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Perspectives in modeling wall effects in the RANS approach

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Outline

- Identification of problems with the RANS approach
- Possible directions for their solution

RANS approach: identity crisis

General belief: RANS models are one- or two-equation turbulence models that require the modeling of wall effects

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial < u_i u_j >}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j}$$

$$< u_i u_j > = -\nu_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}$$

$$k-l, \quad k-\omega, \quad k-\varepsilon, \quad k-\xi, \quad k-\varphi, \quad k-\varepsilon / k-\omega, \quad multi-scale....$$

RANS approach: reality

RANS model is any statistical closure obtained from the infinite set of the Reynolds-Averaged Navier-Stokes equations

1st order

$$\frac{\bar{D}U_i}{Dt} = -\frac{\partial < u_i u_j >}{\partial x_j} + \cdots$$

2nd order

$$\frac{\bar{D} < u_i u_j >}{Dt} = -\frac{\partial < u_i u_j u_k >}{\partial x_k} + \Pi_{ij} - \varepsilon_{ij} + \cdots$$

3rd order

$$\frac{\bar{D} < u_i u_j u_k >}{Dt} = -\frac{\partial < u_i u_j u_k u_l >}{\partial x_l} + \Pi_{ijk} - \varepsilon_{ijk} + \cdots$$

The order of a closure is determined by the order of the moments for which the equations are solved.

Higher the order, higher the model fidelity.

The infinite set of the Reynolds-Averaged Navier-Stokes equations describes completely the turbulent flow physics from the statistics point of view

- Two-equation models are the *first-order closures.* They are the simplest from the RANS family of models with not so much physics left.

- The purpose of any corrections in such models is not to bring more physics, but to compensate for its lack.

Why not to increase the closure order instead?

Second-order closures (RSTMs)

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial < u_i u_j >}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j}$$

$$\frac{\partial < u_i u_j >}{\partial t} + U_k \frac{\partial < u_i u_j >}{\partial x_k} = -\left[< u_j u_k > \frac{\partial U_i}{\partial x_k} + < u_i u_k > \frac{\partial U_j}{\partial x_k} \right] \qquad P_{ij}$$

$$-\frac{\partial < u_i u_j u_k >}{\partial x_k} - \frac{1}{\rho}\left[< u_j \frac{\partial p}{\partial x_i} > + < u_i \frac{\partial p}{\partial x_j} > \right] \qquad D^{(T)}{}_{ij} + \Pi_{ij}$$

$$+\nu \frac{\partial^2 < u_i u_j >}{\partial x_k \partial x_k} - 2\nu < \frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k} > \qquad D^{(M)}{}_{ij} - \varepsilon_{ij}$$

Algebraic models did not show much potential in flows of practical importance

Current state-of-the art in RSTMs

Terms to model	Modeling directions
$D^{(T)}{}_{ij} = \frac{\partial < u_i u_j u_k >}{\partial x_k}$	$< u_i u_j u_k >$
$\Pi_{ij} = -\frac{1}{\rho}(< u_i \frac{\partial p}{\partial x_j} > + < u_j \frac{\partial p}{\partial x_i} >)$	$\Phi_{ij} = \Pi_{ij} - \frac{1}{\rho}(\frac{\partial < p u_i >}{\partial x_j} > + \frac{\partial < p u_j >}{\partial x_i})$
ε_{ij}	

Third-order moments modeling

A tensor can only be modeled as a tensor of the same tensor rank, index order, covariance, and symmetry

$< u_i u_k u_l >$ is a *symmetric* tensor, so its model should be *symmetric*

Standard model: Daly & Harlow (1970): $< u_i u_k u_l > \sim < u_l u_m > \dfrac{\partial < u_i u_k >}{\partial x_m}$

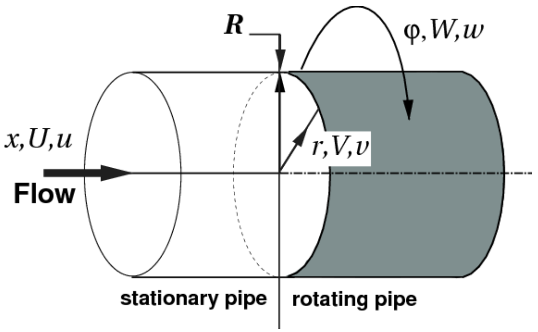
Better choice: Hanjalić & Launder (1972):

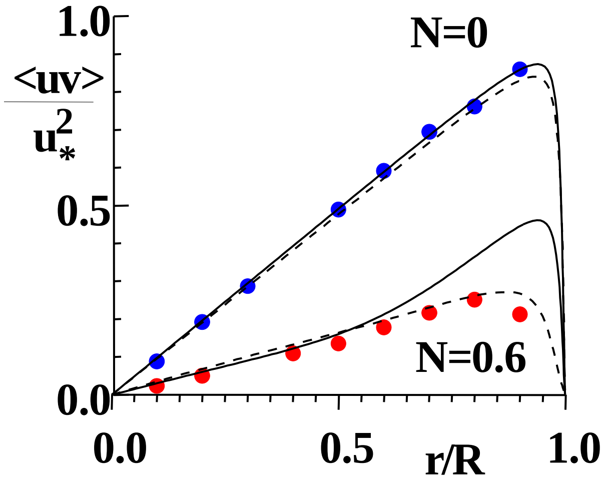
$$< u_i u_k u_l > \sim < u_l u_m > \frac{\partial < u_i u_k >}{\partial x_m} + < u_i u_m > \frac{\partial < u_k u_l >}{\partial x_m} + < u_k u_m > \frac{\partial < u_i u_l >}{\partial x_m}$$

