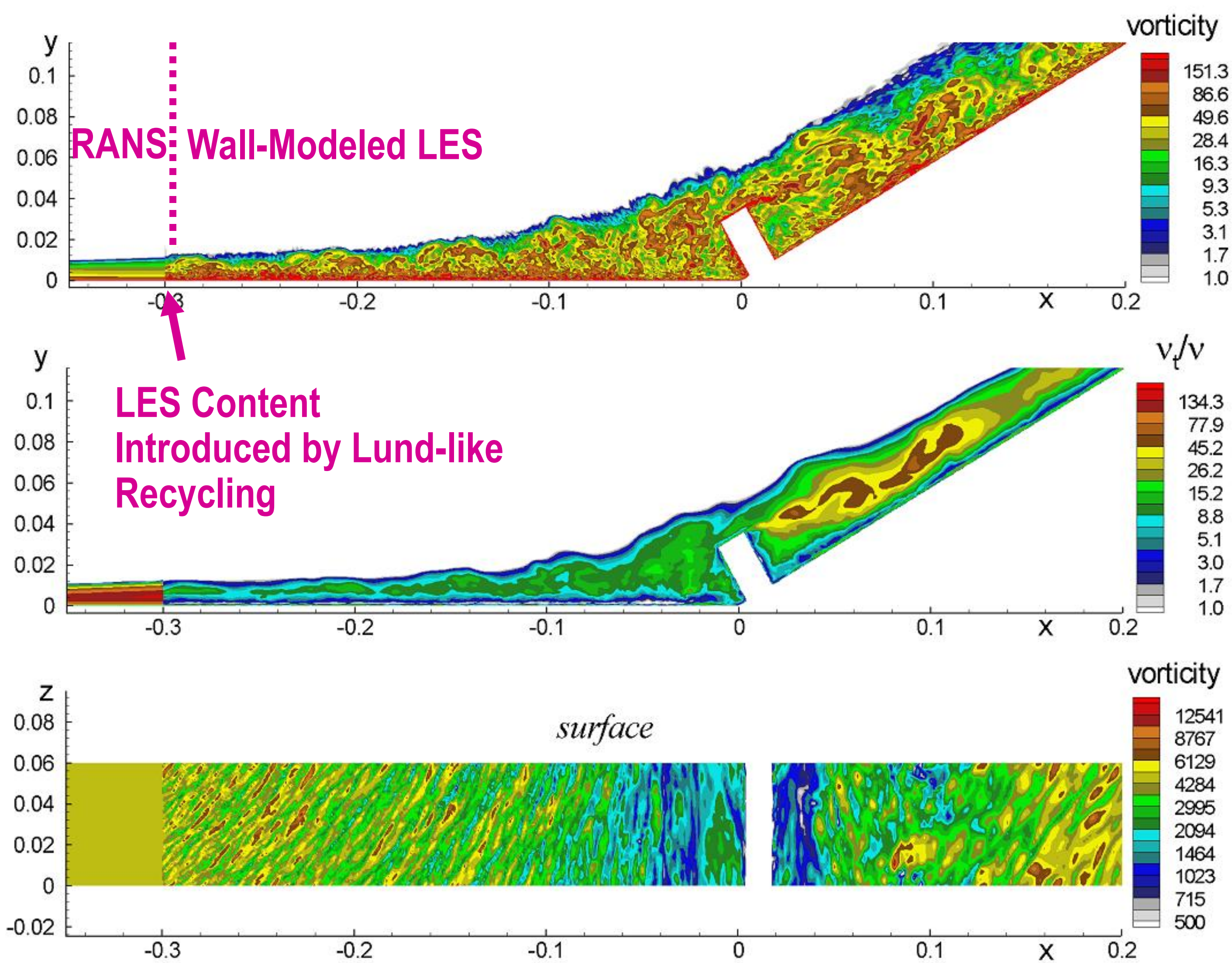


a) View of entire vortex wake.

Simulation of a Small Separation Region

Purpose: predict noise for pilots, caused by reattachment on windshield

RANS-to-LES Switch in Attached Boundary Layer



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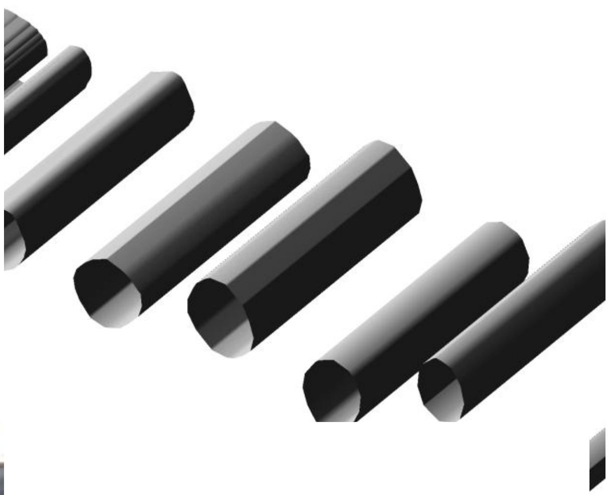
Four Types of Bluff-Body Simulations

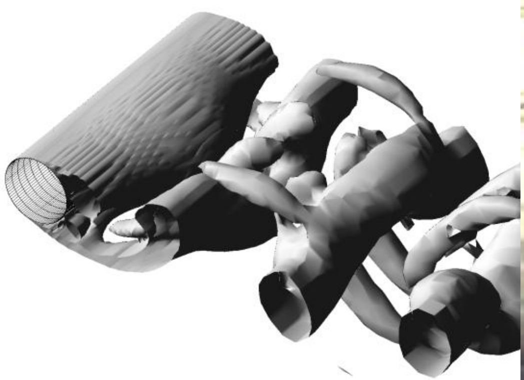
All cases with *laminar* se

steady RANS, C_d ~ 1.73

2I



Experiment



3D Unsteady RANS, C_d ~ 1.24

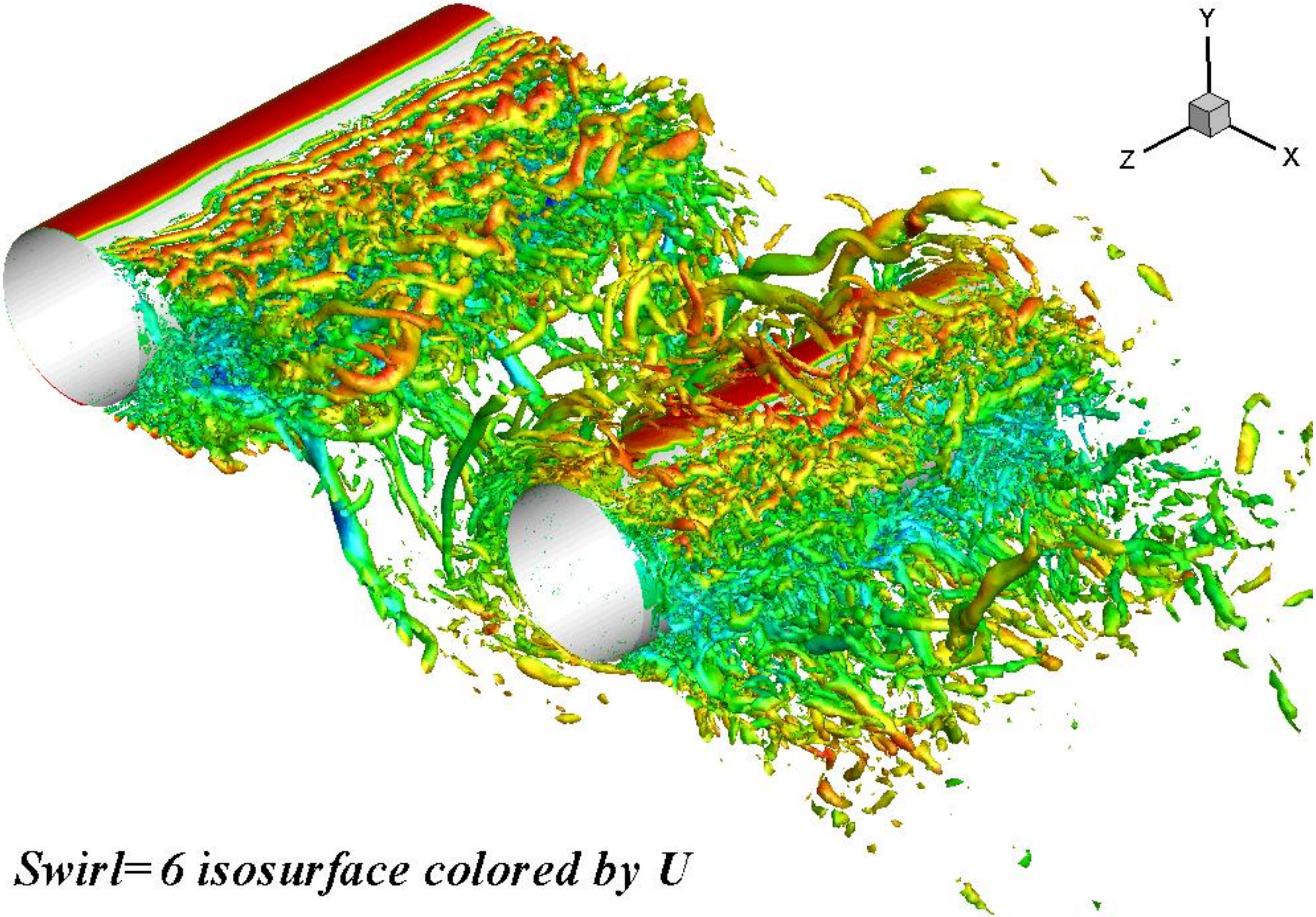
DES, C_d ~ 1.26

Spectrum of Approaches to Turbulence

Name	DNS	LES	DES	RANS
Empiricism	No	Low	Medium	High
Unsteady	Yes	Yes	Yes	No (can be)
# of points (Boeing wing)	10^{20}	10^{11}	10^{7} to 10^{8}	10^{7}
In Service (Boeing)	2080*	2045*	2010 (sub-regions)	1995
Vibration, Noise	Yes	Yes	Yes	No (buffet maybe)

*Assuming Moore's Law holds!

