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# VORTEX WAVE INTERACTION:

# ROLLS, WAVES, STREAKS AND THE EMERGENCE OF SPOTS

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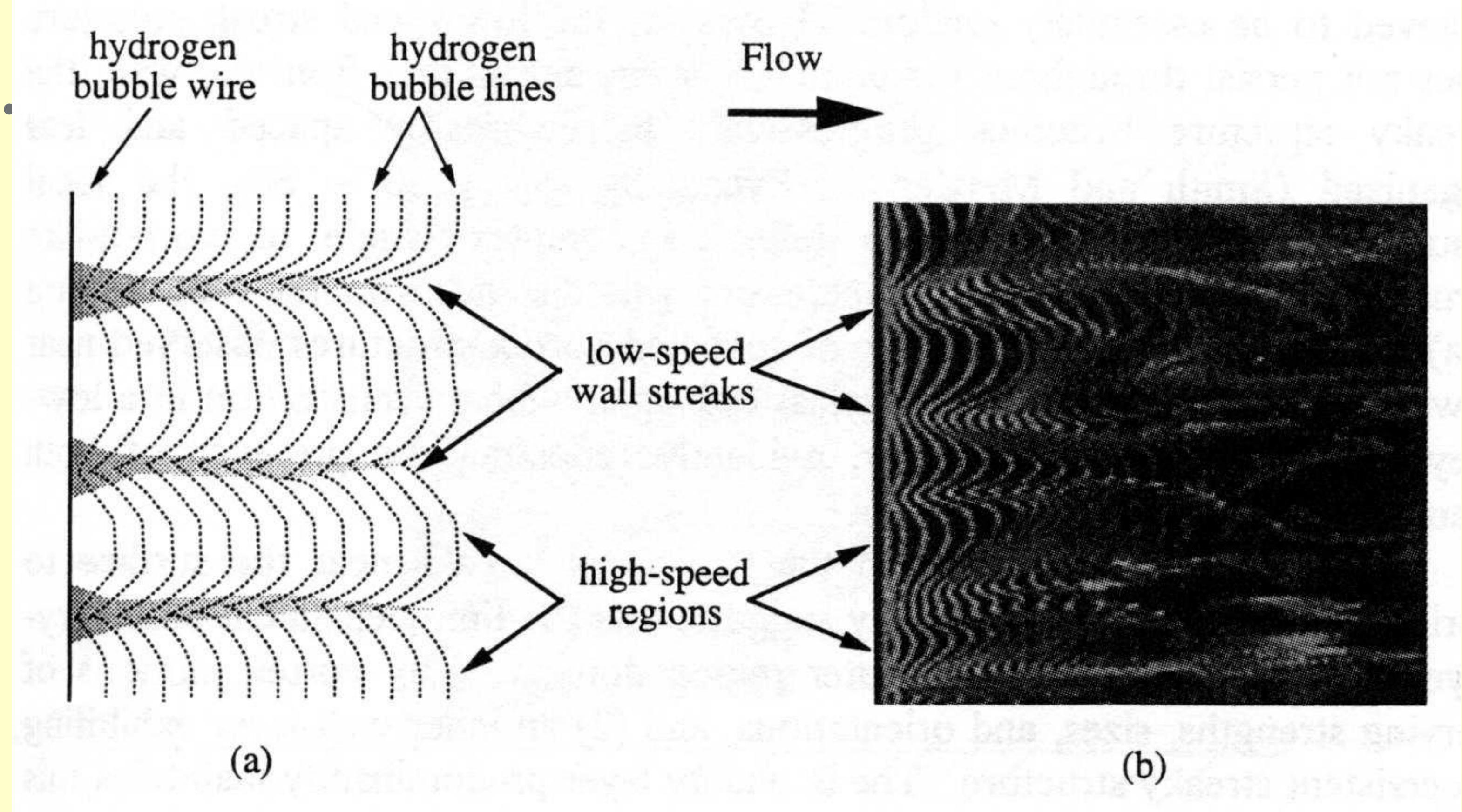
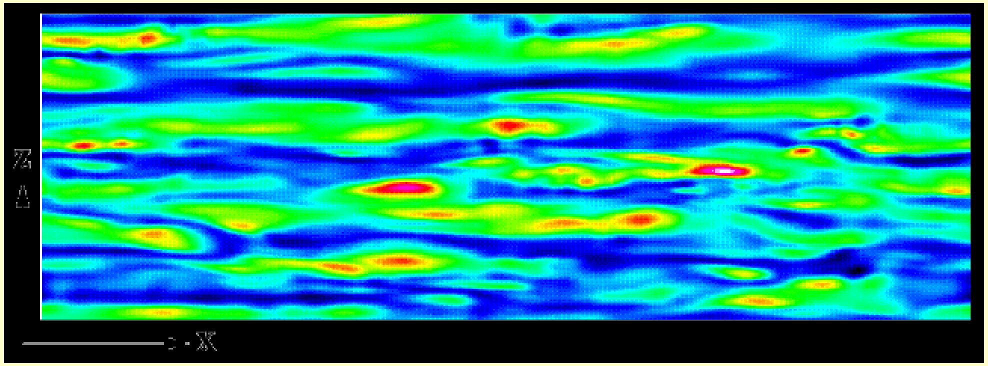
*Joint work with Spencer Sherwin, Hugh Blackburn,*
*Kengo Deguchi, Andrew Walton*

