On time domain methods for Computational Aeroacoustics

Fang Q. Hu and Ibrahim Kocaogul
Old Dominion University, Norfolk, Virginia

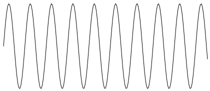
Xiaodong Li, Xiaoyan Li and Min Jiang
Beihang University, Beijing 100191, China

Noise prediction by linear acoustic wave propagation

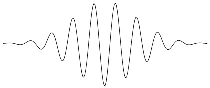
Noise source modelling + Noise propagation

FW−H equation
Kirchhoff integral
Linearized Euler Equations
Green's function
......

Time Domain Wave Packet (TDWP) method



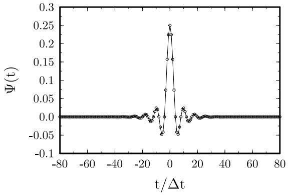
sinusoidal wave
(single frequency)

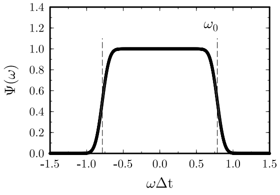


wave packet
(broadband and mulch-frequency)

Proposed broadband acoustic test pulse function for source:

$$\Psi(t) = \frac{\Delta t \sin(\omega_0 t)}{\pi t} e^{(\ln 0.01)(t/M\Delta t)^2}, \quad |t| \le M\Delta t$$





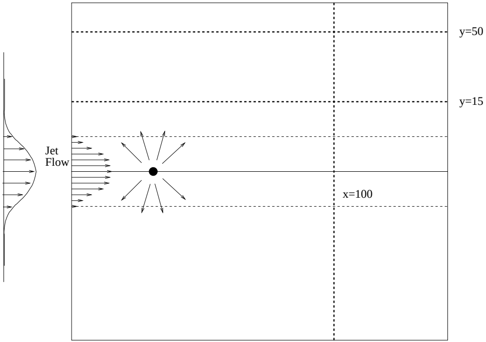
Advantages of Time Domain Wave Packet (TDWP) method

- One computation for all frequencies (within numerical resolution) for linear problems
- Ability to synthesize broadband noise sources
- Acoustic source has a short time duration, so computation is more efficient than driving a time domain calculation to a time periodic state
- **Separation of acoustic and hydrodynamic instability waves becomes possible**
- **Long numerical transient state is avoided**

Application Examples

1. Sound propagation through shear flows
2. Vortical gust-blade interaction
3. Duct sound radiation problem

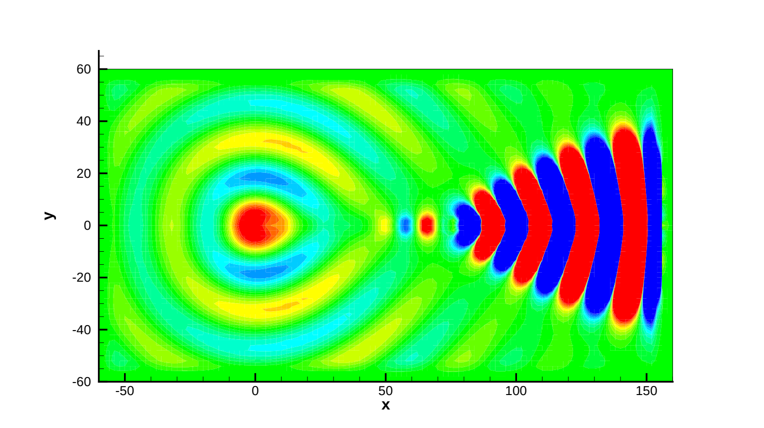
1. Sound source in a jet flow (a CAA Benchmark problem)



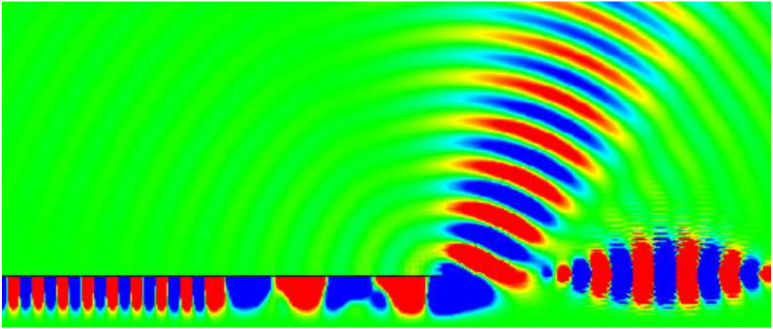
Single frequency source function:

$$S(x,y,t) = \sin(\Omega t) e^{-(\ln 2)(B_x x^2 + B_y y^2)}$$

Instability wave in a shear flow

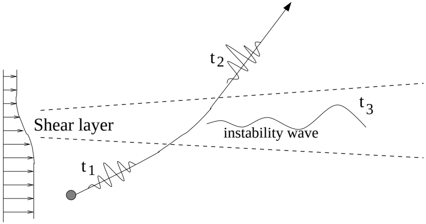


Instability wave in duct radiation computation



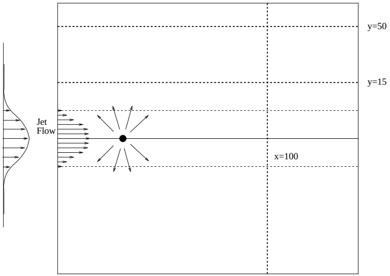
Time Domain Wave Packet (TDWP) method

Separation of acoustic and instability waves:



Acoustic and instability waves travel at different speeds. An acoustic wave packet has a short time duration, it will be separated from the instability wave in time domain calculation, here $t_1 < t_2 < t_3$.

Time Domain Wave Packet approach

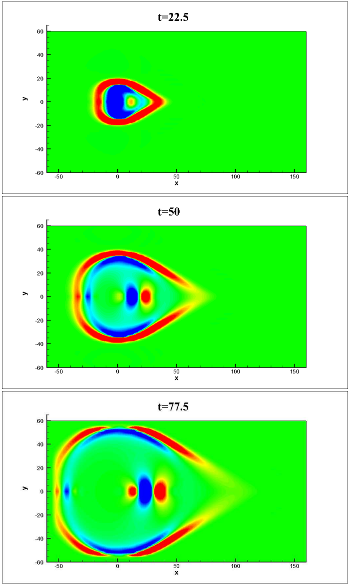


Single frequency source function:

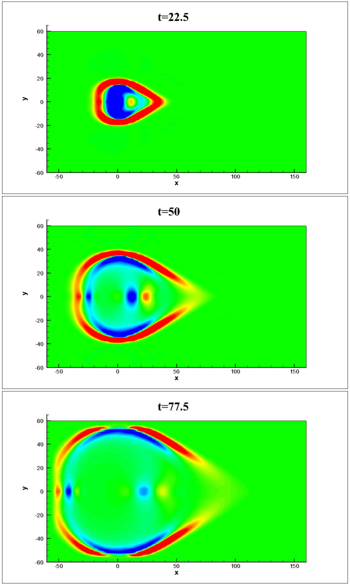
$$S(x, y, t) = \sin(\Omega t) e^{-(\ln 2)(B_x x^2 + B_y y^2)}$$

TDWP source function:

$$S(x, y, t) = \Psi(t) e^{-(\ln 2)(B_x x^2 + B_y y^2)}$$



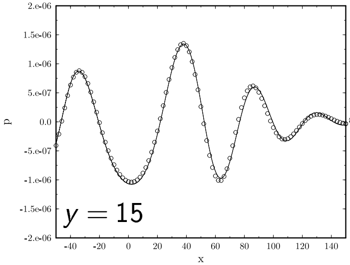
without suppression

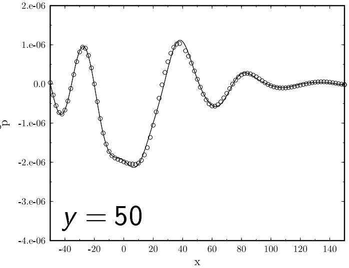


with suppression

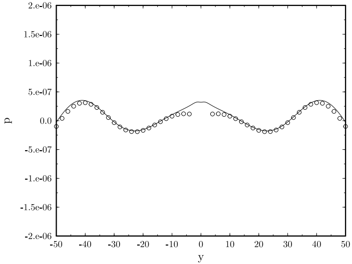
Frequency domain solution recovered by FFT

(Symbol: analytical; Line: computation)