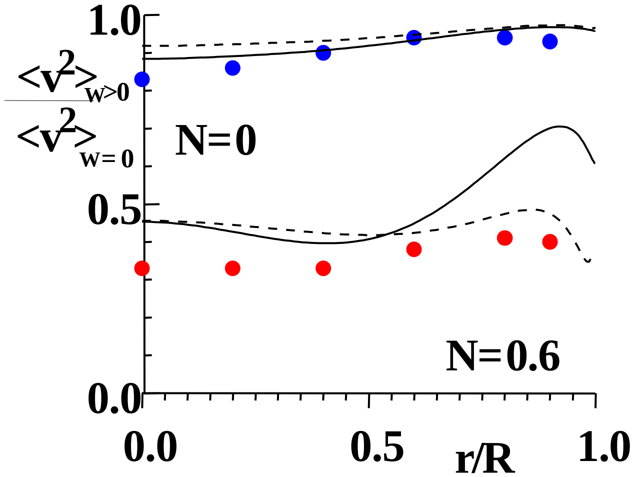
Relevance to wall effects

Rotating pipe flow:

$$N = W_0 \,/\, U_0$$





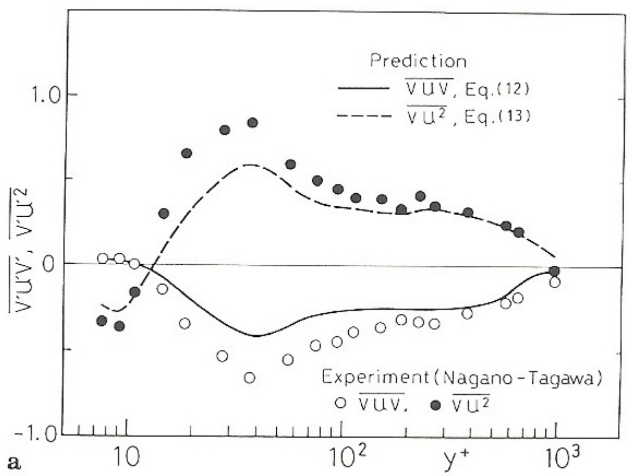


Daly & Harlow (1970) ———

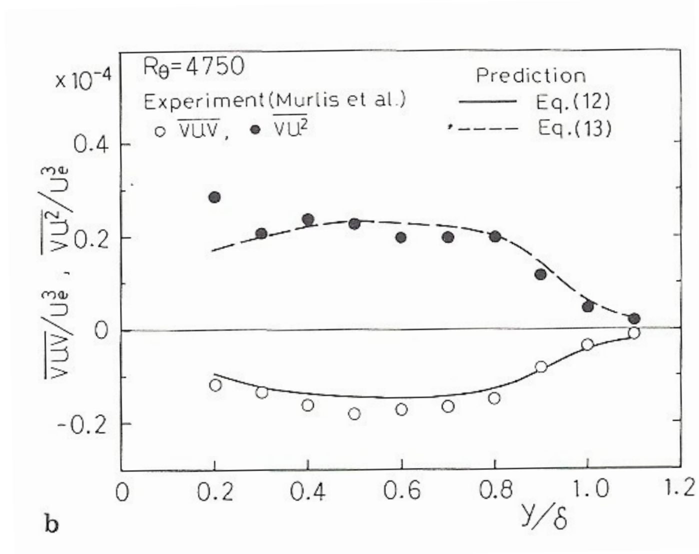
Hanjalić & Launder (1972) - - -

Kurbatskii & Poroseva, *Int. J. Heat Fluid Flow*, 1999

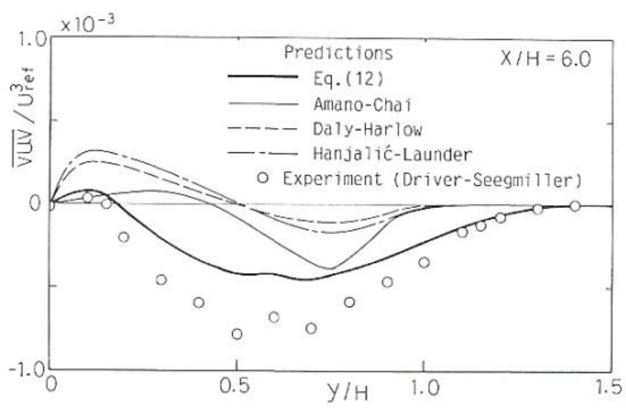
Pipe

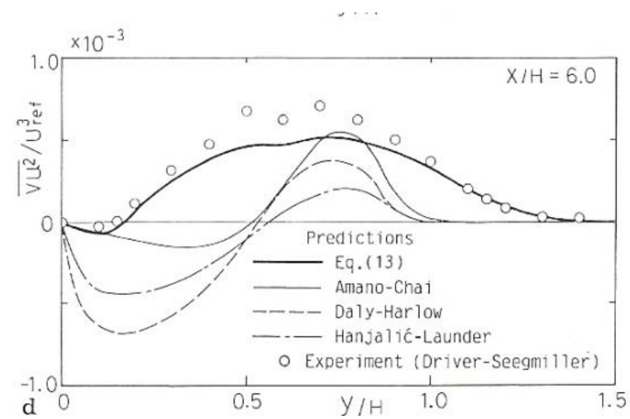


Flat plate



Back-step



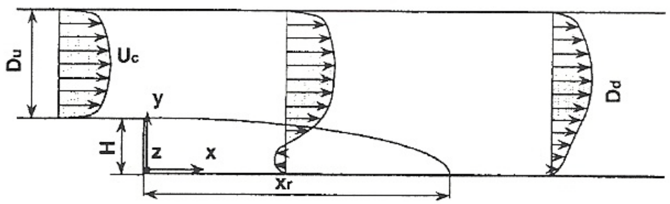


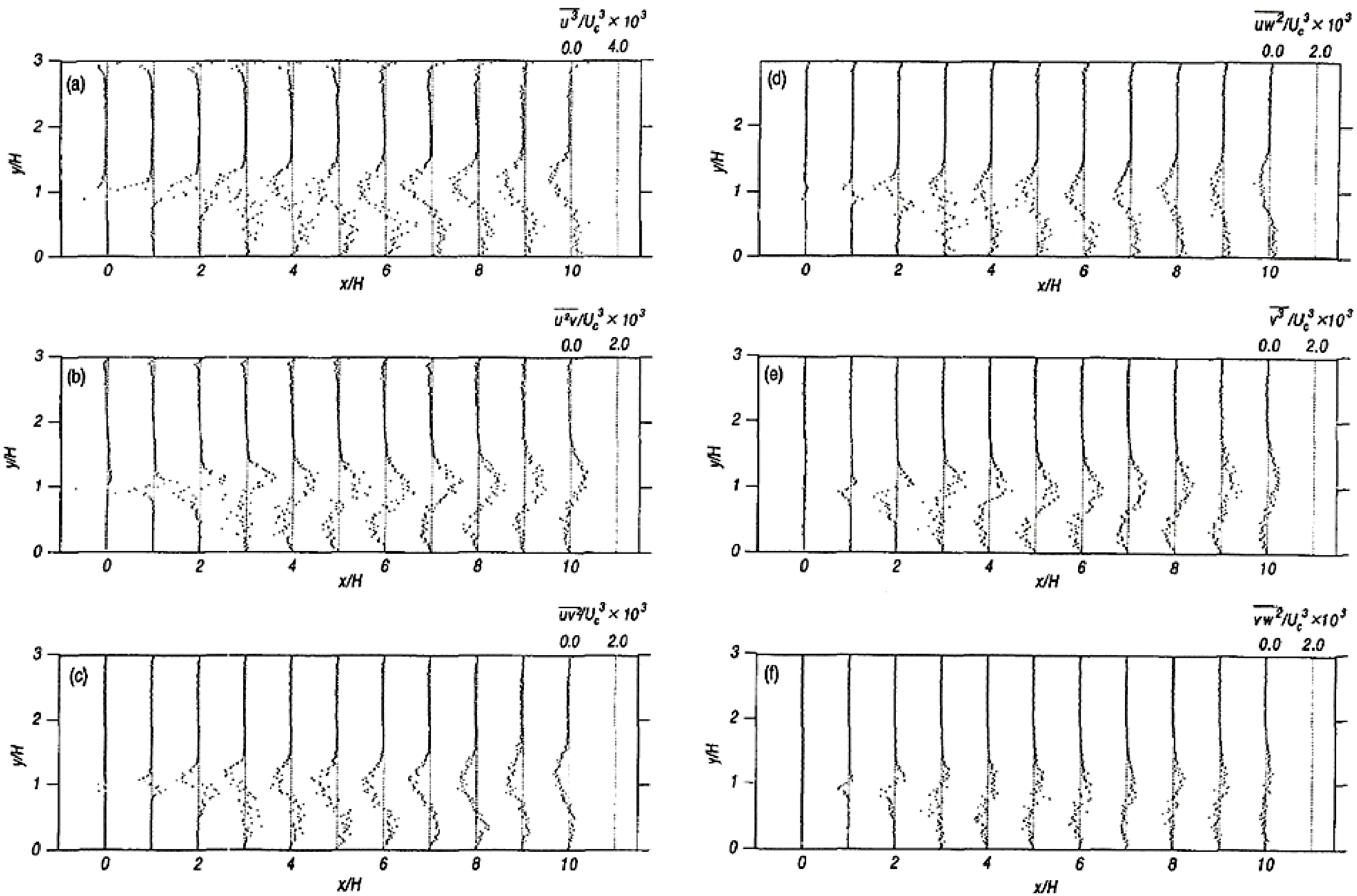
Nagano & Tagava, *TSFP 7*, 1991

Relevance to wall effects

Back-step flow: wall +separation effects

Kasagi & Matsunaga, *Int. J. Heat Fluid Flow*, 1995





Solution for turbulent diffusion

Higher-order closures can be a required choice:

$$\frac{\overline{D} < u_i u_j u_k >}{Dt} = -\frac{\partial < u_i u_j u_k u_l >}{\partial x_l} + \Pi_{ijk} - \varepsilon_{ijk} + \cdots$$

Pressure-containing correlations modeling

$$\Phi_{ij} = \Pi_{ij} - \boxed{\frac{1}{\rho}\left(\frac{\partial < pu_i >}{\partial x_j} > + \frac{\partial < pu_j >}{\partial x_i}\right)}$$

Choices for modeling the pressure diffusion

- neglect
- absorb in a model for the turbulent diffusion
- model separately from the pressure-strain correlations, somehow

Relevance to wall effects

$$u_i(x_2,t) = a_i + b_i x_2 + c_i x_2^2 + d_i x_2^3 + \cdots$$

Table 6.1 *Wall-limiting behaviour of the leading terms in the Reynolds stress budget*

ij	Φ_{ij}	$\Pi_{ij} \equiv (\mathcal{D}^p_{ij} + \Phi_{ij})$	$\mathcal{D}^p_{ij}$	$\mathcal{D}^\nu_{ij}$	$-\varepsilon_{ij}$