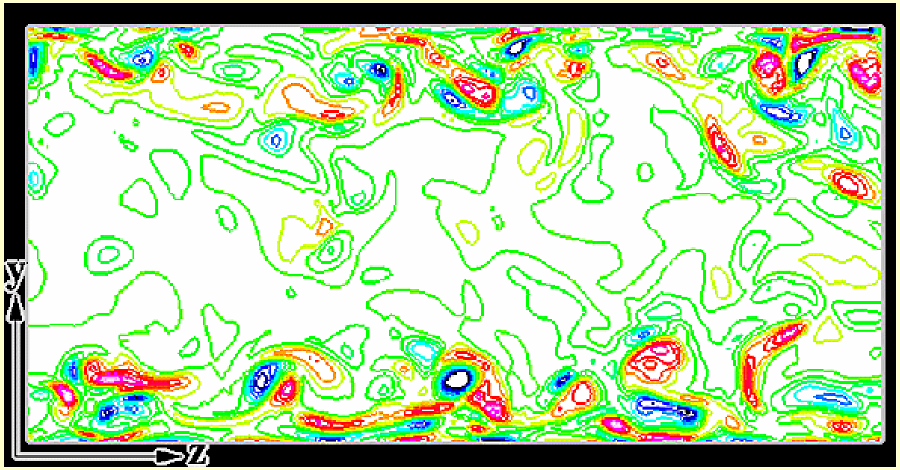
# Turbulent boundary layer




•Kline, Reynolds, Schraub & Runstadler, JFM 1967

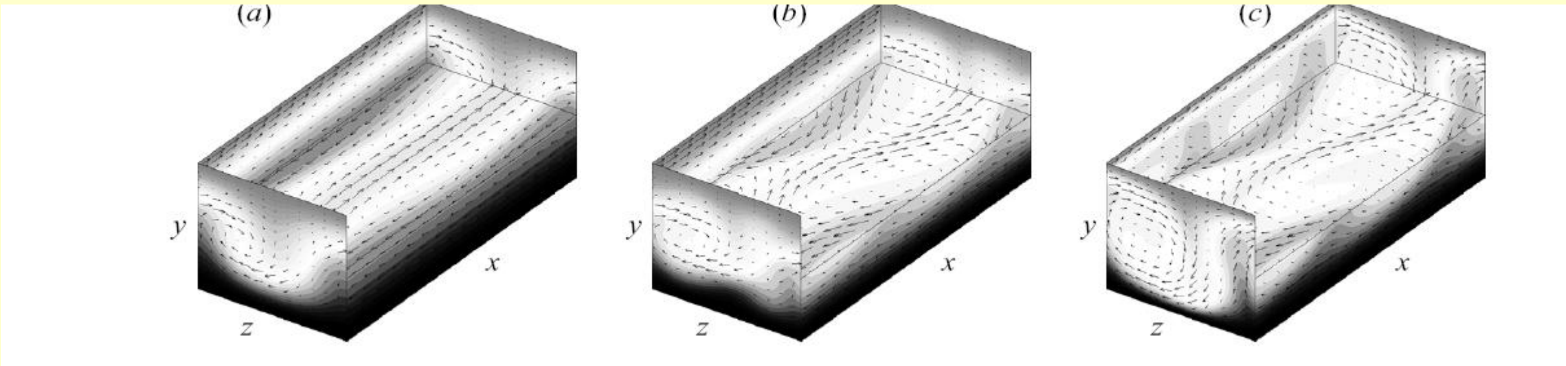
•<u>NUMERICAL SIMULATIONS OF PPF</u>

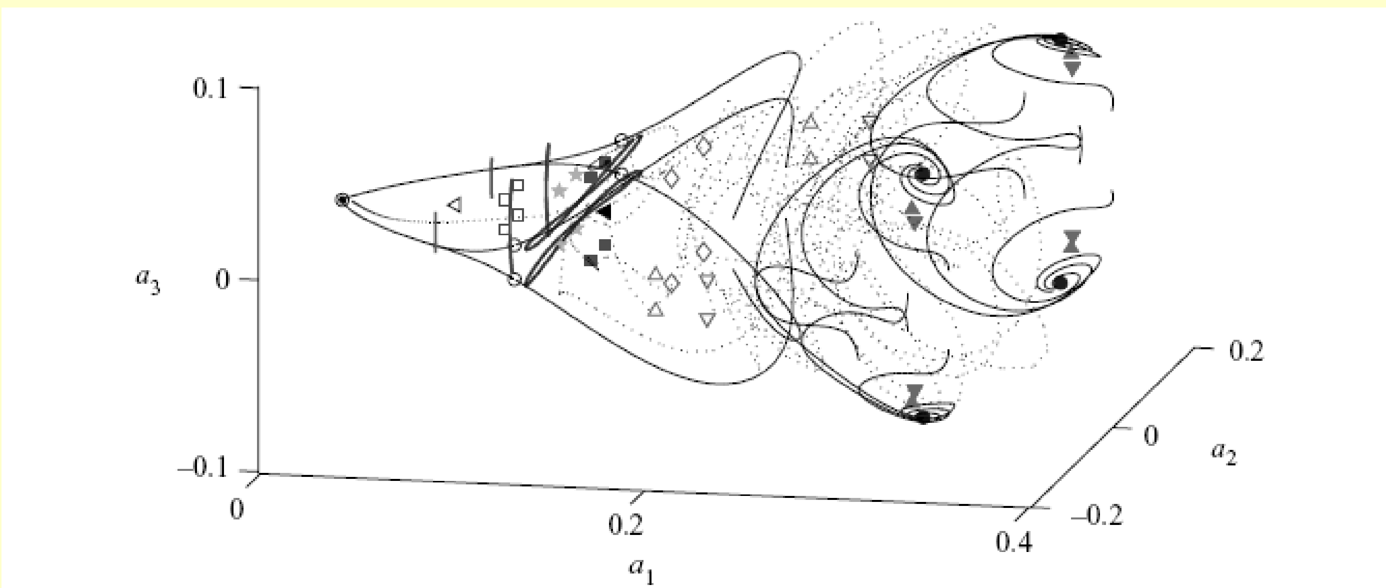



•Kim, Moin & Moser, JFM 1987, R  180, Rm  5600

# Self Sustained processes: Waleffe, Nagata, Kim, Eckhardt, ...




*Numerical simulations of turbulent flow*
*Regularly visit equilibrium structures*

# VORTEX WAVE INTERACTIONS,SELF SUSTAINED PROCESSES ,EXACT COHERENT STRUCTURES: WHATS THE DIFFERENCE?

D.J. Benney

The evolution of disturbances in shear flows at high Reynolds numbers, Stud. Appl. Math. 70, 1-19 (1984).

Hall, P., Smith, F.T.

Vortex-wave interaction theory, formal high R description of rolls, waves and streaks interacting.1987,1988,1991

Nagata. M.

Finite R finite amplitude numerical equilibrium solutions of Plane Couette flow.…1990. Followed by Waleffe, Kerswell, Eckhardt, …… pipes and channels. Numerical results show equilibrium structures with rolls driving streaks which are themselves unstable to waves which drive the rolls and so on ……. “Self sustained processes”

# Nonlinear equilibrium states- Wang, Gibson, Waleffe 2007

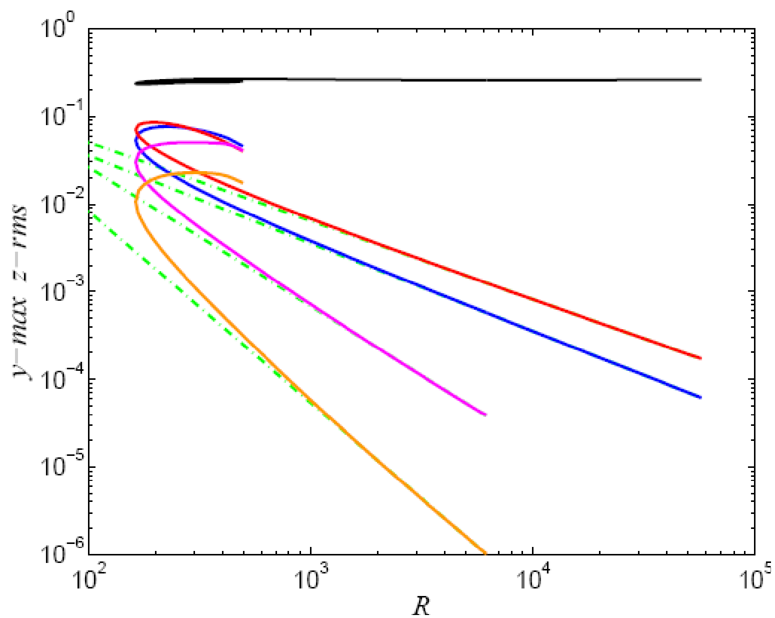



FIG. 1: Amplitude of $x$-Fourier modes for a 3D steady stat in plane Couette flow *vs.* $R$ for $(\alpha, \gamma) = (1, 2)$. Top to bot tom: $O(1)$ streak $u_0(y,z) - \bar{u}(y)$, $O(R^{-0.9})$ fundamental mod $|w_1|$, $O(R^{-1})$ streamwise rolls $(v_0, w_0)$ and $o(R^{-1})$ $|v_2|$ anc $|v_3|$. Continued beyond $R = 6168$ by dropping all harmonics $R_{sn} \approx 164$ is the turning point where lower and upper branc solutions coalesce.