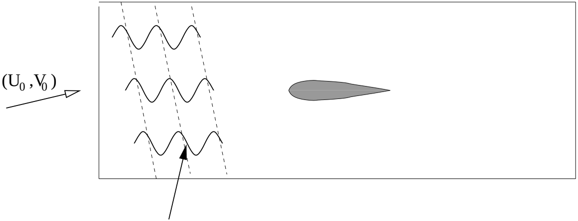




$x = 100$



2. Vortical gust-blade interaction

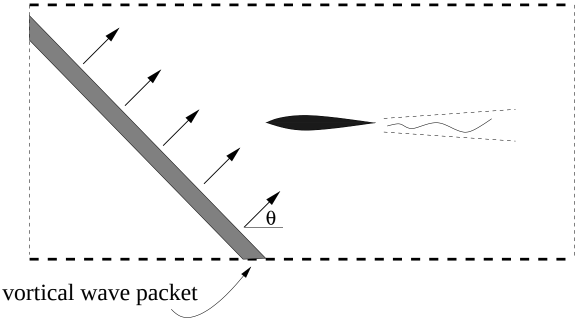


Incident Vortical Gust :

$$u_g = -\frac{V\beta}{\alpha}\cos(\alpha x + \beta y - \omega t)$$

$$v_g = V\cos(\alpha x + \beta y - \omega t)$$

Vortical gust imposition in TDWP



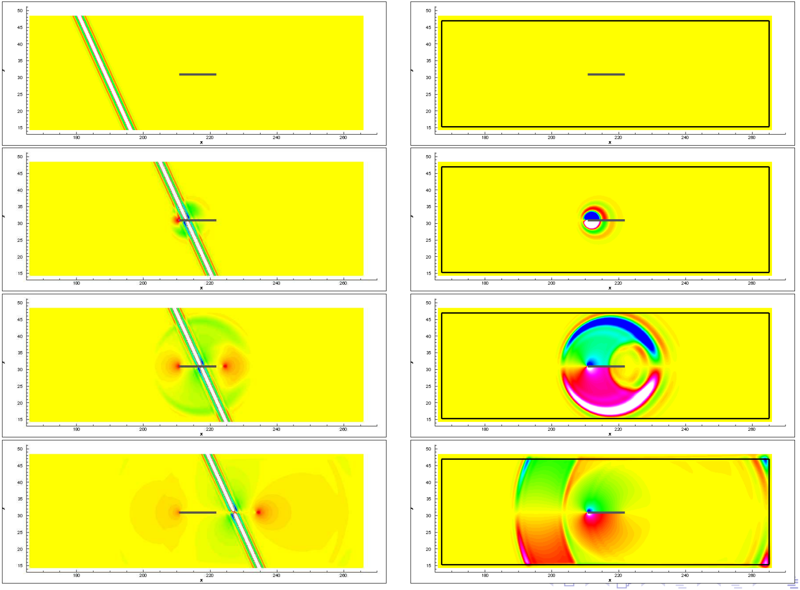
$$u_g(x,y,t) = -B\Psi(t - Ax - By)$$

$$v_g(x,y,t) = A\Psi(t - Ax - By)$$

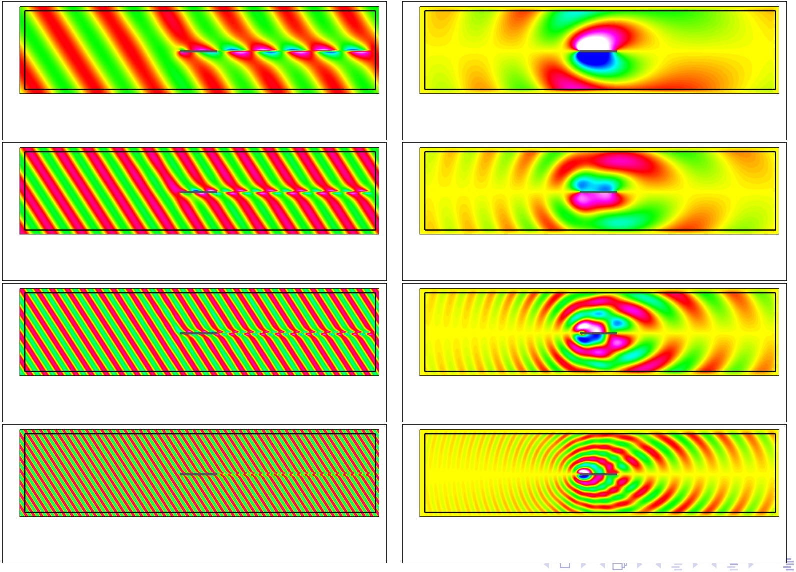
$$A = \frac{\cos(\theta)}{\bar{u}_0\cos(\theta) + \bar{v}_0\sin(\theta)}, \quad B = \frac{\sin(\theta)}{\bar{u}_0\cos(\theta) + \bar{v}_0\sin(\theta)} \implies \frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = 0$$