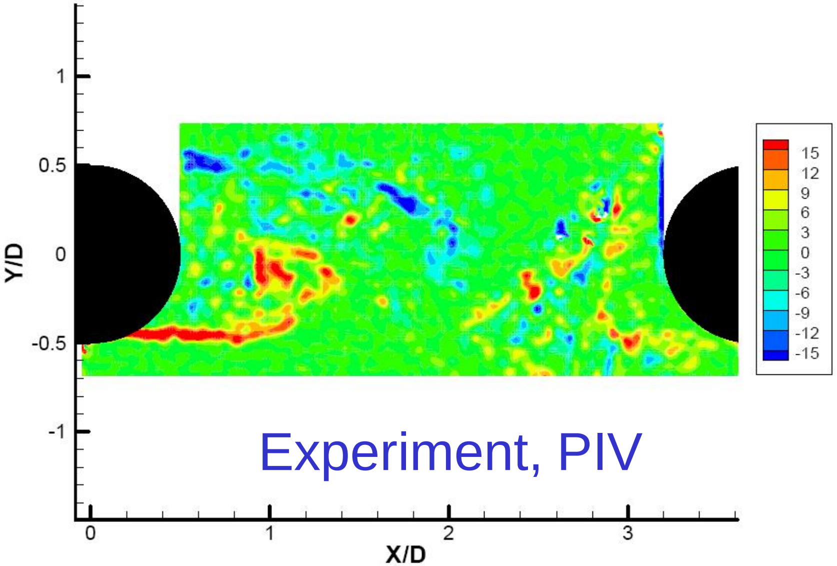
DES of Tandem Cylinders



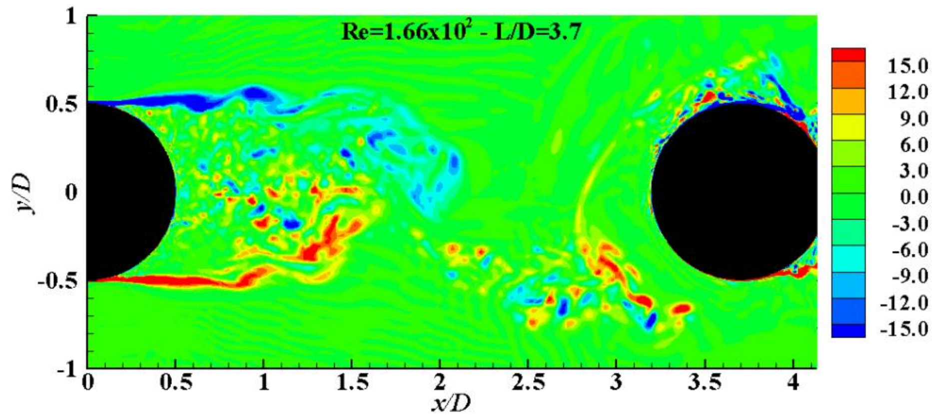
Swirl=6 isosurface colored by U

Comparison with NASA Experiment

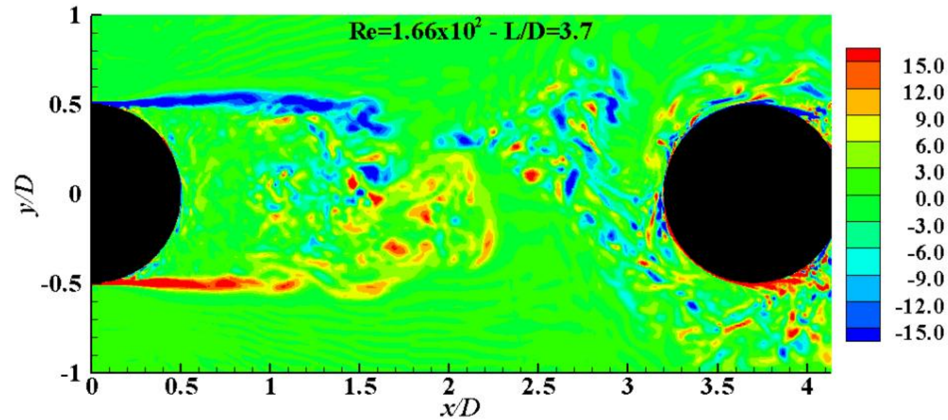
Snapshots of Spanwise Vorticity



Experiment, PIV



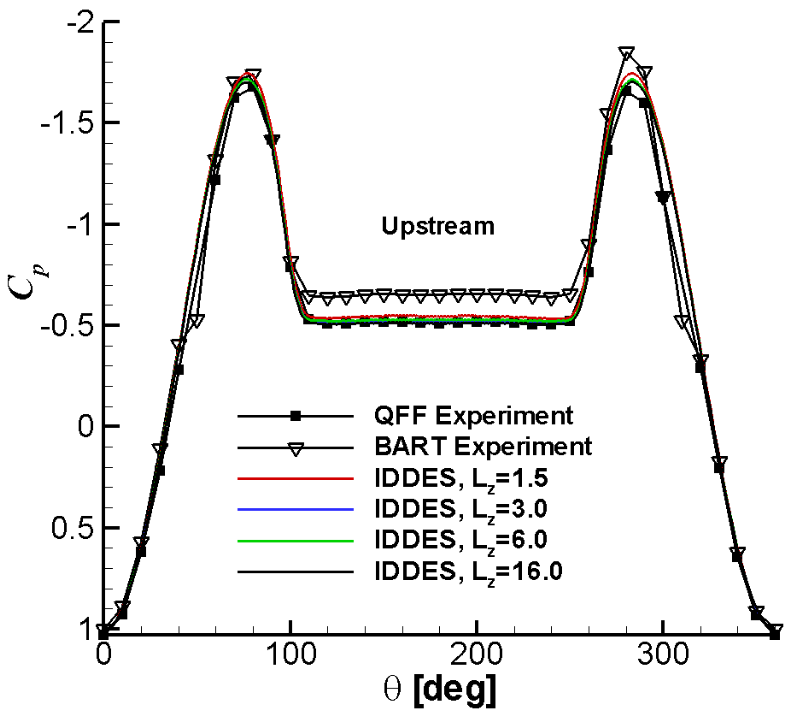
DDES, L_z=3D

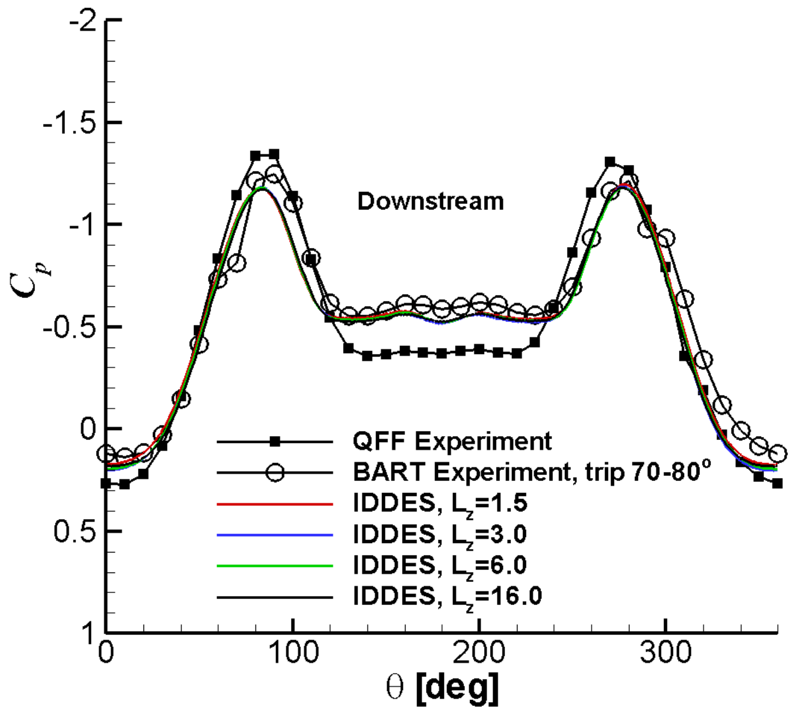


DDES, L_z=16D

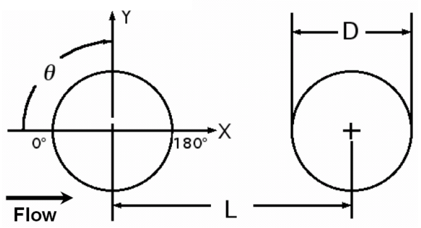
Comparison with NASA Experiment

Surface Pressure Coefficient





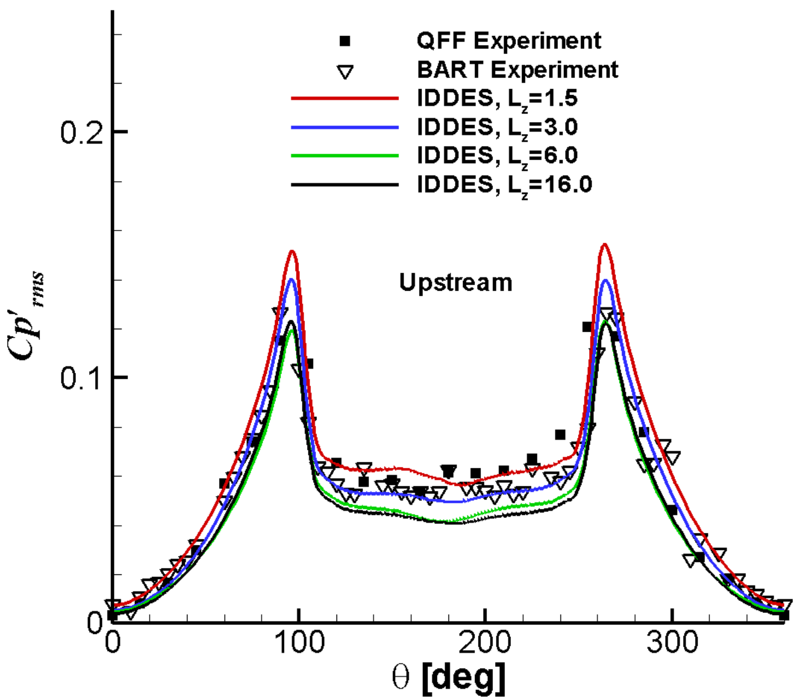
Upstream

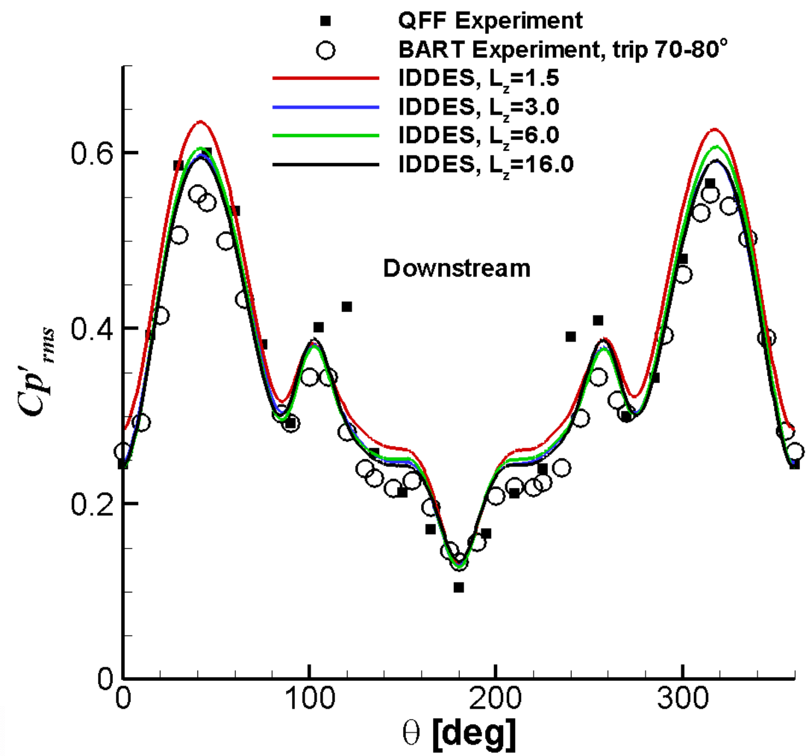


Downstream

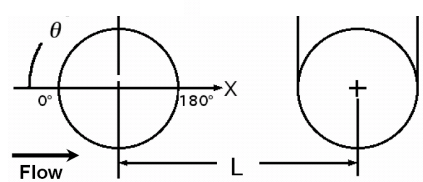
Comparison with NASA Experiment

RMS of Surface Pressure





Upstream



Downstream

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The Fundamental Paradox of Turbulence Modeling?

1) Turbulence does not exist at a point *(x,y,z,t)*

- It can be understood and predicted only in a *region* of space and time,
  - Large enough for some repeatable behavior to take place
  - Such as establishing a $k^{-5/3}$ spectrum, or logarithmic layer
- E.g., an entire boundary layer that has developed normally for at least $x = 10\ \delta$ (δ the BL thickness)

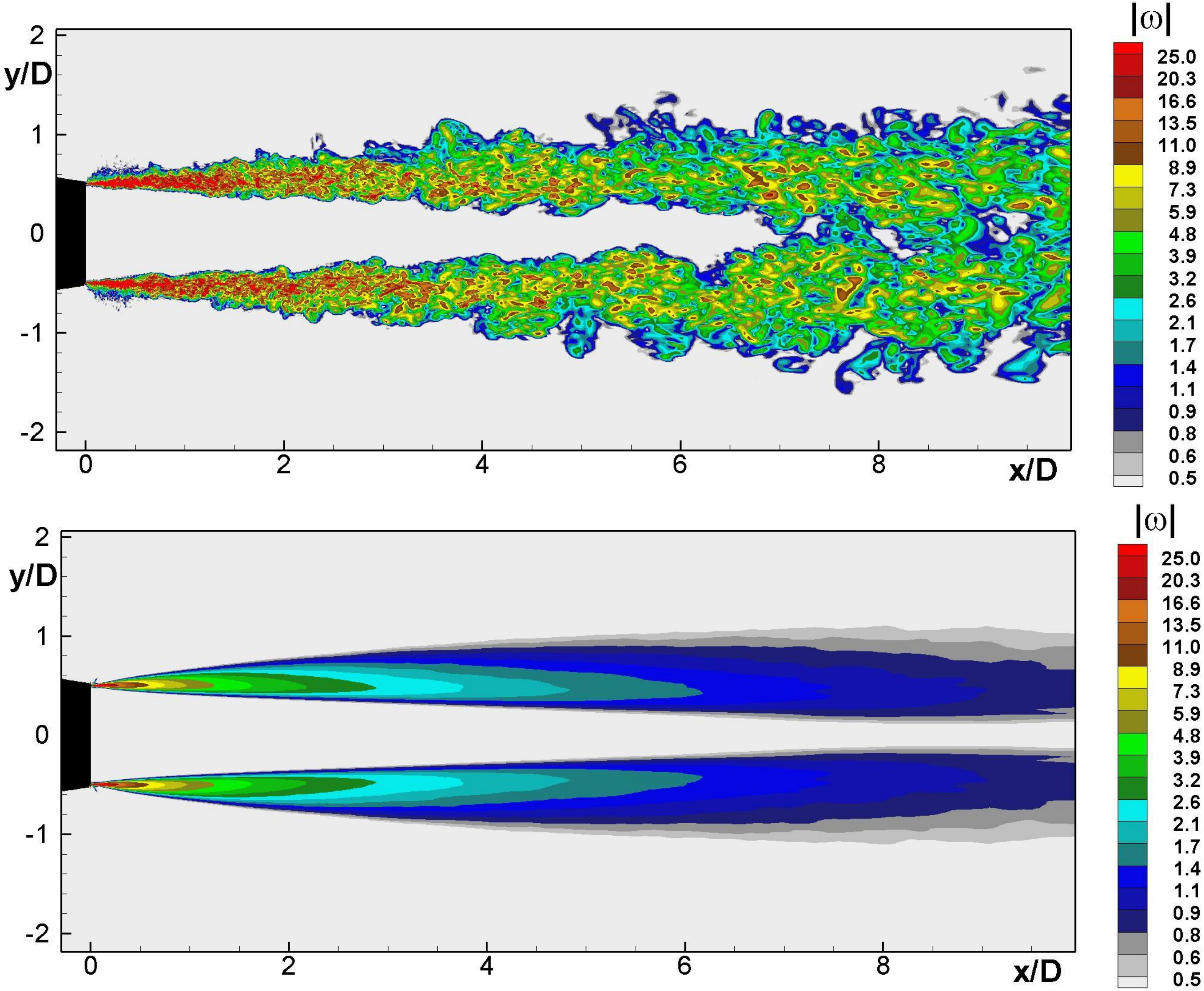
2) Defining "turbulence at a point" is the basic demand of CFD!

- Not only at a point, but using a *small* number of variables
- The solution to this impossible problem will not be pure
  - Non-local "wall-blockage" terms have a lot to offer,
    - But they cannot be derived from the Reynolds-Stress transport equations
- Algebraic RANS models such as Cebeci-Smith treated entire regions at once
- Modern differential RANS models do not
  - For compatibility with unstructured grids and parallel machines

- **In the end, transport and diffusion "glue" the region together, and we *test* the model over a large region in *(x,y,z,t)***

Ideas refined with J. D. McLean

Reynolds Averaging



State of RANS Modeling in Aeronautics