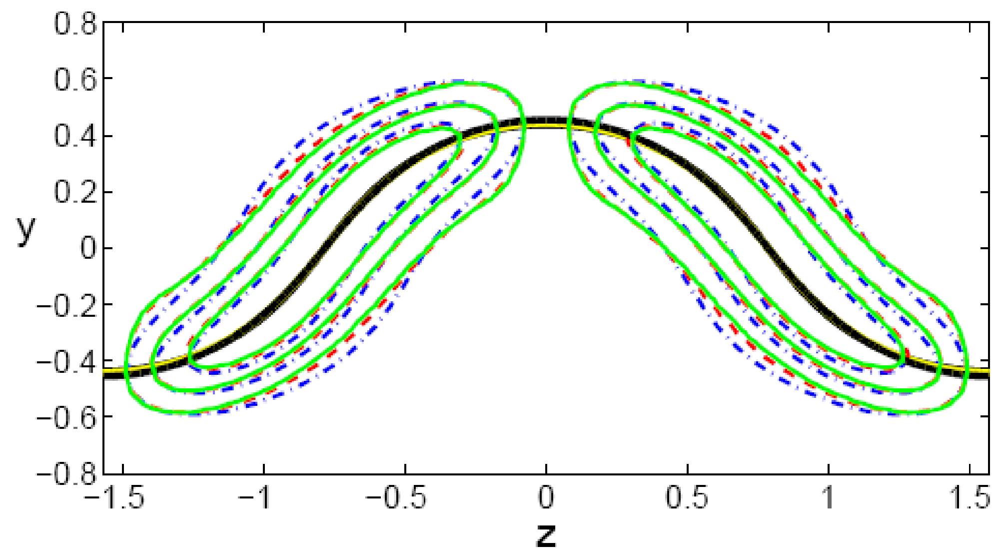



FIG. 3: Contours of $|v_1|$ for $(\alpha, \gamma) = (1, 2)$ at $R = 50171$ (solid) and 12637 (dashed) stretched by $R^{1/3}$ factors along curves normal to $u_0$-contours to match $|v_1|$ contours at $R = 3079$ (dash-dot). The (almost overlapping) black and yellow solid curves show $u_0(y, z) = 0$ at the 3 $R$'s.

*WALEFFE REALIZED CRITICAL LAYER STRUCTURE PRESENT BUT…..*

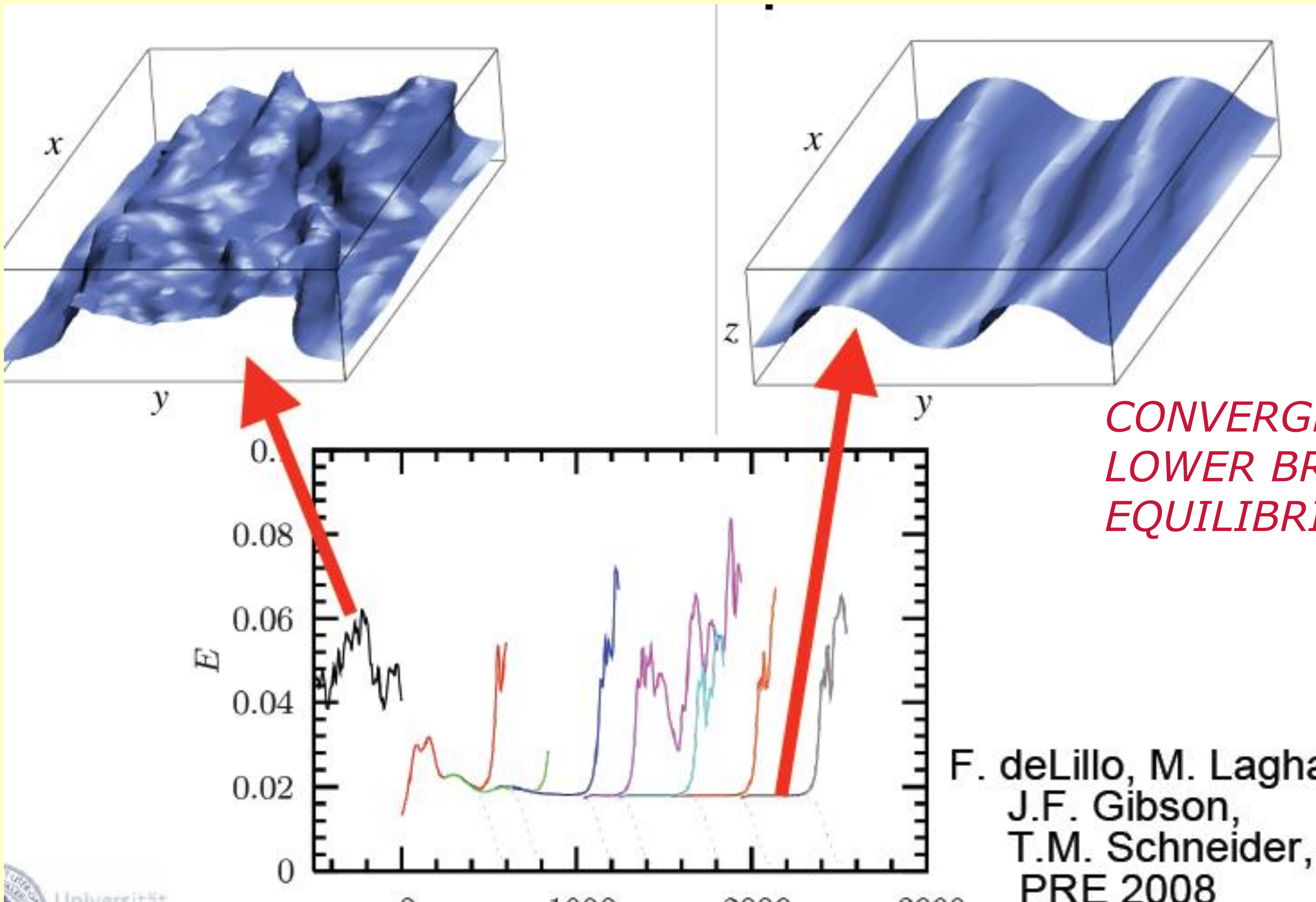
# EDGE STATES: LOWER BRANCH SSP, Eckhardt, Schneider, Kerswell,……

Take initial disturbance of given size and march Navier Stokes equations forward in time.

Adjust initial size so solution stays between ‘laminar’ and ‘turbulent state as long as possible- defines an edge state.

Numerically find edge states are lower branch self sustained processes.

# EDGE STATE




CONVERGES TO
LOWER BRANCH
EQUILIBRIUM STATE

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# LIMITATIONS OF FULL NUMERICAL APPROACH

For R increasing small scale structures develop and cannot find equilibrium states- limited to R<20K. Calculations very expensive, few solutions known.

Physics has to be interpreted from numerical data

Solutions hard to find, upper branch not understood, possibly only lower branch solutions at high R. BUT lower branch play role of ‘gatekeeper’ between laminar and turbulent attractors and DNS simulations of turbulence regularly visit equilibrium. Localized solutions found , similar to turbulent spots found in full DNS simulaitons.