Example of time domain solution

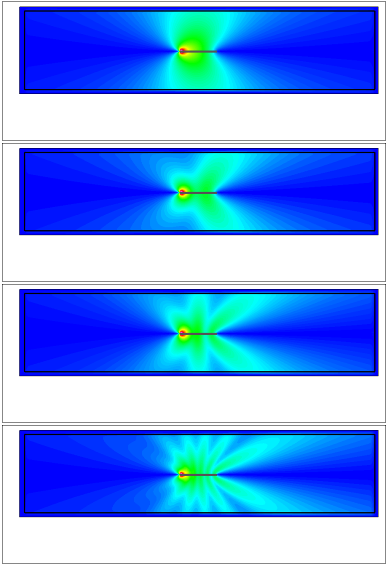
(v-velocity and pressure, $\bar{u}_0 = 0.45$)



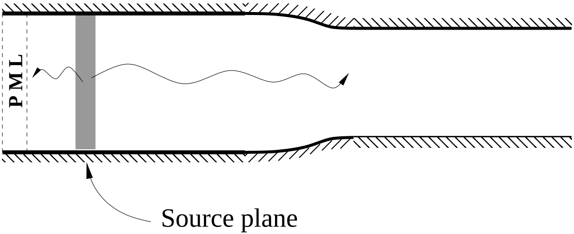
Frequency domain solution by FFT (u-velocity and pressure)



Frequency domain solution (pressure magnitudes)



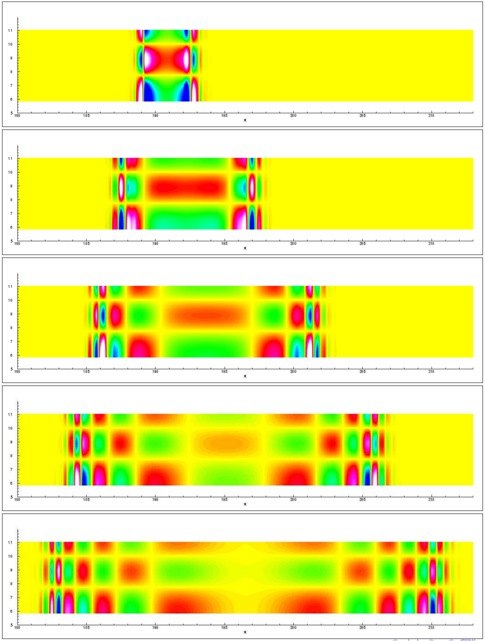
3. Duct mode imposition in TDWP



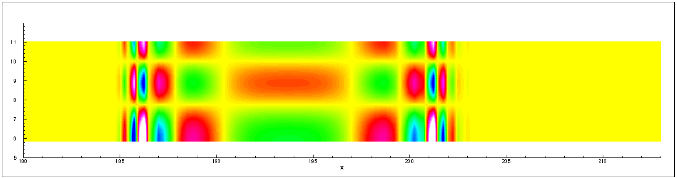
Pressure equation at in-flow region:

$$\frac{\partial p}{\partial t} + \bar{u}_0 \frac{\partial p}{\partial x} + \gamma \bar{p}_0 \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right) = \phi_{mn}(y,z)\Psi(t)e^{-\sigma x^2}$$