- **Turbulence Modeling Working Group, led by Brian Smith**
  - http://turbmodels.larc.nasa.gov/, created by Rumsey
  - Principal models fully documented, give same answer in all codes
- **Large market share for two models, SA and SST, by F. Menter**
  - Both from 1992, both pragmatic, both pretty much NASA Ames products!
  - Small number of versions
  - Both use wall distance
  - Improvements: curvature/rotation, roughness, compressibility, nonlinear…
  - k-ε is alive and has prestige, although it is quite poor for separation
  - Heat transfer is lagging
- **Apparent failure of rigorous thinking (based on Reynolds equations)**
  - It leads to more complex models; Full Reynolds stress or Algebraic Stress
    - No consistent accuracy advantage in thin shear flows
    - Rebellious at times
  - More systematic two-equation model design (k-ϕ) at ONERA
    - Optimize choice of second variable. Has not spread into codes
  - Difficulty matching DNS Reynolds stresses, which violate the law of the wall
    - Dependence on Reynolds number and pressure gradient

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Novel uses of DNS/LES for RANS Modeling

- **Create the "target" model quantities from unsteady field**
  - Traditionally, *k* and ε
    - But errors can compensate, and give the right ν_t (e.g. in log layer)
  - Eddy viscosity directly, with $$\nu_{eff} == \frac{-<u_i u_j> S_{ij}}{2 S_{kl} S_{kl}}$$
    - Interpreted as least-squares fit, or a TKE production match
  - Also constants in QCR and other nonlinear constitutive relations
    - With similar least-squares formulations
    - An alternative to Rodi-type derivations of Algebraic-Stress Models
- **Test the model equations in the simulation field**
  - Work by NTS-Boeing, and by Leschziner's group
  - Freeze the mean flow field, and solve the model in it
  - Advantage: see the error immediately, instead of only seeing it after it modifies the velocity field