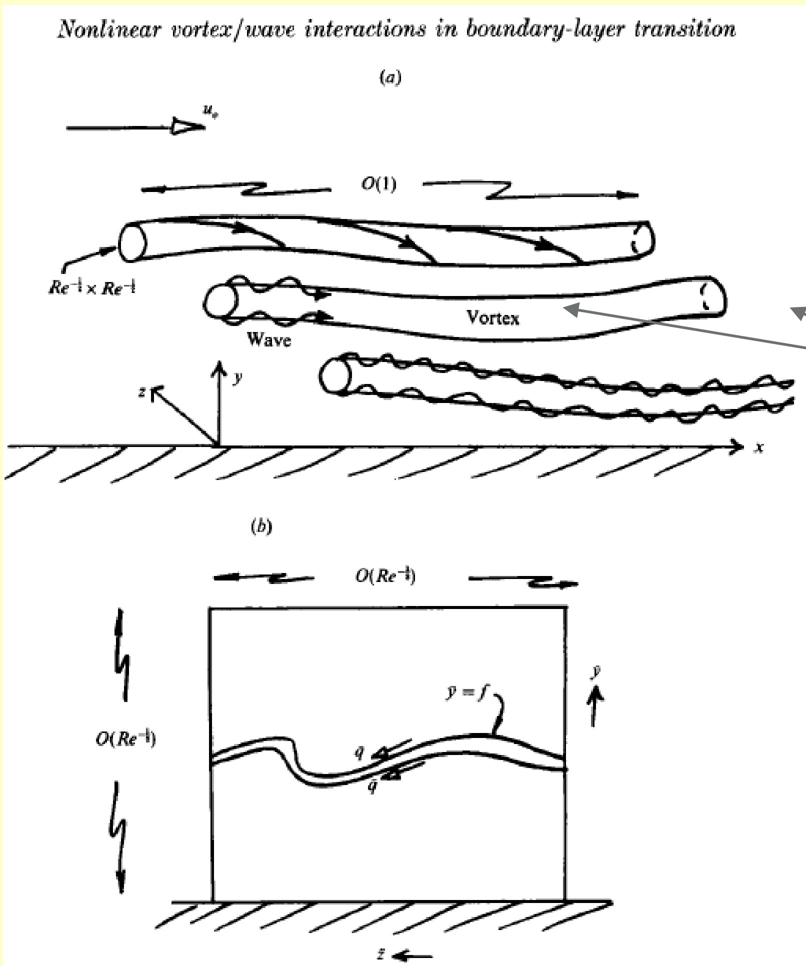
## VORTEX WAVE INTERACTION THEORY

- USE large R assumption so necessarily no difficulty reaching high R, and shows why numerical approach will always fail.

- VWI -Vortex wave interaction theory – VWI. Hall-Smith 89,90,91,92.

- Wave system and longitudinal vortex structure strongly coupled and evolve in time and/or space. VWI suggests wave interaction applied to Couette flow~ 1/R^(5/6). Effect of wave on roll calcualted analytically based on large R, gives jump conditions for roll flow across critical layer.

# Vortex wave interaction theory Hall-Smith 1988-




Longitudinal vortices driven in critical layer or wall layer locally neutrally stable

TS or Rayleigh waves Locally neutral driving vortex in critical or wall layer

Spanwise view of critical layer

Now following Hall-Smith (1991) expand flow in terms of a roll, streak, and wave with wave size fixed in order to drive the roll flow, roll flow decoupled from streak flow and driven by wave Reynolds stresses, roll drives streak then wave lives as instability of streak

$$\widehat{u} = u(y,z,\tau) + R^{-\frac{7}{6}}U(y,z,\tau)E + C.C + ....,$$

$$v = \frac{\overline{v}(y,z,\tau)}{R} + R^{-\frac{7}{6}}V(y,z,\tau)E + C.C. + ....,$$

$$\widehat{w} = \frac{w(y,z,\tau)}{R}) + R^{-\frac{7}{6}}W(y,z,\tau)E + C.C. + ....,$$

$$\widehat{p} = \frac{p(y,z,\tau)}{R} + R^{-\frac{7}{6}}P(y,z,\tau)E + C.C. + ....,$$

# VORTEX WAVE INTERACTION EQUATIONS

$$[w_y]_-^+ = \frac{[v_y]_-^+}{f_z} = J(z)$$

$$J = \rho^2 \left(\frac{2}{3}\right)^{\frac{2}{3}} \left(-\frac{2}{3}\right)! \frac{2\pi}{a^{\frac{5}{3}}\Delta^5} \left\{ \left( -\frac{7\Delta_z}{2\Delta} - \frac{5a_z}{3a} \right) |Q_z|^2 + \frac{\partial}{\partial z} |Q_z|^2 \right\},$$

$$[p]_-^+ = K = -\rho^2 \left(\frac{2}{3}\right)^{\frac{2}{3}} \left(-\frac{2}{3}\right)! \frac{2\pi}{a^{\frac{5}{3}}\Delta^5} f_{zz} |Q_z|^2 .$$

Nonlinear eigenvalue Problem for rho in terms of alpha and spanwise wavenumber

$$\begin{aligned} vu_y + wu_z &= u_{yy} + u_{zz}, \\ vv_y + wv_z + p_y &= v_{yy} + v_{zz}, \\ vw_y + ww_z + p_z &= w_{yy} + w_{zz}, \\ v_y + w_z &= 0. \end{aligned}$$

$$P_{yy} + P_{zz} - \alpha^2 P - \frac{2u_y P_y}{u-c} - \frac{2u_z P_z}{u-c} = 0,$$

$$P_y = 0, y = \pm 1$$

So SSP (Waleffe, Kerswell, Eckhardt….from mid 2001… =finite Reynolds number version of VWI

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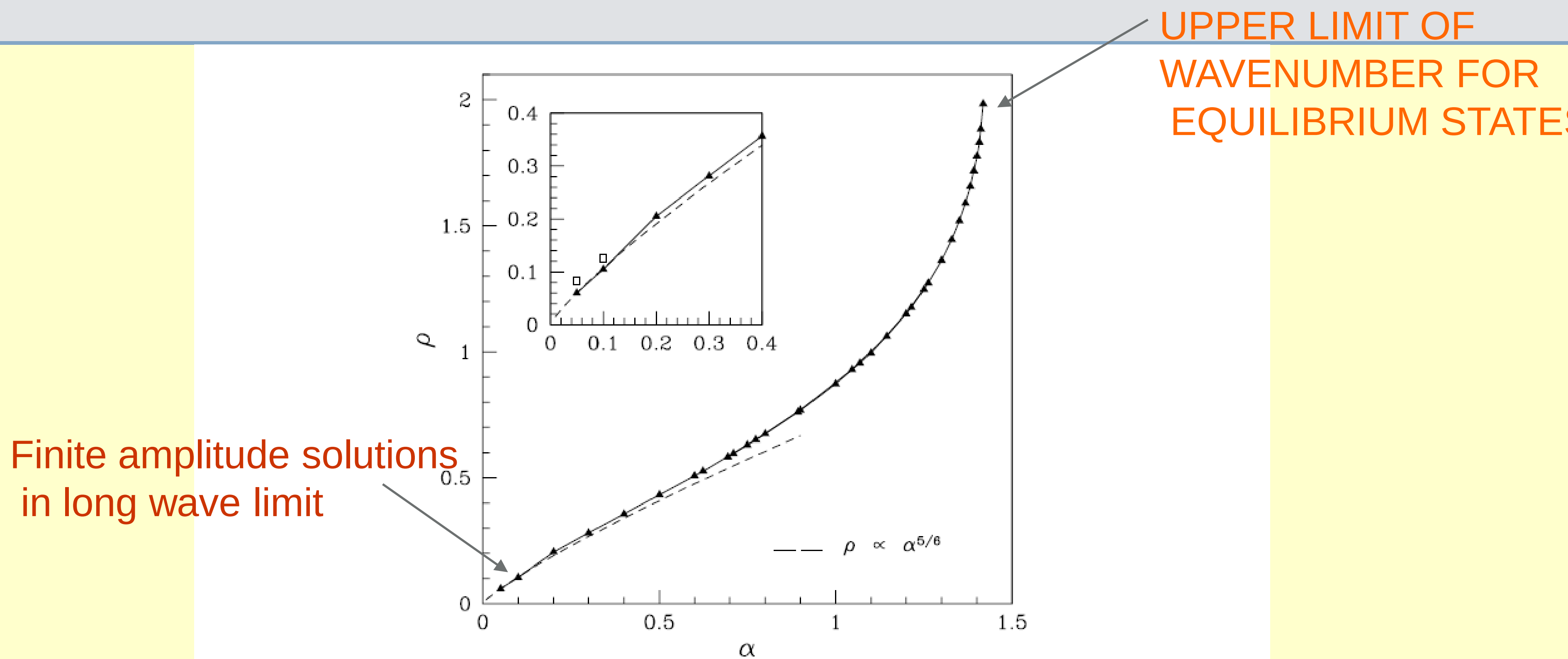



FIGURE 5. Data of $\rho$ versus $\alpha$ at with a zero growth rate is achieved in the non-linear iterations of section 3. Also shown as a dashed line is the plot of $\rho = C\alpha^{5/6}$ where the constant $C$ is fixed by the data at $\alpha = 0.05$.