11	$2\overline{a_p \partial b_1/\partial x_1}\, x_2$	$-4\nu\, \overline{b_1 c_1}\, x_2$	$-\left(2\, \overline{a_p \partial b_1/\partial x_1} + 4\nu\, \overline{b_1 c_1}\right) x_2$	$2\nu\, \overline{b_1 b_1} + 12\nu\, \overline{b_1 c_1}\, x_2$	$-2\nu\, \overline{b_1 b_1} - 8\nu\, \overline{b_1 c_1}\, x_2$
22	$4\, \overline{a_p c_2}\, x_2$	$-4\nu\, \overline{c_2 c_2}\, x_2^2$	$-\left(4\, \overline{a_p c_2}\, x_2 + 4\, \nu \overline{c_2 c_2}\, x_2^2\right)$	$12\nu\, \overline{c_2 c_2}\, x_2^2$	$-8\nu\, \overline{c_2 c_2}\, x_2^2$
33	$2\overline{a_p \partial b_3/\partial x_3}\, x_2$	$-4\nu\, \overline{b_3 c_3}\, x_2$	$-\left(2\, \overline{a_p \partial b_3/\partial x_3} + 4\nu\, \overline{b_3 c_3}\right) x_2$	$2\nu\, \overline{b_3 b_3} + 12\nu\, \overline{b_3 c_3}\, x_2$	$-2\nu\, \overline{b_3 b_3} - 8\nu\, \overline{b_3 c_3}\, x_2$
12	$\overline{a_p b_1}$	$-2\nu\, \overline{b_1 c_2}\, x_2$	$-\left(\overline{a_p b_1} + 2\nu\, \overline{b_1 c_2}\, x_2\right)$	$6\nu\, \overline{b_1 c_2}\, x_2$	$-4\nu \overline{b_1 c_2}\, x_2$
23	$\overline{a_p b_3}$	$-2\nu\, \overline{b_3 c_2}\, x_2$	$-\left(\overline{a_p b_3} + 2\, \nu\, \overline{b_3 c_2}\, x_2\right)$	$6\nu\, \overline{b_3 c_2}\, x_2$	$-4\nu\, \overline{b_3 c_2}\, x_2$
13	$\overline{a_p(\partial b_1/\partial x_3 + \partial b_3/\partial x_1)} x_2$	$-2\nu\left(\overline{b_1 c_3} + \overline{b_3 c_1}\right) x_2$	$-\overline{a_p(\partial b_1/\partial x_3 + \partial b_3/\partial x_1)} x_2$ $-2\nu\left(\overline{b_1 c_3} + \overline{b_3 c_1}\right) x_2$	$2\nu \overline{b_1 b_3} + 6\nu\, \left(\overline{b_1 c_3} + \overline{b_3 c_1}\right) x_2$	$-2\nu\, \overline{b_1 b_3} - 4\nu\, \left(\overline{b_1 c_3} + \overline{b_3 c_1}\right) x_2$