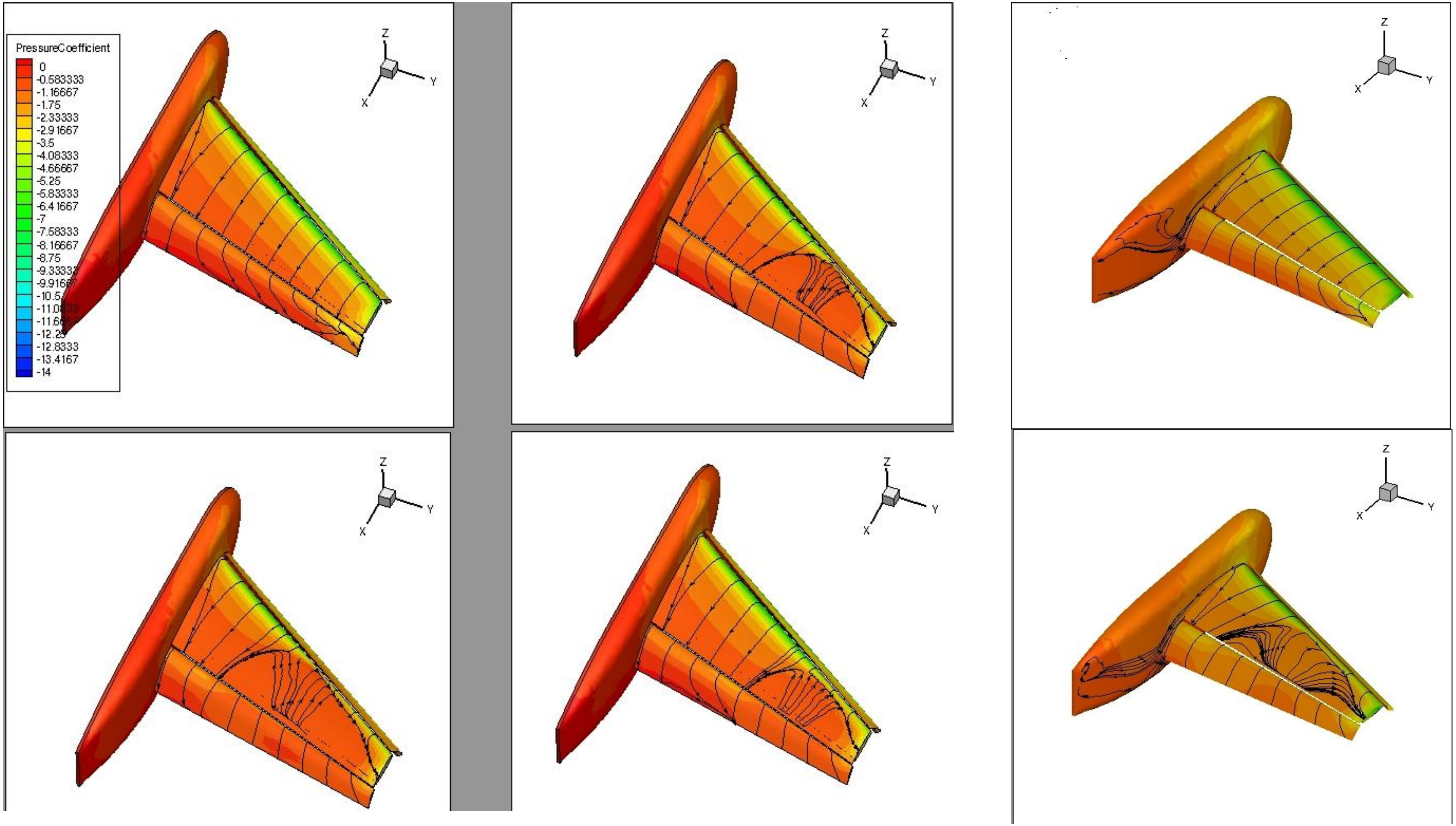
Progress in RANS Modeling?

- **The adventures of the Karman constant**
  - Current range of serious experimental values for κ : 0.38 to 0.42
    - **Compensation by *C* makes the effect small until $y^+ \sim 10^4$**
  - Less important than the model's reaction to pressure gradient
  - New proposal to have different Karman constants in different flows!
    - **Such in a pipe and in a boundary layer**
    - **Would be the death of the Law of the Wall… and of turbulence modeling!**
- **Little input from DNS**
  - Flows too simple
  - Reynolds number too low (e.g., NO impact of DNS on Karman-constant debate)
  - LES is starting to be used well, e.g. on "hill flows"
- **Can a better model be accepted tomorrow?**
  - Difficulty in getting published and (more important) added to mainstream codes
- **Can the best existing model be determined today?**
  - Are experiments good enough?
    - No "perfect" measurements
    - Lack of detail, so that testing is indirect (for instance, shock position)
  - Is RANS CFD good enough to judge models with full precision?
    - Grid convergence not certain, even for a simple wing-body case!
    - Multiple solutions, when separation gets interesting

Multiple Solutions on Trap Wing at 28°

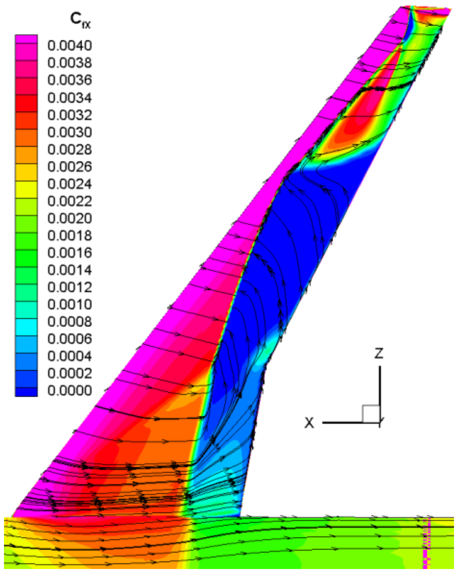
S-A Model

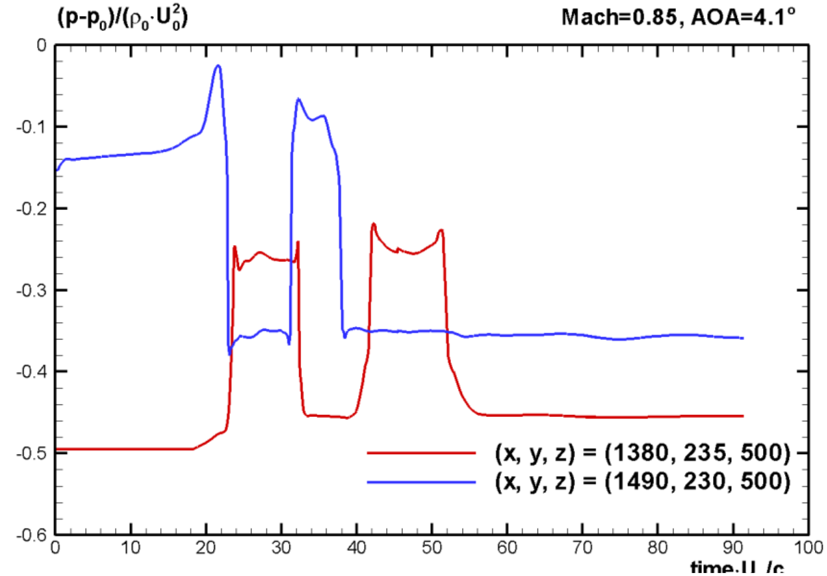
k - ω

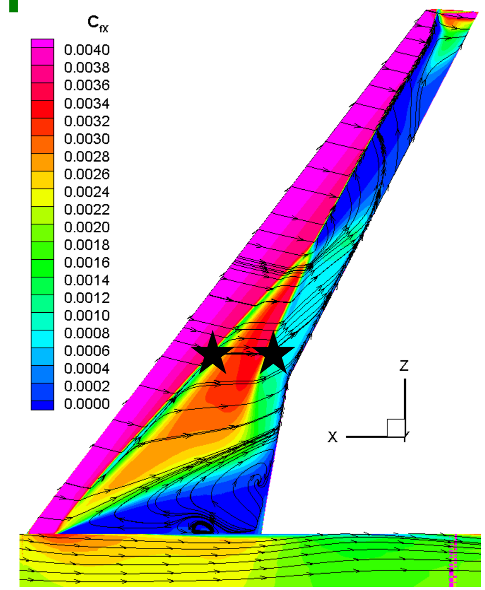


GGNS code, fixed grid, fully turbulent. All iteration-converged to machine zero. Overflow and NTS have similar "stories."

“Steady” RANS with Separation







- NTS code: high-order, structured, verified in unsteady flows. Fully turbulent
- CRM case, M = 0.85 and α = 4.1°
- Left: steady code, with QCR. No side-of-body separation; mid-wing separation
- Middle: unsteady code, started from steady solution
  - Pressure histories at two ★ field points reveal shock rearrangement
- After 60 chords of flight the flow is in a new “settled” state… and radically different!
  - The residuals are calculated the same way in both cases
- This flow was given back to steady-state code… which slowly returned to first state!
- Notice the solution did NOT enter a limit cycle; i.e., the answer is not “buffet”
- Other codes (GGNS, Overflow) also give solutions which depend on initial state

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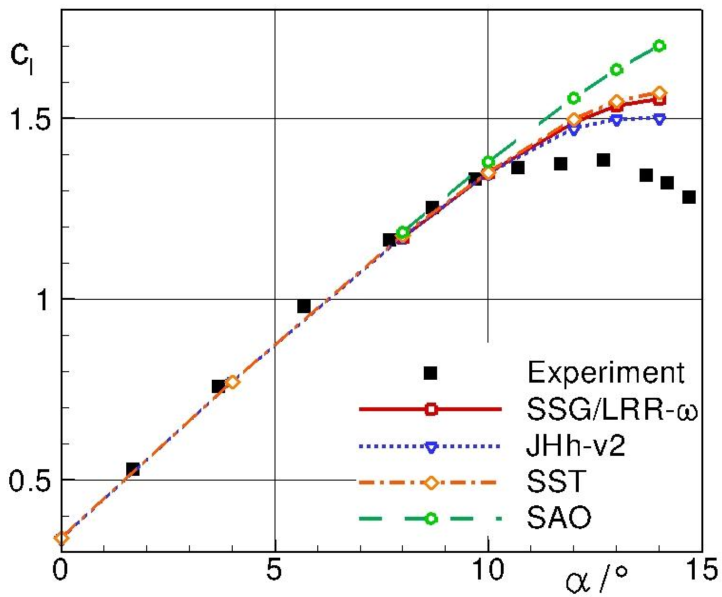
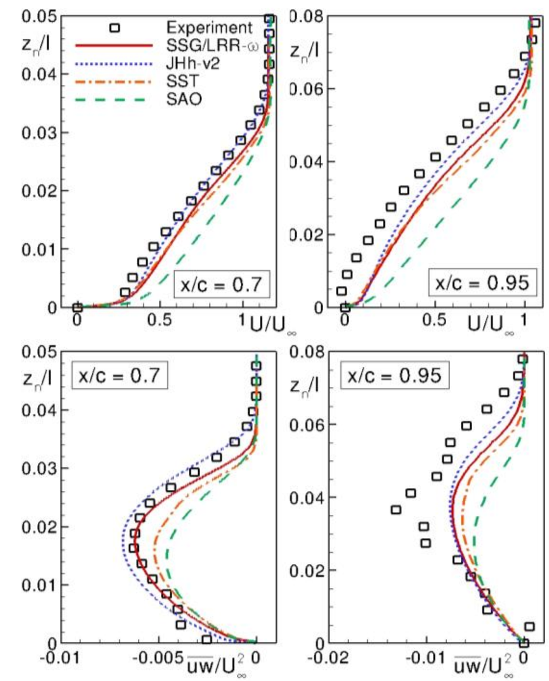
Reynolds-Stress-Transport Modeling