- Each point on curve generates a familiy of
- solutions over a range of Reynolds numbers
- and small scale structure has been taken out

# Variation of drag with wavenumber



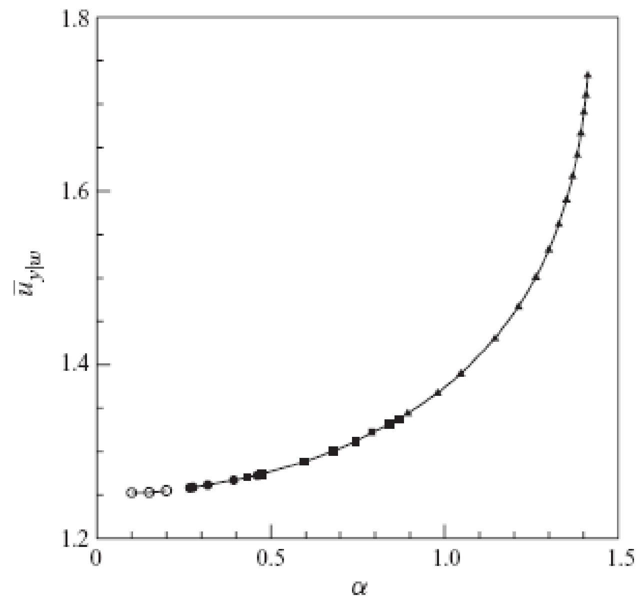
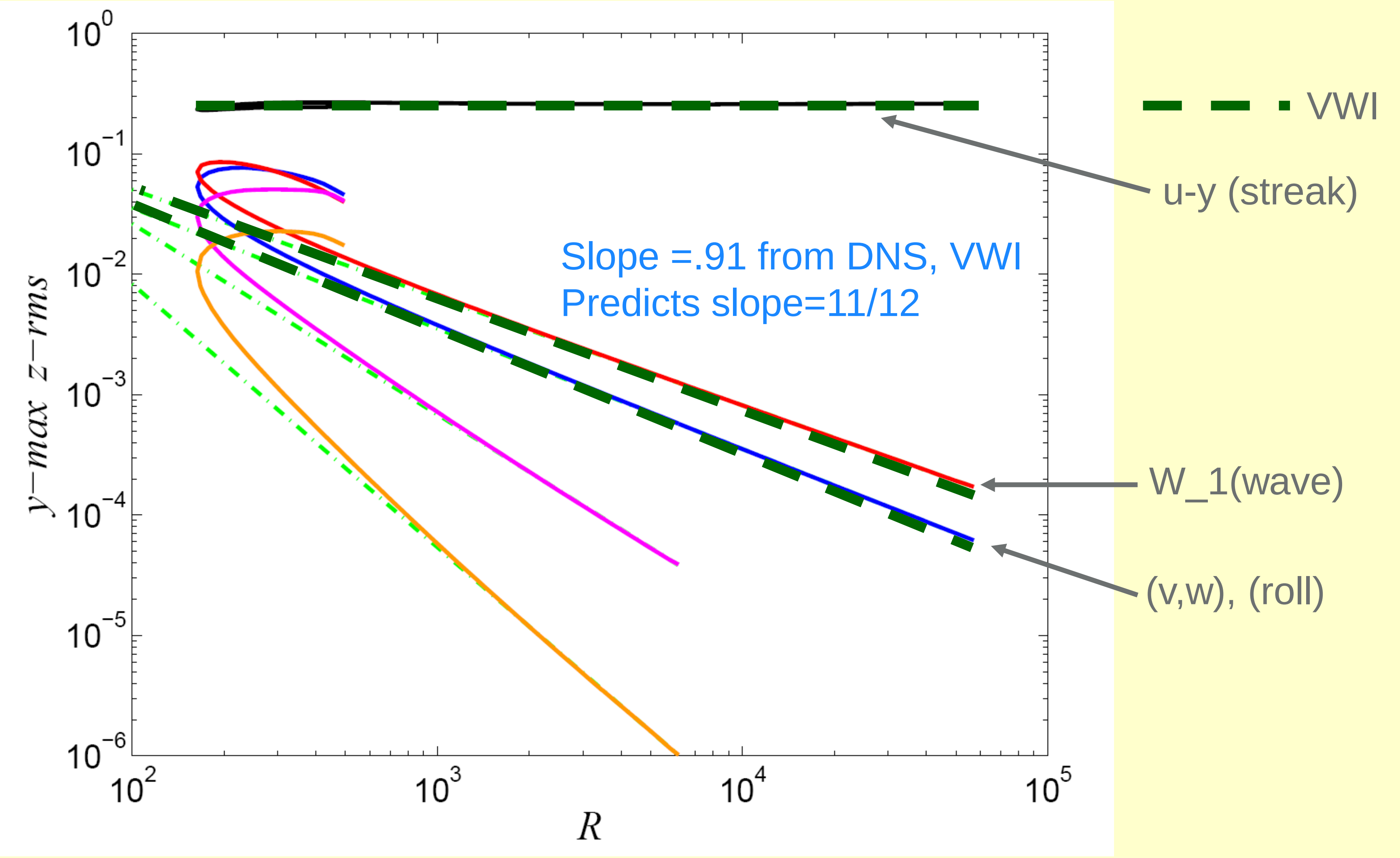


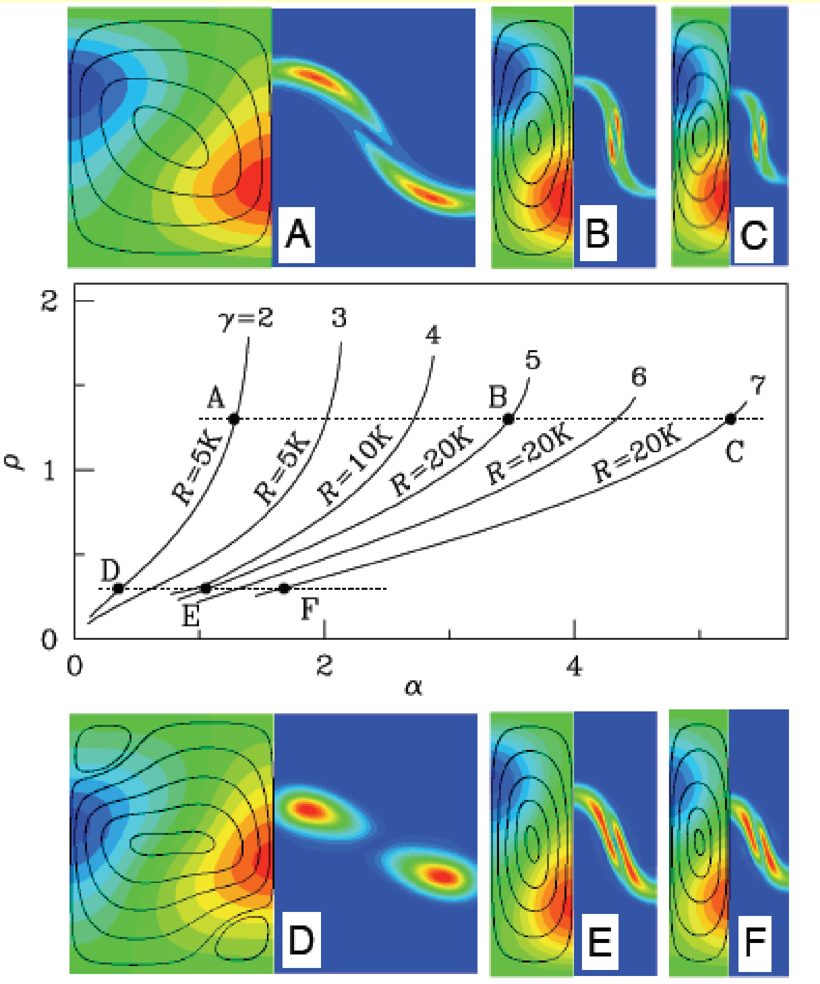

FIGURE 8. Variation of the mean wall shear rate $\bar{u}_{y|w}$ as a function of $\alpha$ evaluated for the values of $\rho$ leading to $c=0$. The different symbols correspond to the search method adopted in obtaining data of figure 5.