Hanjalić & Launder, 2011

cannot be neglected and particularly near a wall

Relevance to wall effects

Flat plate, ZPG

Spalart, *JFM*,1988

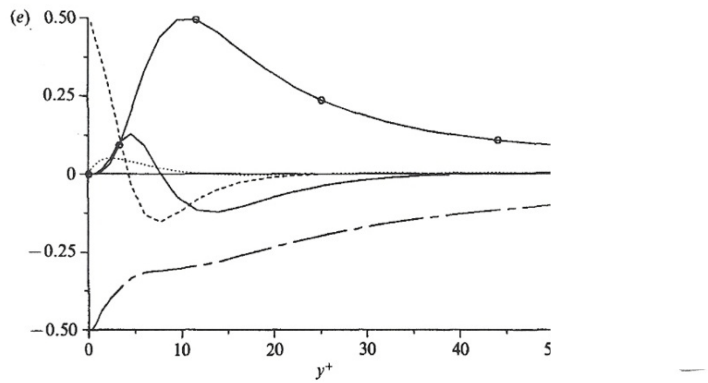


FIGURE 24. Reynolds-stress budget terms near wall. Normalized by u_τ^4/ν. (a) u^2; (b) v^2; (c) w^2; (d) $-uv$; (e) $u^2+v^2+w^2$. —○—, production; ——, turbulent diffusion; ---, viscous diffusion; —–—, dissipation; ···, pressure.

$$-\frac{1}{\rho}\times \left| \; \frac{\partial < pu_i >}{\partial x_i} = \frac{\partial < pu_i >^{(r)}}{\partial x_i} + \boxed{\frac{\partial < pu_i >^{(s)}}{\partial x_i}} + \frac{\partial < pu_i >^{(w)}}{\partial x_i} \right.$$

$$\frac{1}{5} < u_m u_m u_i >_{,i}$$

(Lumley, *Adv. Appl. Mech*.,1978; Poroseva, *THMT*, 2000)

cannot be absorbed into a turbulent diffusion model

Pressure-strain correlations modeling

$$\boxed{\Phi_{ij}} = \Pi_{ij} - \frac{1}{\rho}\left(\frac{\partial < pu_i >}{\partial x_j} + \frac{\partial < pu_j >}{\partial x_i}\right) = \Phi^{(r)}{}_{ij} + \Phi^{(s)}{}_{ij} + \Phi^{(w)}{}_{ij}$$

Choices for modeling Φ_{ij}

- linear
- non-linear
- other approaches (*Q*-model)

Relevance to wall effects

Rotating pipe flow: IP (Naot et al., 1970), LRR (Launder et al., *JFM*, 1975), SSG (Speziale et al., *JFM*, 1991), LSSG (Gatski & Speziale, *JFM*, 1993), Q-model (Kassinos et al., Int. J. Heat Fluid Flow, 2000)

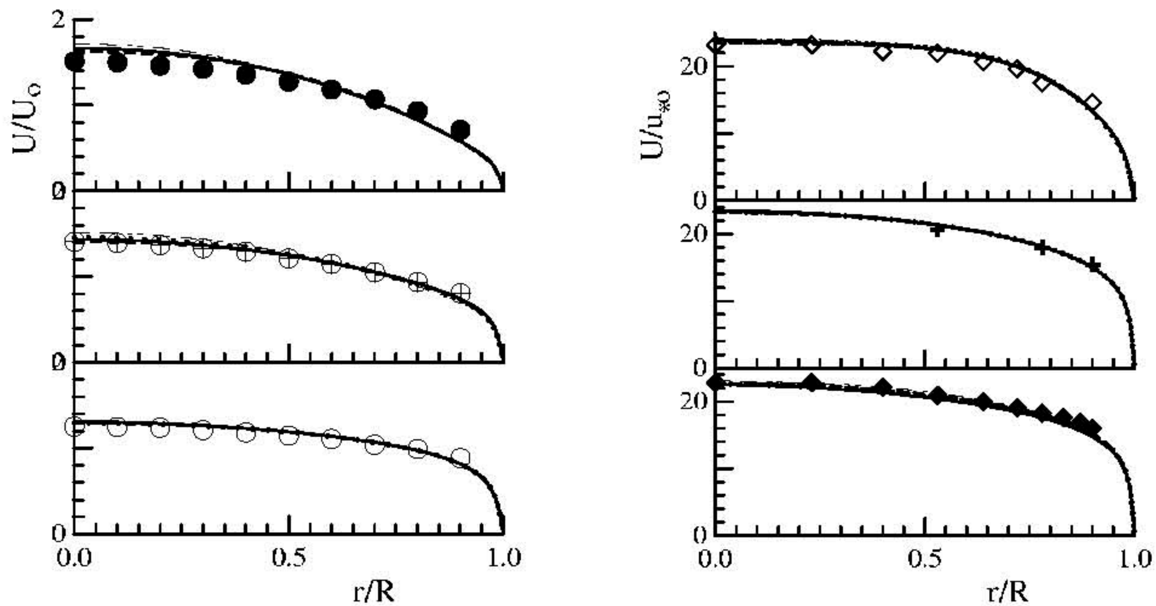


FIGURE 3. Axial mean velocity at **a)** $Re = 2 \times 10^4$, **b)** $Re = 4 \times 10^4$. Calculations: (———) LRR, (- - - -) LSSG; (········) SSG; (—··—) Q. Experiments: **a)** (○) $N = 0.$, (⊕) $N = 0.5$, (●) $N = 1.$; **b)** (♦) $N = 0.$, (+) $N = 0.15$, (**x**) $N = 0.3$, (◇) $N = 0.6$