- **Arguably THE sound basis for RANS modeling**
  - Starts from exact equations
    - Subject to Closure Problem
    - More exact terms than with 1-2 eqs, first of all, the individual productions
    - Modeling “should matter less” if pushed to higher-order terms
  - However, quickly uses “plausible approximations”
    - Such as anisotropic dissipation tensor (shift to pressure term)
    - The more successful models tend to use wall distance and wall-normal
- **Two-equation modeling considered a “poor cousin”**
  - Let alone one-equation modeling! (-:
  - Simpler models are “fighting back”
    - SST, SARC, QCR, other corrections
    - Not the linear k-ε model of the 1970’s!
- **Success stories, relative to eddy viscosity:**
  - Mostly in thin shear flows with “extra strains”
  - None for massive separation (e.g., SRANS of cylinder?)
  - Not sure of curvature effects (Coanda), corner flows (CRM), even simple vortex

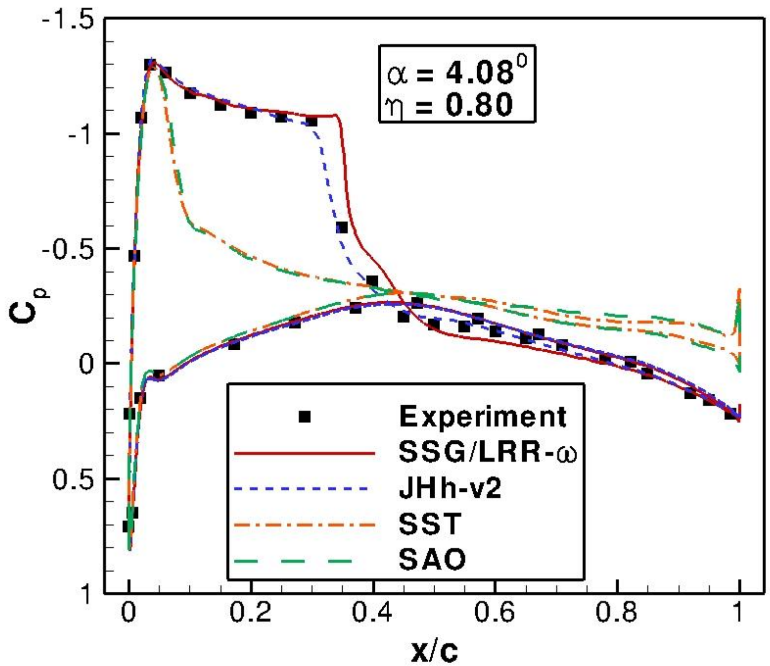
Reynolds-Stress-Transport Modeling

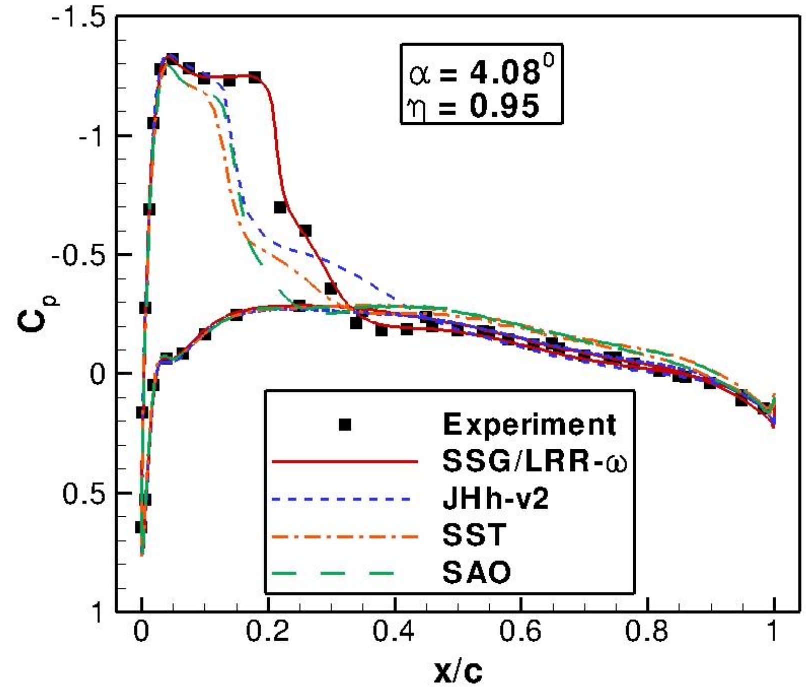




- **AIAA 2012-0465 by Cécora et al, Braunschweig**
  - Two modern RST models compared with SST and SA
  - TAU unstructured code of DLR. The only major aero code with RST?
  - CPU cost double of SST cost, slower convergence; needs higher-quality grid
- **Airfoil case**
  - Differences appear near Clmax
  - RST models do not beat SST
  - 2D CFD versus 3D wind-runnel test. But 3D CFD had far too much separation

Reynolds-Stress-Transport Modeling





- **ONERA M6 wing**
  - Cécora *et al*. results again
  - Relevant to Boeing wing shock position
- **Two sections:**
  - Left: success story for both RST models
  - Right: success for only one of the RST models

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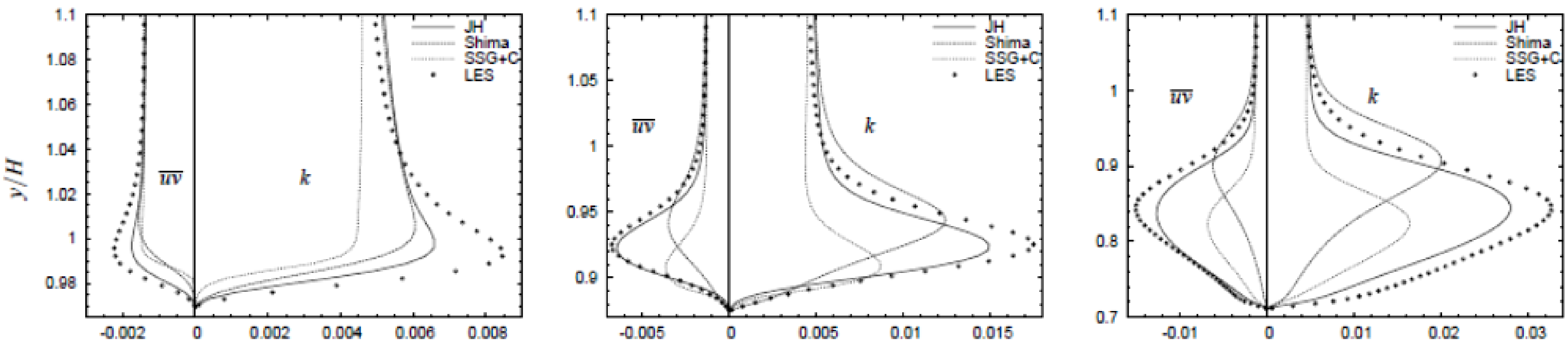
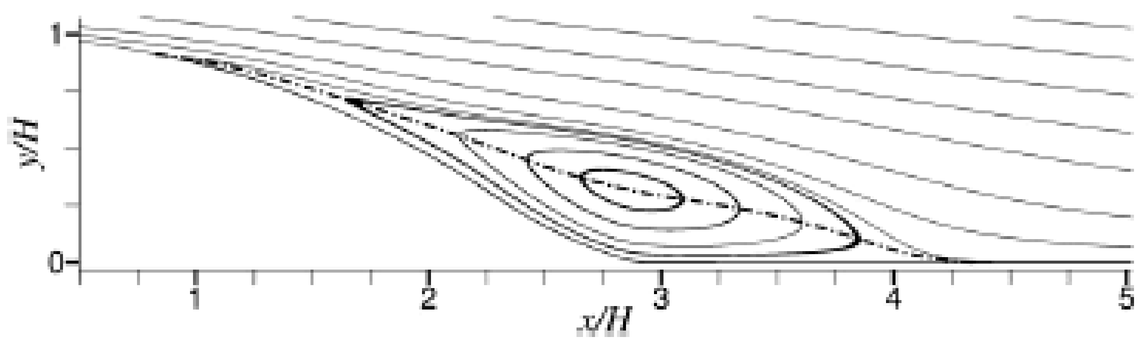
Reynolds-Stress-Transport Modeling