Minimum drag at zero wavenumber! Actually not quite zero new problem emerges
At alpha~1/R, critical layer now fills channel, also upper limit for alpha for existence of nonlinear solution, correlates well with minimum box size for sustained turbulence

# Comparison with Wang, Gibson, Waleffe



# Effect of spanwise wavenumber



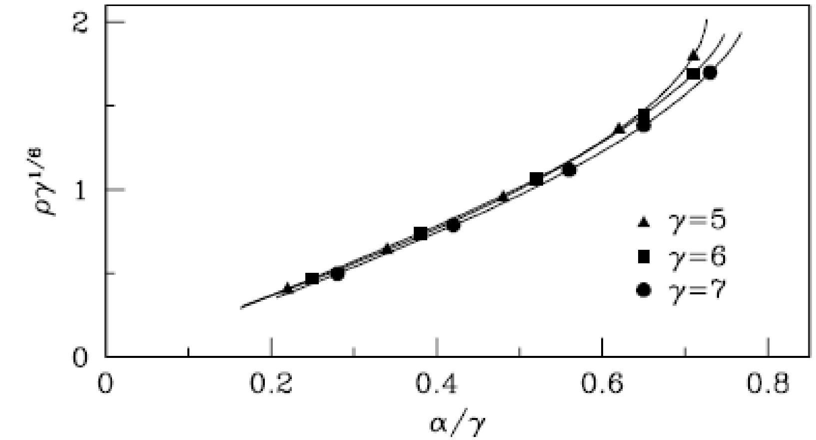




FIG. 5. Collapse of the nonlinear eigenrelation data from Fig. 2 b., demonstrating the relationship (6).

$\rho = \rho(\alpha, \gamma)$ takes the form

$$\rho = \gamma^{-1/6}\,[\alpha/\gamma]^{5/6}\, S\,(\alpha/\gamma)\,. \qquad (6)$$

*So small spanwise limit gives structures independent of walls and gives new canonical state applicable to ANY shear flow. And satisfies Kolmogorov 5/3 scaling!! And generalizes to basic flows u(y,z), so can do Hagen-Poisseuille flow*

# SMALL WAVELENGTH LIMIT, MINIMUM DRAG CONFIGURATION

Theory shows minimum drag occurs when alpha tends to zero, in fact new regime develops when alpha ~ 1/R, critical layer then fills whole channel and all harmonics in streamwise direction matter. (Deguchi, Hall, Walton 2012)

$$\widehat{\mathbf{u}} = (u(x,y,z,\tau), R^{-1}v(x,y,z,\tau), R^{-1}w(x,y,z,\tau))(1+O(R^{-2})),$$
$$\widehat{p} = R^{-2}p(x,y,z,\tau) + O(R^{-4}),$$
$$\tau = \widehat{t}/R, \quad x = \widehat{x}/R, \quad y = \widehat{y}, \quad z = \widehat{z},$$

and substitution into (2.1) yields

$$\frac{D\mathbf{u}}{D\tau} = -(0, p_y, p_z) + (\partial^2_{yy} + \partial^2_{zz})\mathbf{u},$$
$$\nabla \cdot \mathbf{u} = 0,$$

for $\mathbf{u} = (u, v, w)$, where

$$\frac{D}{D\tau} \equiv \frac{\partial}{\partial\tau} + \mathbf{u}\cdot\nabla, \quad \nabla \equiv (\partial_x, \partial_y, \partial_z).$$

*i.e. just the NONLINEAR GORTLER VORTEX EQUATIONS OF HALL 1988 with G=0*

# Nonlinear Equilibrium states for long waves

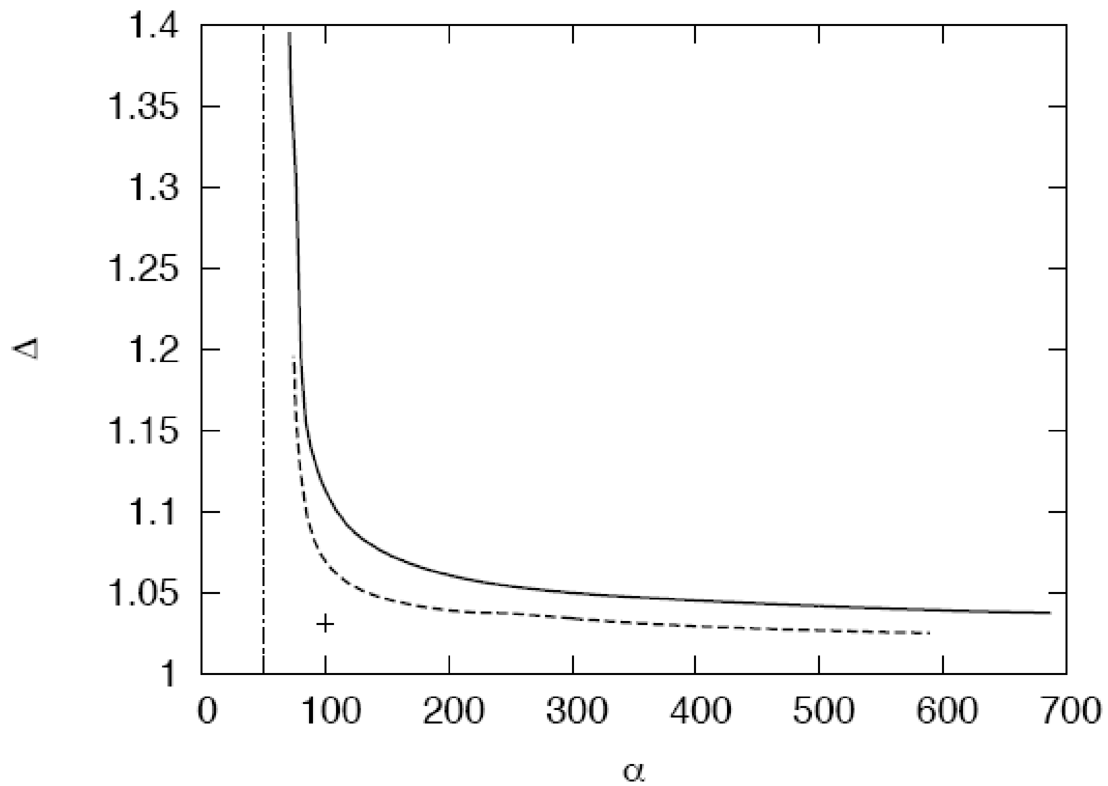



FIGURE 4.1. The mean wall shear $\Delta$. The solid and dashed curves are $\beta = 1.37$ and 1 respectively. The vertical dash-dotted line is the 2D energy threshold. The cross represents the single result for $(\alpha, \beta) = (100, 0.5)$.