(Poroseva, *CTR Ann. Res. Briefs*, 2001)

Only the simplest linear model required wall corrections

TCL model (Craft & Launder, 1996):

$$
\begin{aligned}
\Phi^{(r)}_{ij} = {} & c_2^* k \left(a_{ik} S_{jk} + a_{jk} S_{ik} - \tfrac{2}{3} a_{kl} S_{kl} \delta_{ij}\right) + c_3^* k \left(a_{ik} W_{jk} + a_{jk} W_{ik}\right) \\
& + c_4^* k S_{ij} - c_5^* a_{ij} \mathcal{P}_{kk} + c_6^* k \left(a_{ik} a_{kl} S_{jl} + a_{jk} a_{kl} S_{il} - 2 a_{kj} a_{li} S_{kl} - 3 a_{ij} a_{kl} S_{kl}\right) \\
& + c_7^* k \left(a_{ik} a_{kl} W_{jl} + a_{jk} a_{kl} W_{il}\right) \\
& + c_8^* k \left[a_{mn}^2 \left(a_{ik} W_{jk} + a_{jk} W_{ik}\right) + 3/2 a_{mi} a_{nj} \left(a_{mk} W_{nk} + a_{nk} W_{mk}\right)\right]
\end{aligned}
$$

linear			quasi-linear	quadratic		cubic
c_2^*	c_3^*	c_4^*	c_5^*	c_6^*	c_7^*	c_8^*
0.6	0.866	0.8	0.3	0.2	0.2	1.2

Solution for pressure terms

- Non-linear and Q models are too complex, and still need wall corrections

- Derived under assumption of turbulence homogeneity.

- It is unphysical to model the pressure diffusion separately from the pressure-strain correlations.

- Finite boundary conditions for both: pressure-strain correlations and the pressure diffusion.

Solution: modeling Π_{ij} instead

Table 6.1 *Wall-limiting behaviour of the leading terms in the Reynolds stress budget*

ij	Φ_{ij}	$\Pi_{ij} \equiv \left(\mathcal{D}^p_{ij} + \Phi_{ij}\right)$	$\mathcal{D}^p_{ij}$	$\mathcal{D}^\nu_{ij}$	$-\varepsilon_{ij}$
11	$2\overline{a_p \partial b_1/\partial x_1}\, x_2$	$-4\nu\,\overline{b_1c_1}\, x_2$	$-\left(2\,\overline{a_p \partial b_1/\partial x_1} + 4\nu\,\overline{b_1c_1}\right) x_2$	$2\nu\,\overline{b_1b_1} + 12\nu\,\overline{b_1c_1}\, x_2$	$-2\nu\,\overline{b_1b_1} - 8\nu\,\overline{b_1c_1}\, x_2$
22	$4\,\overline{a_pc_2}\, x_2$	$-4\nu\,\overline{c_2c_2}\, x_2^2$	$-\left(4\,\overline{a_pc_2}\, x_2 + 4\,\nu\overline{c_2c_2}\, x_2^2\right)$	$12\nu\,\overline{c_2c_2}\, x_2^2$	$-8\nu\,\overline{c_2c_2}\, x_2^2$
33	$2\overline{a_p \partial b_3/\partial x_3}\, x_2$	$-4\nu\,\overline{b_3c_3}\, x_2$	$-\left(2\,\overline{a_p \partial b_3/\partial x_3} + 4\nu\,\overline{b_3c_3}\right) x_2$	$2\nu\,\overline{b_3b_3} + 12\nu\,\overline{b_3c_3}\, x_2$	$-2\nu\,\overline{b_3b_3} - 8\nu\,\overline{b_3c_3}\, x_2$
12	$\overline{a_pb_1}$	$-2\nu\,\overline{b_1c_2}\, x_2$	$-\left(\overline{a_pb_1} + 2\nu\,\overline{b_1c_2}\, x_2\right)$	$6\nu\,\overline{b_1c_2}\, x_2$	$-4\nu\overline{b_1c_2}\, x_2$
23	$\overline{a_pb_3}$	$-2\nu\,\overline{b_3c_2}\, x_2$	$-\left(\overline{a_pb_3} + 2\,\nu\,\overline{b_3c_2}\, x_2\right)$	$6\nu\,\overline{b_3c_2}\, x_2$	$-4\nu\,\overline{b_3c_2}\, x_2$
13	$\overline{a_p(\partial b_1/\partial x_3 + \partial b_3/\partial x_1)} x_2$	$-2\nu\left(\overline{b_1c_3} + \overline{b_3c_1}\right) x_2$	$-\overline{a_p(\partial b_1/\partial x_3 + \partial b_3/\partial x_1)} x_2$ $-2\nu\left(\overline{b_1c_3} + \overline{b_3c_1}\right) x_2$	$2\nu\overline{b_1b_3} + 6\nu\left(\overline{b_1c_3} + \overline{b_3c_1}\right) x_2$	$-2\nu\,\overline{b_1b_3} - 4\nu\left(\overline{b_1c_3} + \overline{b_3c_1}\right) x_2$

Hanjalić & Launder, 2011

- Π_{ij} are originally present in the RANS equations
- easy boundary conditions
- No need to model the pressure diffusion separately

Modeling ideas

$$\Pi_{ij} = -\frac{1}{\rho}\left(< u_j \frac{\partial p}{\partial x_i} > + < u_i \frac{\partial p}{\partial x_j} > \right) = \Pi_{ij}^{(r)} + \Pi_{ij}^{(s)} + \Pi_{ij}^{(w)}$$

$$-\frac{1}{\rho} < u_i \frac{\partial p}{\partial x_j} > = -\frac{1}{2\pi}\iiint \left[U'_{\ m,n} < u'_n u_i >'_{,m} \right]'_{,j} \frac{1}{r} dV' \; - \; \frac{1}{4\pi}\iiint < u'_m \, u'_n u_i >'_{mn} \, \frac{1}{r} dV'$$

$$-\frac{1}{4\pi}\iint \left\{ \frac{1}{r} \frac{\partial < p'_{,j} u_i >}{\partial n'} - < p'_{,j} u_i > \frac{\partial}{\partial n'}\left(\frac{1}{r}\right) \right\} dS'$$

$$\Pi^{M(r)}{}_{ij} = (a_{nmij} + a_{nmji}) U_{m,n}$$

this is NOT an assumption of turbulence homogeneity

$$a_{nmji} = -\frac{1}{2\pi}\iiint \frac{\partial < u'_n u_i >}{\partial x'_m \, \partial x'_j} \frac{1}{r} dV'$$

$$\left\{ \begin{array}{l} m - j; \\ \text{if } \; m = n, \text{ then } a_{nmji} = 0; \\ \text{if } \; m = j, \text{ then } a_{nmji} = 2 < u_n u_i > \end{array} \right.$$

Poroseva, THMT, 2000

Modeling ideas (Cont.)