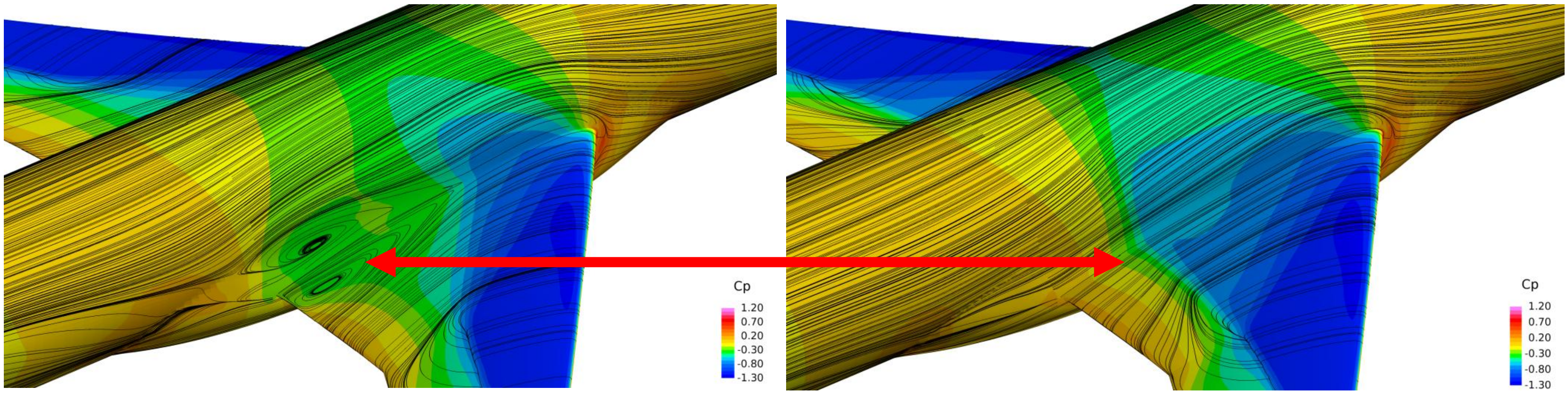


(a) $x/H = 0.5$ (b) $x/H = 1.0$ (c) $x/H = 1.5$

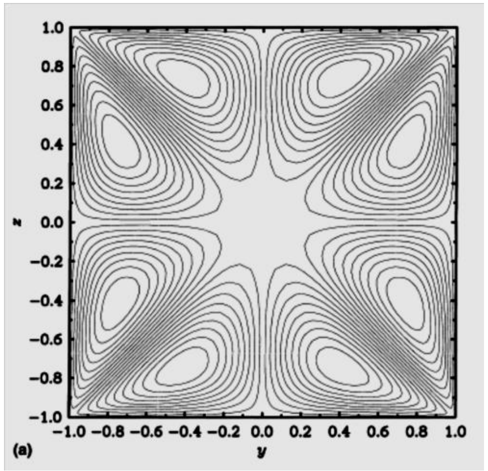
- **Work of Bentaleb, Lardeau & Leschziner at TSFP7**
  - TKE and Reynolds shear stress after separation from a smooth surface (subsonic)
- **Three reputed second-moment closures give three widely different results**



Quadratic Constitutive Relation, QCR

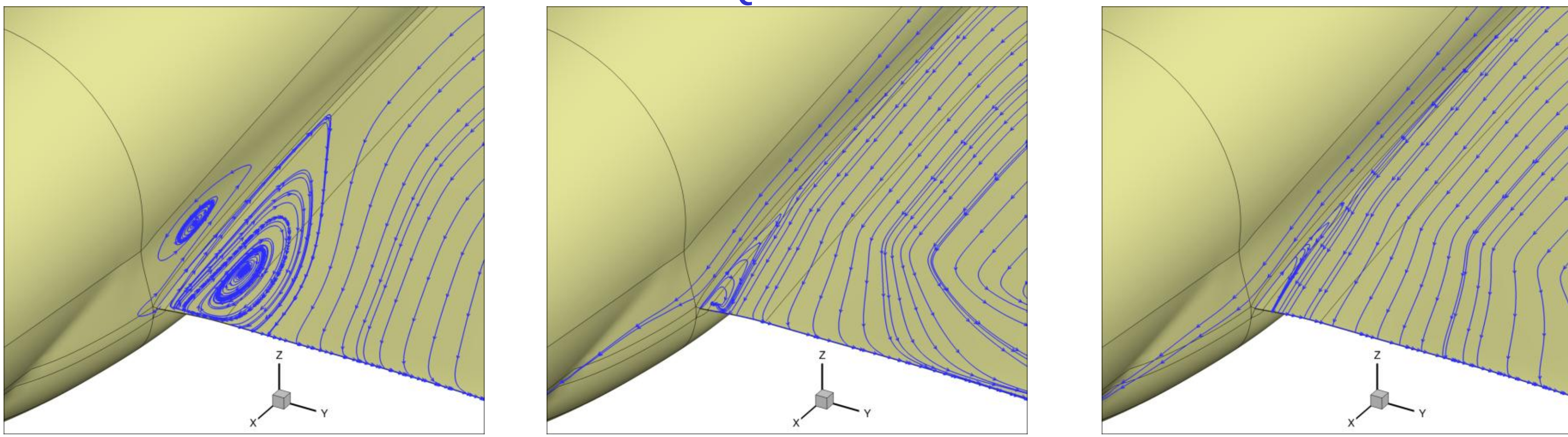


- **Results of Yamamoto *et al* at 2012 Drag Prediction Workshop**
- **Called "Nonlinear Constitutive Relation" in author's 2000 IJHFF review**
  - Prefer acronym QCR now
  - A "simple man's EARSM"
  - Similar to a model of Wilcox & Rubesin
  - Applicable to any eddy-viscosity model, e.g., SST
- **Gives Turbulent Secondary Flows in square pipes**
- **Strongly reduces corner separation (without adjustment)**
- **Example of "easy" improvement**
- **Based on intuition**



Preliminary: Effect of Nonlinear Terms on Side-of-Body Separation

SA model | SA+QCR model | EASM k-ω model



CRM configuration from DPW-IV Multiblock grid from JAXA, AoA=4 deg, M=0.85, Re_{MAC}=5 million

- **Results of Rumsey and team at Langley**
- **QCR and EARSM have similar effect on corner flow**
- **Full Reynolds-Stress model performance unknown as of now**
- **Race will continue…**

Research team leaders: Chris Rumsey (NASA LaRC)

Prediction of Natural Transition

- Need: 2D Tollmien-Schlichting and 3D Cross-Flow modes
  - + Transition due to separation
  - + By-pass transition, esp. for internal flows
- Classic: e^n method in boundary layer profiles
  - Near-classic: database/neural-network methods in same profiles (Drela, ONERA, Boeing…)
- Typical until now: run Boundary-Layer code in NS pressure distribution, and run near-classics in BL profiles
  - NS velocity profiles hard to use directly
  - Give transition line back to NS code
- New wave: PDE method inside NS code
  - Langtry-Menter model, SST + two equations
  - Very convenient, rather robust and successful…
  - Still lacks 3D CF mode (and high Mach?)
- Future: healthy rivalry between e^n and PDE
  - Both need info about surface and ambient perturbations
- Relaminarisation: RANS models miss it

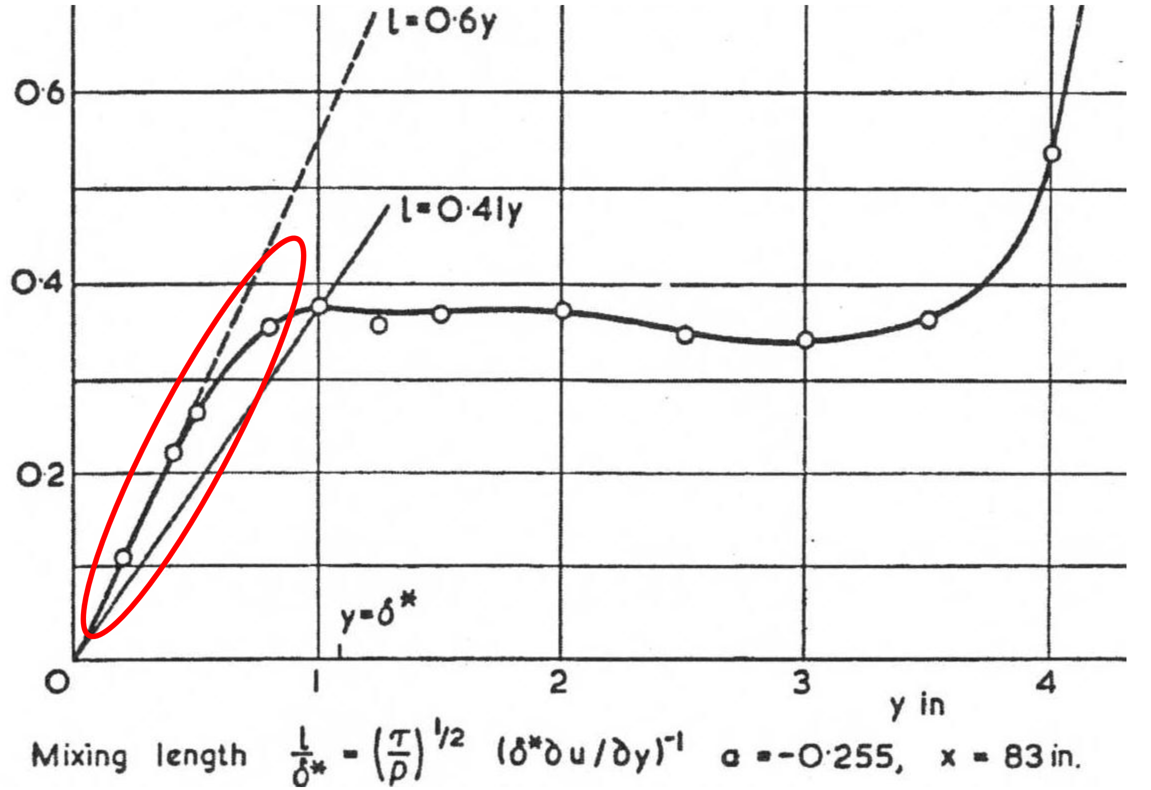
Outline

- **RANS is still in high demand, and will be for 50+ years**
  - Re-visit feasibility of Large-Eddy Simulation (LES) in real life
- **RANS and LES are not enemies, but partners**
  - Covering different regions in a Detached-Eddy Simulation (DES)
  - Direct Numerical Simulation and LES "educating" RANS models
- **Steady and Unsteady RANS, DES, for massive separation**
  - No simple answers, and many purposes
  - All simulation modes need to be understood
- **Progress in practical RANS models slight since 1990's**
  - Many impediments to decisive progress
  - The "Fundamental Paradox" of RANS modeling
  - New issue of multiple solutions
- **Comments on Reynolds-Stress-Transport Modeling**
  - Successes, but mostly away from aeronautical flows
- **Resilience of Logarithmic Law in pressure gradient: a DNS**
  - Example of "what we don't know" about turbulence
- **Summary and Grand Plan**