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# EQUILIBRIUM STATES

16 *Kengo Deguchi*[1], *Philip Hall*[2] *and Andrew Walton*[3]

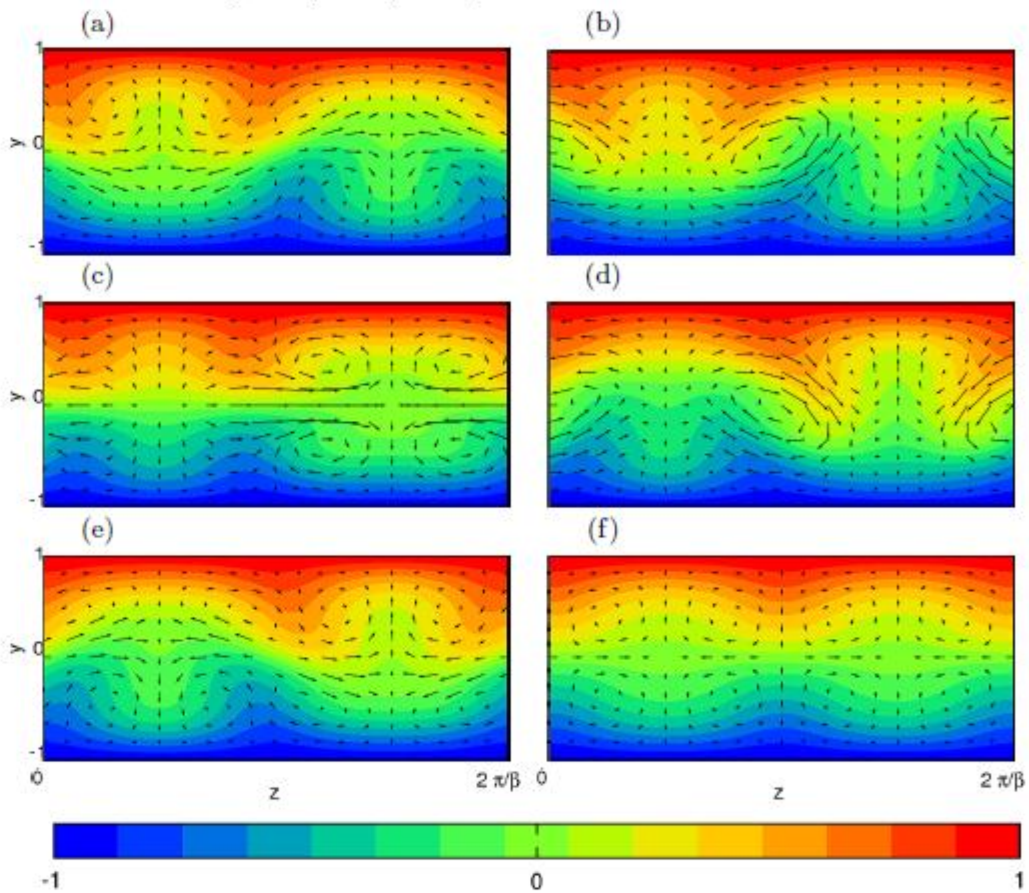


FIGURE 4.3. The velocity field **u** for $(\alpha, \beta) = (71, 1.37)$. The colours represent the streamwise component $u$ whereas the arrows are cross-sectional components $v, w$. (a), (b), (c), (d) and (e) correspond to streamwise cross-sections at $x = (n/8)(2\pi/\alpha)$ with $n = 0, 1, 2, 3$ and 4 respectively. (f) is the streamwise-averaged field.

# DEVELOPMENT OF SPOTS- locally in x for fixed beta, then x and x as beta decreases

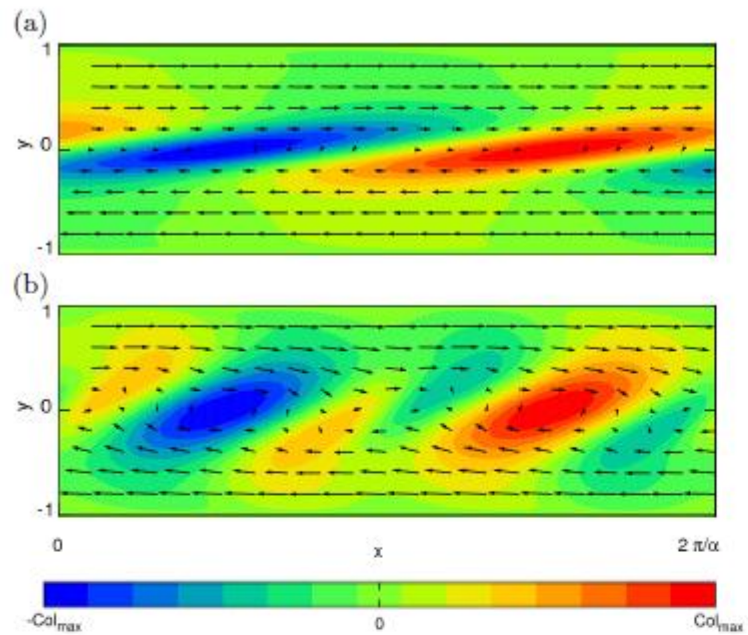


FIGURE 4.4. The velocity field $\mathbf{u}$ at $z = 0$. The colours represent the spanwise component $w$ whereas the arrows are cross-sectional components $u, v/\alpha$. (a): $(\alpha, \beta) = (500, 1.37)$, $\mathrm{Col}_{\max} = 18.9$. (b): $(\alpha, \beta) = (71, 1.37)$, $\mathrm{Col}_{\max} = 36.7$.

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# DEVELOPMENT OF SPOTS- locally in x for fixed beta, then x and x as beta decreases



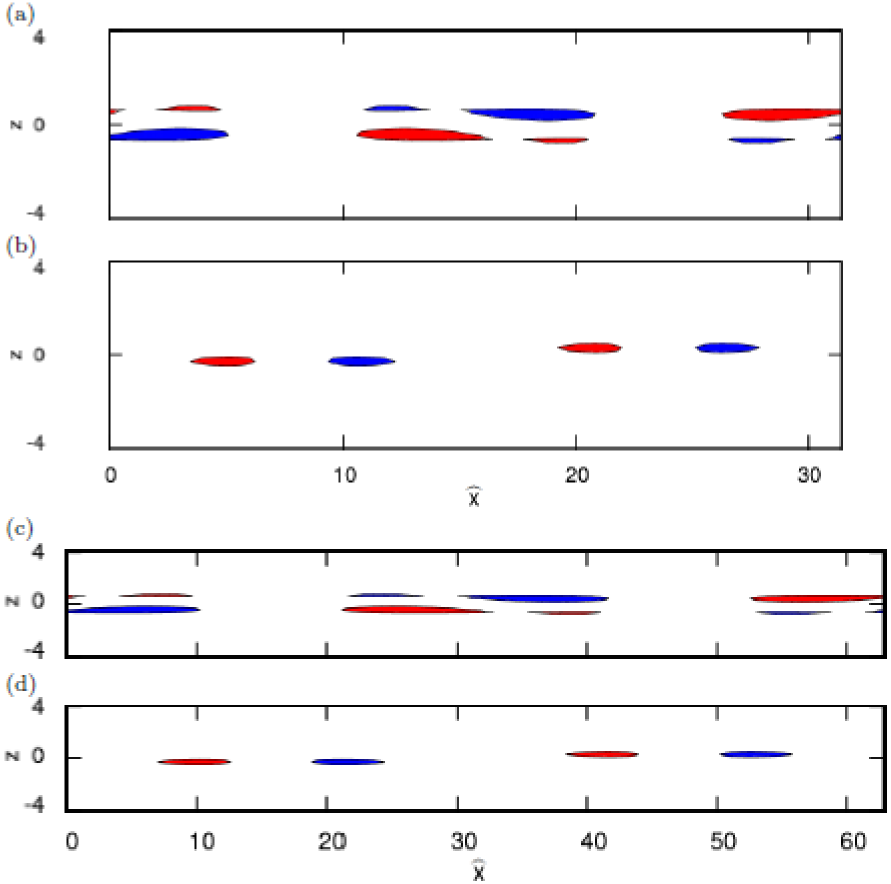



FIGURE 4.10. Red/blue represent the regions where the velocity exceeds 80% of its in-plane maximum/minimum. (a) Streamwise component $u$ on $y = 0$; (b) wall-normal component $v$ on $y = 0$. The solution at $(\alpha, \beta) = (100, 0.5)$ is used. The streamwise coordinate is rescaled to the original variable $\hat{x}$ by using $R = 500$. (c) & (d): same as (a) & (b) but for $R = 1000$.