In modeling $\Pi^{(r)}_{ij}$:

$$a_{nmji} = -\frac{1}{2\pi}\iiint \frac{\partial < u'_n u_i >}{\partial x'_m \partial x'_j}\frac{1}{r}dV'$$

In modeling $\Phi^{(r)}_{ij}$:

$$a_{nmji} = -\frac{1}{2\pi}\iiint < \frac{\partial u'_n}{\partial x'_m}\frac{\partial u_i}{\partial x_j} > \frac{1}{r}dV'$$

Rotta (1951), Launder, Reece, Rodi (1975)

To apply the Green's theorem to this expression, one has to assume the turbulence homogeneity

Different integrals have different properties and their analysis results in different models

Linear model

Poroseva, *THMT,* 2000

$$\Pi^{M(r)}_{ij} = a_{nmij} + a_{nmji}) \; U_{m,n}$$

$$= -\frac{1}{5}\left(\langle u_i u_m \rangle U_{m,j} + \langle u_j u_m \rangle U_{m,i}\right) + \frac{4}{5}\left(\langle u_i u_m \rangle U_{j,m} + \langle u_j u_m \rangle U_{i,m}\right) + R_1$$

$$R_1 = k\,(C_1 + C_2)\,(U_{i,j} + U_{j,i}) + \left(-\frac{1}{2}C_1 - C_2\right)\left(\langle u_i u_m \rangle U_{m,j} + \langle u_j u_m \rangle U_{m,i}\right)$$

$$+\left(-C_1 - \frac{1}{2}C_2\right)\left(\langle u_i u_m \rangle U_{j,m} + \langle u_j u_m \rangle U_{i,m}\right) + (-4C_1 - C_2)\langle u_m u_n \rangle U_{m,n}\delta_{ij},$$

Application to limiting states

- Satisfies the exact solution for isotropic turbulence subjected to sudden distortion with any value of C_1 and C_2.

$$a_{nmji} = k\left(\frac{8}{15}\delta_{ni}\delta_{mj} - \frac{2}{15}(\delta_{nm}\delta_{ji} + \delta_{nj}\delta_{mi})\right)$$

- Transforms to LRR model in homogeneous turbulence:

$$a_{nmji} = -\frac{1}{2\pi}\iiint \frac{\partial < u'_n u_i >}{\partial x'_m \partial x'_j}\frac{1}{r}\,dV'$$

$$n - i$$

$$\frac{1}{5} - \frac{5}{2}C_1 - C_2 = 0$$

$$C_1 = \frac{6}{55} + \frac{4}{11}C'$$

- Two-component turbulence ($<\tilde{u}_1^2>=0$, β = 2 or 3)

$$C_1 = -C_2 \frac{\tilde{U}_{1,1}(4k - <\tilde{u}_\beta^2>) + 2\tilde{U}_{\beta,\beta}(k - <\tilde{u}_\beta^2>)}{\tilde{U}_{1,1}(10k - 4<\tilde{u}_\beta^2>) + 8\tilde{U}_{\beta,\beta}(k - <\tilde{u}_\beta^2>)}$$

- Two-component axisymmetric turbulence ($<\tilde{u}_2^2>=<\tilde{u}_3^2>=k$)

$$C_1 = -0.5 \cdot C_2$$

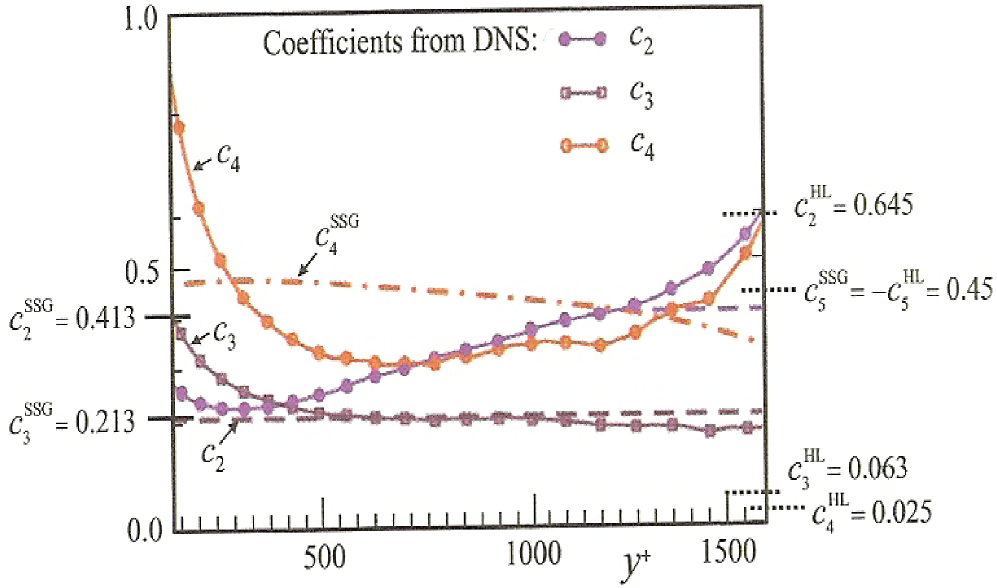
- Two-component axisymmetric homogeneous turbulence

$$C_2 = 0.4$$

C_1, C_2 can be kept as const in a given flow, but vary depending on the flow geometry and some other parameters.

More research and data are required to suggest their functional form.