Resilience of Log Law to Pressure Gradient

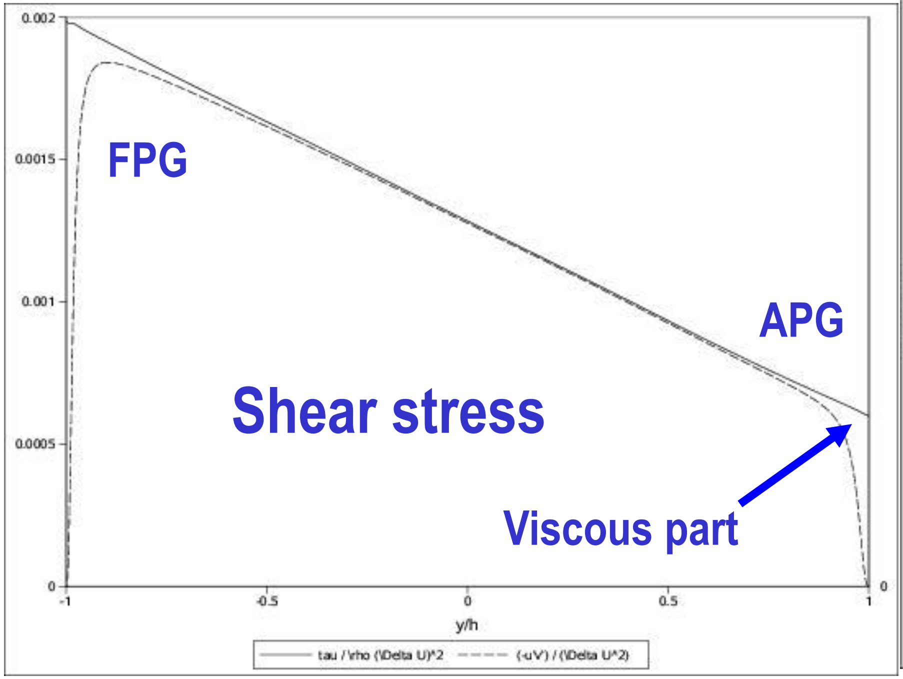
- **In a classical constant-stress layer ($y^+ >> 1$, $\tau^+ = 1$), three length scales are equal and grow linearly:**
  - $u_\tau/(dU/dy) = \kappa y$ log law (1)
  - $l = \kappa y$ mixing length (2)
  - $\nu_t / u_\tau = \kappa y$ eddy viscosity (3)
- **Physically, these hypotheses are equally justified (my opinion)**
- **With τ^+ different from 1 because of PG, they conflict!**
- **In 1975 Galbraith, Sjolander & Head found that (1) is better**
  - APG boundary layers, experiments
- With Johnstone and Coleman, we did a DNS (JFM)
  - Couette-Poiseuille flow
    - It has one FPG wall, Poiseuille-like; and one APG wall, new
- **This question matters a lot to RANS modeling**
  - Algebraic models used (2), for convenience/local character
- **It also matters to theory, or “theory”**
  - (2) is local; (1) and (3) are not: they involve u_τ, a wall quantity. Why?

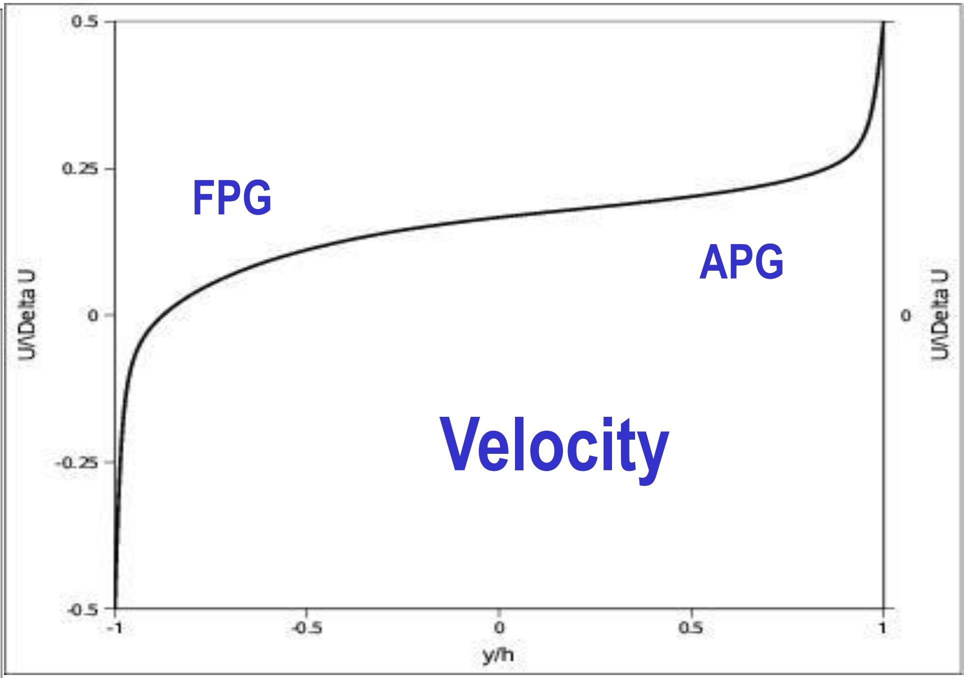
Mixing-Length Concerns



- **Galbraith figure, 1975**
  - Bradshaw-Ferris experiment with strong APG
  - Mixing length far in excess of κ y

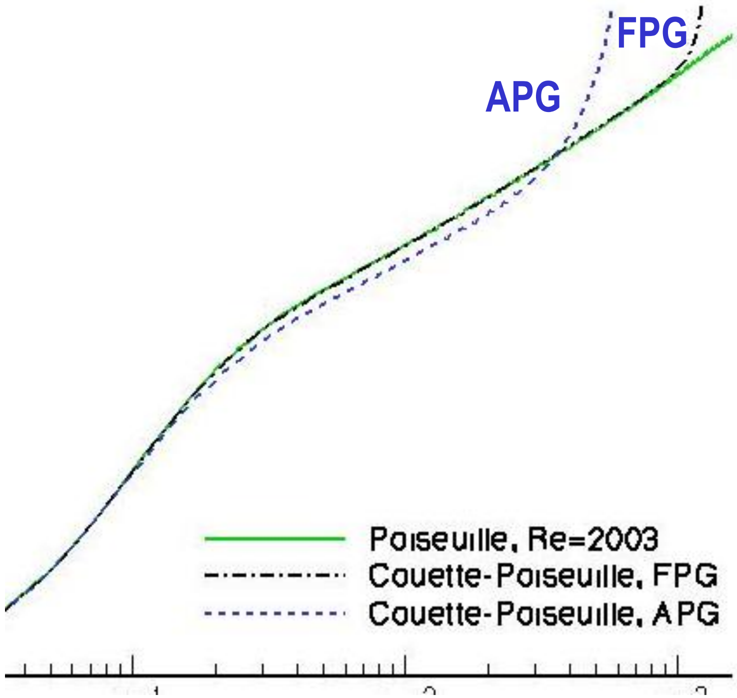
Couette-Poiseuille Flow

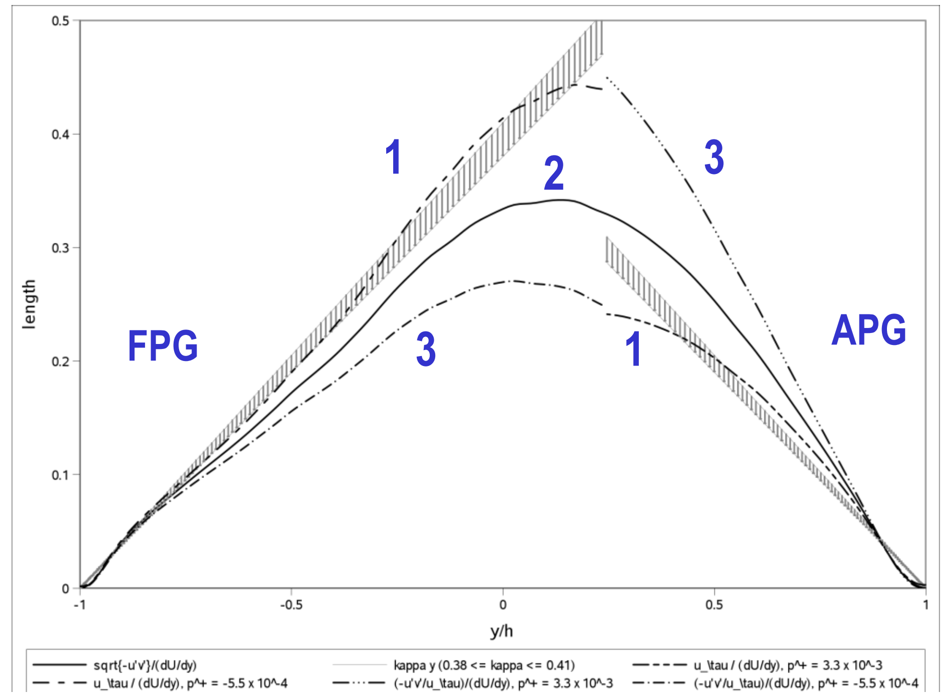




- **Left: Reynolds and total shear stress**
  - Picked a ratio τ_2 / τ_1 of 0.3 between walls
  - Re is not too low ($\Delta U\ h / \nu$ =20,000): buffer layers not too invasive
- **Right: velocity**
  - FPG wall, with higher u_τ, is somewhat dominant

Couette-Poiseuille: the Outcome





- **Left: velocity near both walls "trying hard" to have a log law**
  - FPG in close agreement with Hoyas-Jimenez Poiseuille DNS (sadly, a "curving" log law)
  - APG slightly lower
- **Right: the three lengths scales**
  - Also showing theory, with $\kappa = 0.38$ to 0.41 (roughly the current uncertainty band)
  - (1), the "log law," is clear winner, especially on FPG side (as it was in Poiseuille flow)
  - This is a quantitative, not an asymptotic result (not needing $y \ll \delta$)