# DEVELOPMENT OF SPOTS- locally in x for fixed beta, then x and z as beta decreases

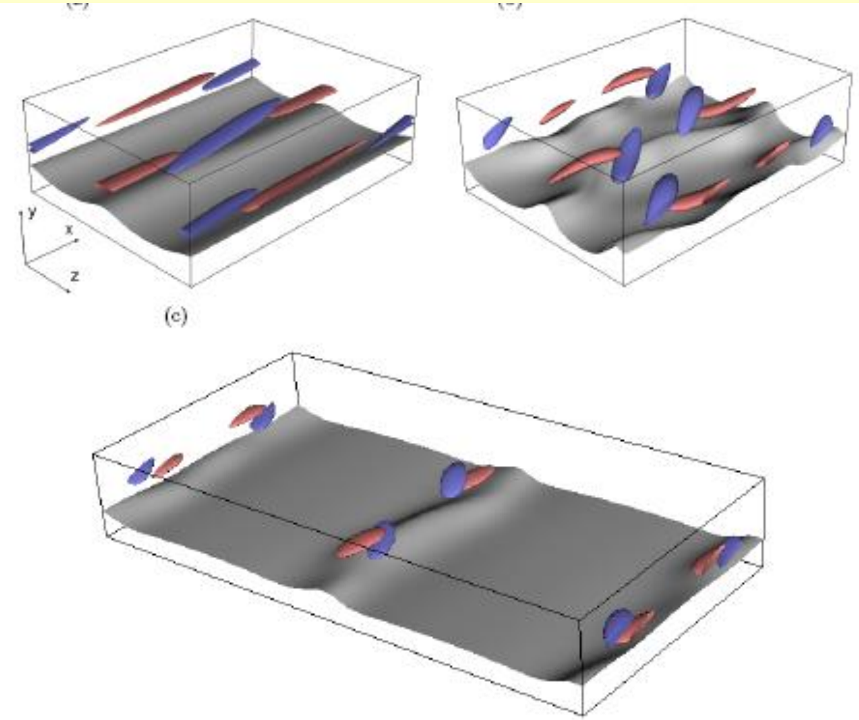
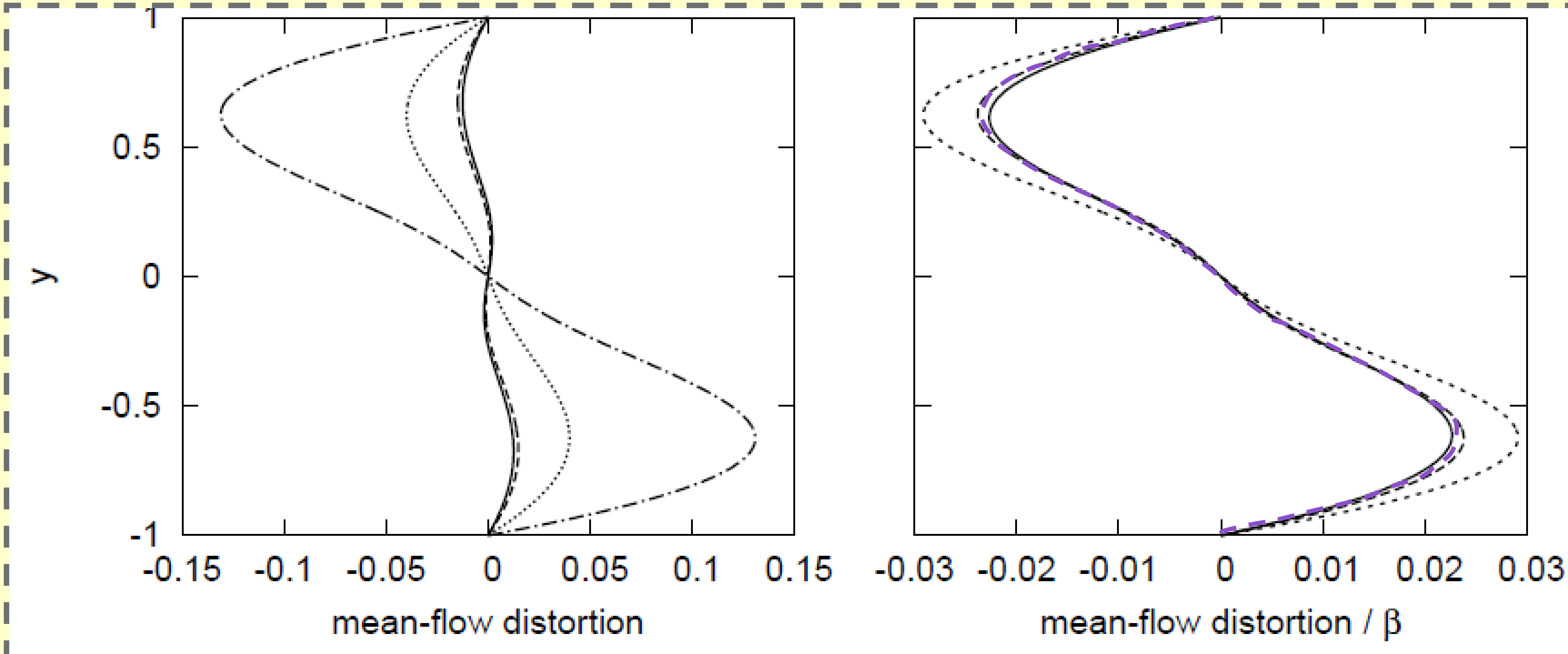



FIGURE 4.8. The grey surface represents an isosurface of 50% of the maximum of the streamwise velocity. The red/blue bubbles are the isosurface of 80% of the maximum/minimum of the streamwise vorticity. (a) $(\alpha, \beta) = (500, 1.37)$; (b) $(\alpha, \beta) = (71, 1.37)$; (c) $(\alpha, \beta) = (100, 0.5)$.

# MEAN FLOW CORRECTION




*Mean flow identical to full NS simulations of turbulence by Kamiano,Lundbladh et al 2001 and others- right hand curve shows mean flow distortion/beta*
*----- DNS calculations of Lundbladh*

# CONCLUSION

VWI captures lower branch equilibrium states basically for all R, so edge state known precisely. Extends to arbitrary shear flows to give new class of equilibrium states, satisfies Kolmogorov 5/3 law.

Upper limit of streamwise wavenumber correlates with minimum box size for sustained turbulence in a box.

Long wave limit predicts minimum box size ~R/10 needed for turbulent spots to develop

Key is using high R asymptotics ot calcualte detailed structure leaving CFD applied to small system with no small scale structure.

Long wavelength limit gives remarkable prediction of turbulent mean flow- means large scale structures fix mean state, can allow for small scale structures