SSG model coefficients:



DNS data: Hoyas & Jimenez, 2006
Plot: Hanjalić & Launder, 2011

Preliminary tests

$$-\frac{1}{\rho}<u_i p>^{(r)}{}_{,j}=(-0.6+C_k)P \qquad C_k=\frac{15}{2}C_1+3C_2$$

$$\frac{Dk}{Dt}=-(0.4+C_k)<u_i u_j>\frac{\partial U_i}{\partial x_j}-\varepsilon+\frac{\partial}{\partial x_i}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right]+wall$$

$$\frac{D\varepsilon}{Dt}=-\frac{\varepsilon}{k}(C_{\varepsilon 1}<u_i u_j>\frac{\partial U_i}{\partial x_j}+C_{\varepsilon 2}\ \varepsilon)+\frac{\partial}{\partial x_i}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right]$$

flow	Wake	ML	RJ	PJ	BL1	BL2	Diff	BS	**stand**
C_k	0.2	0.9	0.6	1	1	0.8	0.6	0.8	**0.6**
$C_{\varepsilon 1}$	0.6	1.9	1.5	2.12	2.2	1.85	1.5	1.85	**1.44**

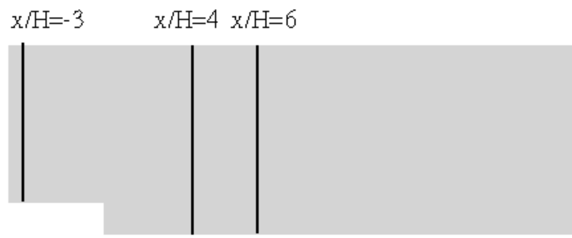
BL1 $\beta=0$

BL2 $\beta=19.6$

$$\sigma_\varepsilon/\sigma_k=1.5,\ C_{\varepsilon 2}=1.92,\ C_\mu=0.09$$

Poroseva, *THMT,* 2000; Iaccarino & Poroseva, *CTR Ann. Res. Briefs*, 2001