BOEING

Length Scales in Hoyas-Jimenez Channel DNS

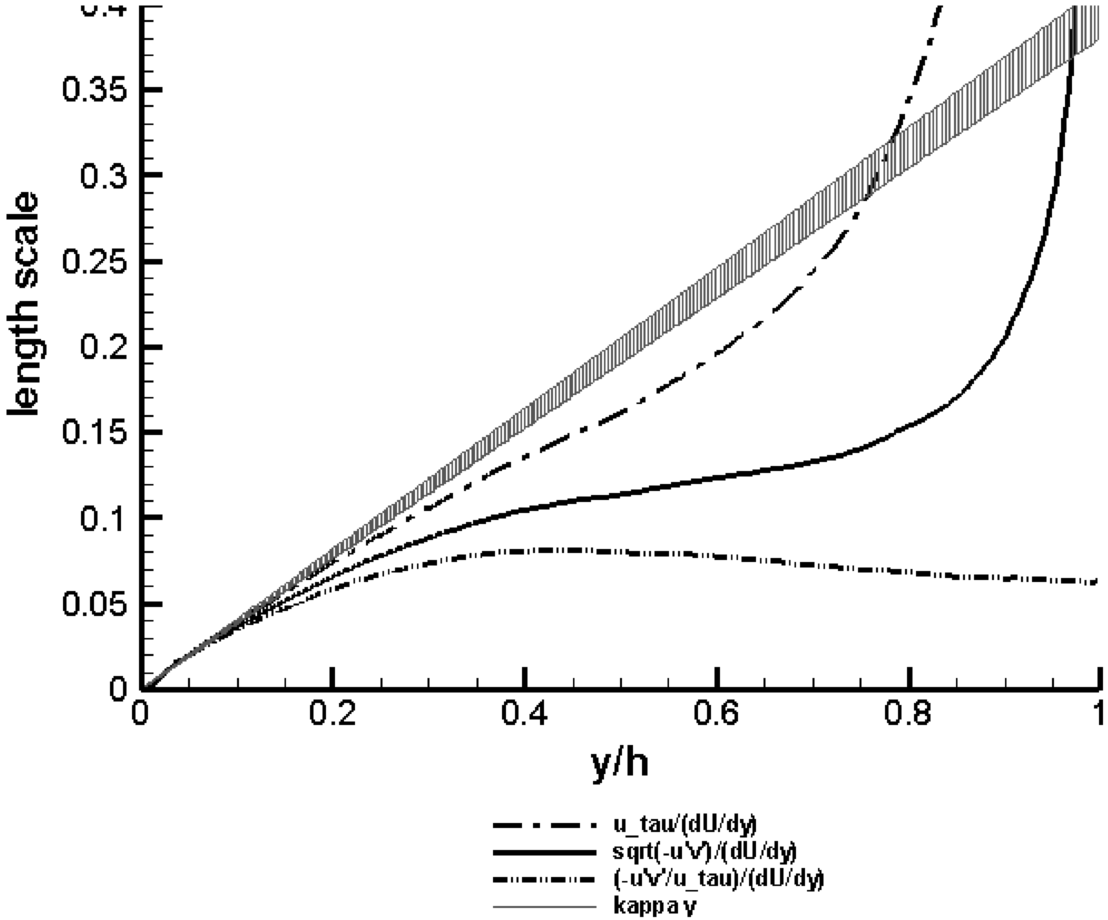


Figure by R. Johnstone

Length Scales in Hoyas-Jimenez Channel DNS

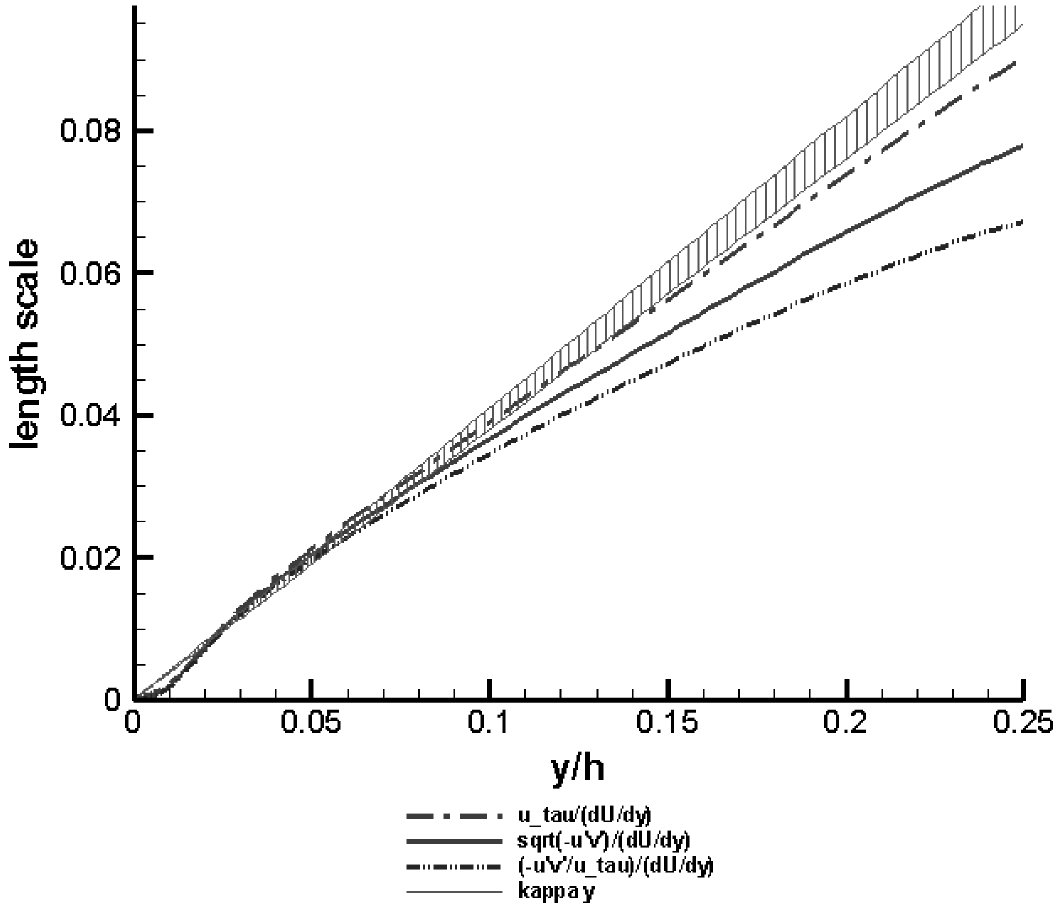


Figure by R. Johnstone

Lessons from Couette-Poiseuille DNS

- **The *velocity* law of the wall appears more accurate**
  - The mixing-length law is local, and more intuitive
    - used in numerous algebraic RANS models, and “wall models” in LES
  - This (likely) fact was observed already in the 1970’s
    - But “inconvenient” in models. Only Johnson & Coakley tried it
    - Except for Wall Functions
  - We only have one new case
    - **Other DNS and experiments desirable,**
    - **Especially with higher Re and weaker Adverse PG: here, $dp/dx^+ = 0.0034$**
- **“Modern” transport-equation models**
  - Do not have a “declaration” in this matter (i.e., between eq. 1, 2, or 3)
  - Would not be easy to re-train if they fail. They are simple and rigid!
  - However, this behavior is at the core of separation prediction
  - Models are being tested
    - Unfortunately, they start with too little skin friction in Couette flow
    - Possibly due to streamwise rollers
- **Observe the very pragmatic view-point**
  - Theory is weak; models are trained from data

Summary

- **RANS Modeling is more important than ever, because the rest of CFD is improving; hardware and software**
  - 3D solutions are everywhere
  - Modeling is less "elegant" than we would like
    - elegance can hide in equations
- **It is possible Moore's Law will saturate, and that the DNS-LES "invasion" will never reach full-size airplanes**
  - Remember, CPU cost = (goodness)^4. $10^{1/4}$ =~ 1.8…
- **RANS is a partner with LES**
  - Hybrid RANS-LES methods are here to stay, but lack foundations
  - The hand-over from RANS to LES will slowly move upstream
  - They are not "push-button" methods. User burden is very high

Summary

- **Progress in pure RANS modeling is held back by:**
  - Lack of new ideas that work better than well-established models
  - Difficulty in improving a given model on enough "fronts" at once
  - Low success of "rigorous" modeling, compared with "intuitive and pragmatic" modeling
  - Low tolerance for complex equations
    - From code writers. Even the SA Rotation-Curvature term gets bugs
    - From RANS modelers. A 7-equation models is very hard to master
  - Lack of perfect, detailed experiments
  - Lack of complex-flow, high-Reynolds-number DNS
  - Lack of perfect CFD (grid convergence) even for a simplified flap system; multiple solutions in fun regions
- **Prediction, even prescription of transition in CFD is delicate**
  - More aircraft with laminar regions are coming (real, and UAV's!)

Summary "by the Template"

- **Status:**
  - RANS modeling remains central to Aerospace and other engineering
  - We make incremental progress; no prospect of paradigm change
  - CPU power and CFD code progress are mildly helpful
- **Challenges:**
  - Field is ideas-limited, problem is "hardened"
    - Systematic approaches to RANS modeling do not win over intuition
    - Exact results (2D2C, HIT, RDT…) are in far corners of the envelope
  - RANS modeling faces a physical "Fundamental Paradox"
  - Pure LES is not "around the corner" at real-life Reynolds numbers
- **Proposed approach:**
  - Draw on the whole planet and on neighboring fields
  - Invest both in RANS and Turbulence-Resolving methods
    - Reward RANS research, even if it sounds funny
    - Solidify hybrid RANS-LES approaches, if possible
  - Nurture research DNS/LES, and detailed experiments
    - Have no patience for "experiments versus CFD" or "LES versus RANS" attitudes

Backup Slide

Formula of Choi & Moin

$$N_{wm} = 54.7 \frac{L_z}{L_x} n_x n_y n_z \, \mathrm{Re}_{Lx}^{\;2/7} \left[\left(\frac{\mathrm{Re}_{Lx}}{\mathrm{Re}_{x0}} \right)^{5/7} - 1 \right]$$

- Assumptions:
  - Number of points inside δ^3: $n_x n_y n_z = 2{,}500$
  - Re_{x0}, the Reynolds number at transition: 5.10^5
- For two sides of wing with aspect ratio 4 and $Re_x = 10^8$ this formula gives $N_{wm} = 9 \cdot 10^9$ (as in article)
- For Boeing wing, $Re_x = 5 \cdot 10^7$, aspect ratio 12, and (more realistic) $20^3 = 8{,}000$ points in δ^3:

$$N_{wm} = 4 \cdot 10^{10}$$

  - Very close to estimate in 1997 DES paper, namely 10^{11} points (this assumed a swept wing with turbulent leading edge)