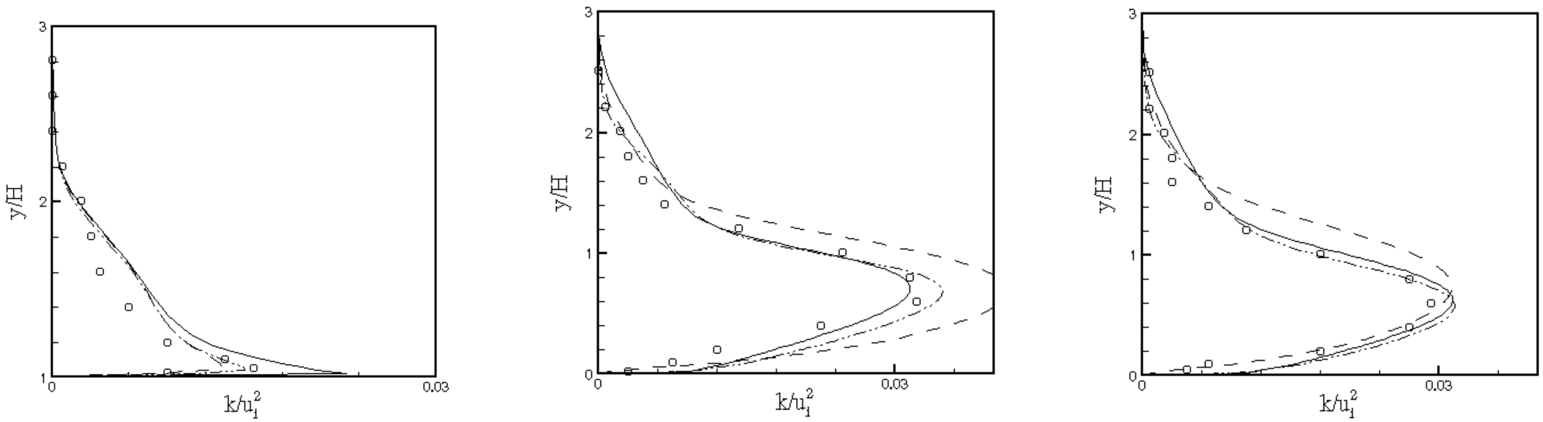
Back-step flow



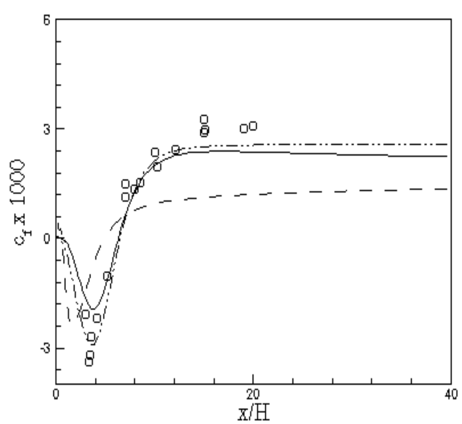
new model

standard *k*-ε model

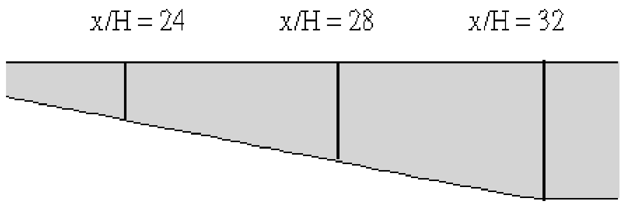
$<v^2>-f$ model



Skin-friction coefficient



Diffuser

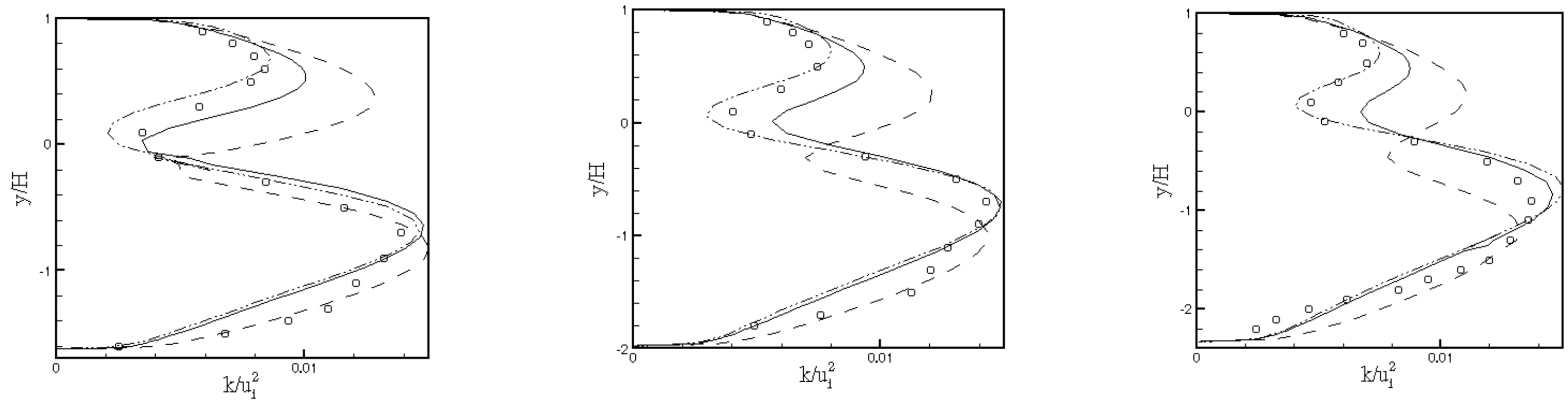


$$C_k = 0.6,\ C_{\varepsilon 1} = 1.5$$

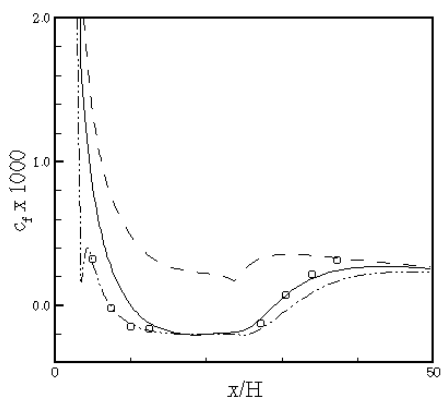
new model

standard *k*-ε model

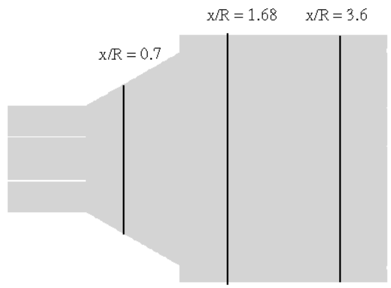
$< v^2 > - f$ model



Skin-friction coefficient



Combustion chamber



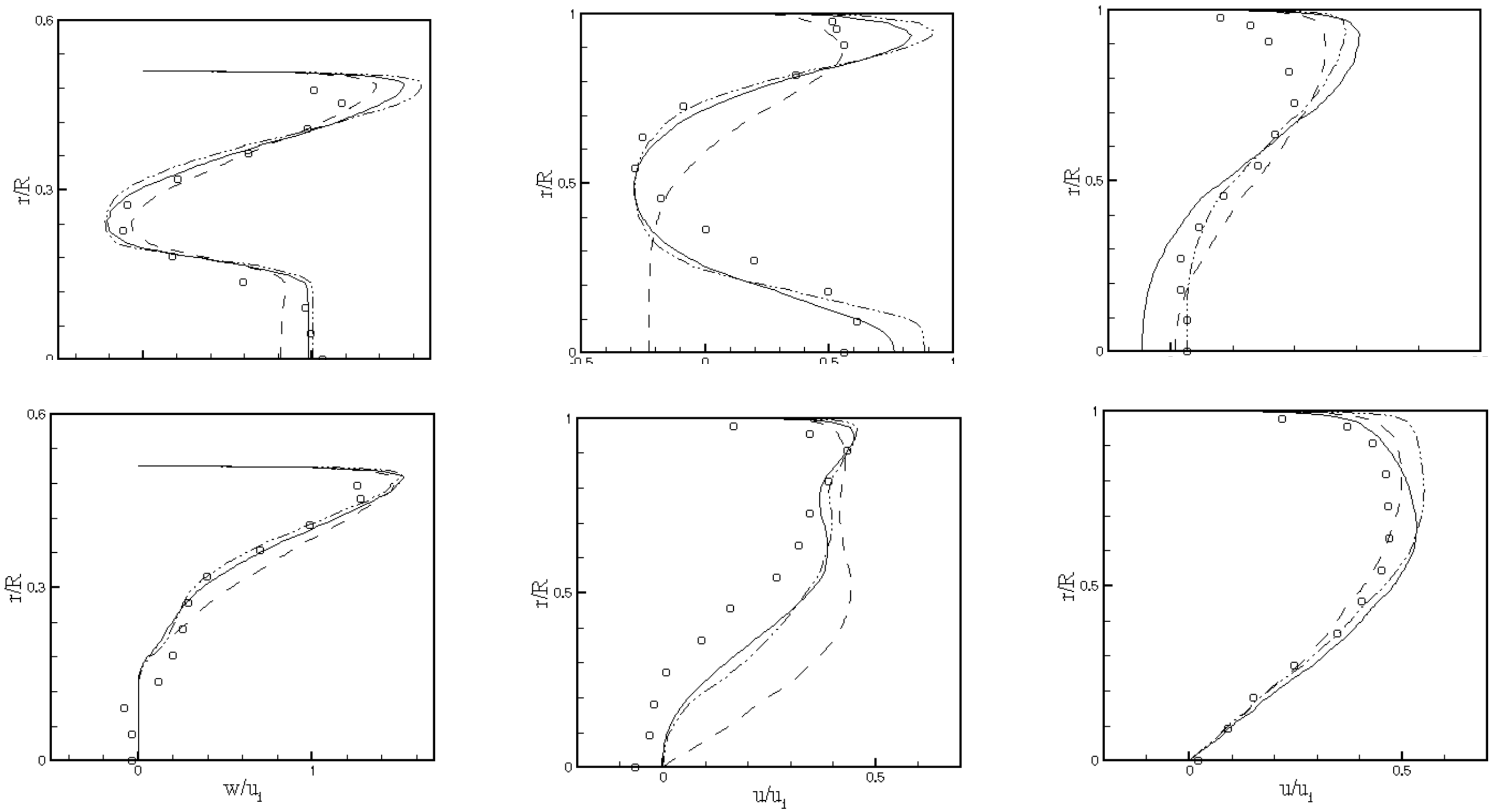
Re = 75,000

——— new $C_{\varepsilon 1} = 1.7$

- - - - standard k-ε model

–··–··– $< v^2 > - f$ model

$r / R = 0.7$ $r / R = 1.68$ $r / R = 3.6$



axial velocity

swirl velocity

Future Direction in Turbulence Modeling

from

the state–of-the-art

(as based on imagination)

to

the state-of–the-science

(*as based on logical reasoning)*

high-order statistical closures

Questions?

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