Example of time domain solution



Duct mode wave packet—exact solution



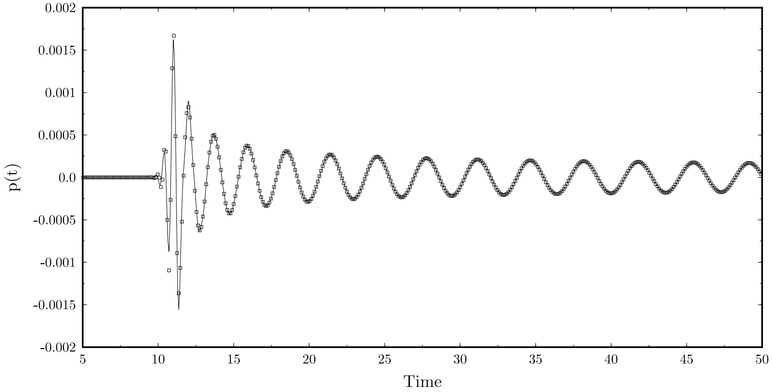
$$p(x,y,z,t) = \phi_{mn}(y,z)\hat{P}(x,t)$$

$$\hat{P}(x,t) = \frac{\alpha^2}{2}\int_{-X}^{X}\int_{-T}^{T'} J_0\left(\bar{\lambda}_{mn}\sqrt{(t-\bar{t}')^2-\bar{x}^2}\right)\frac{D}{Dt}\left[\Psi(t')e^{-\sigma x'^2}\right]dx'dt'$$

$$\hat{P}(x,\omega) = \begin{cases} \frac{\alpha^2}{2}\int_{-X}^{X}\int_{-T}^{T'} \frac{e^{-\bar{x}\sqrt{\bar{\lambda}_{mn}^2-\omega^2}}}{\sqrt{\bar{\lambda}_{mn}^2-\omega^2}} e^{i\omega\bar{t}'}\frac{D}{Dt}\left[\Psi(t')e^{-\sigma x'^2}\right]dx'dt' & 0<\omega<\bar{\lambda}_{mn} \text{ (cut-off)} \\ \frac{\alpha^2}{2}\int_{-X}^{X}\int_{-T}^{T'} \frac{ie^{i\bar{x}\sqrt{\omega^2-\bar{\lambda}_{mn}^2}}}{\sqrt{\omega^2-\bar{\lambda}_{mn}^2}} e^{i\omega\bar{t}'}\frac{D}{Dt}\left[\Psi(t')e^{-\sigma x'^2}\right]dx'dt' & 0<\bar{\lambda}_{mn}<\omega \text{ (cut-on)} \end{cases}$$

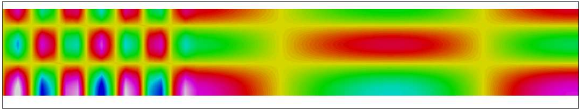
$$\left(\alpha=\sqrt{1-M^2},\ \beta=\frac{M}{1-M^2},\ \bar{t}'=t'-\beta(x-x'),\ \bar{x}=|x-x'|/\alpha^2\right)$$

Time history (line: numerical, symbol:theoretical)

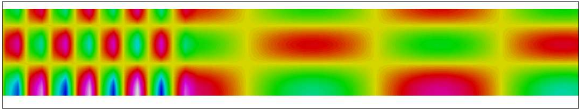


Frequency domain solution

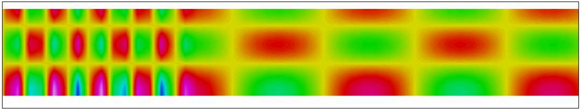
$\omega_1 = 1.6$:



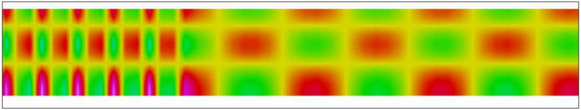
$\omega_2 = 1.9$:



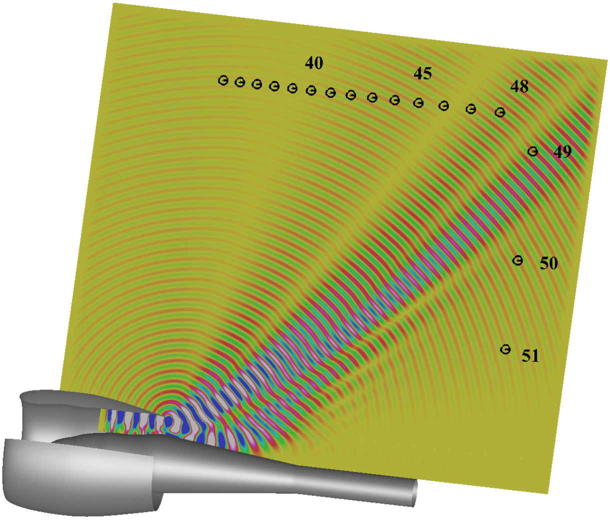
$\omega_3 = 2.3$:



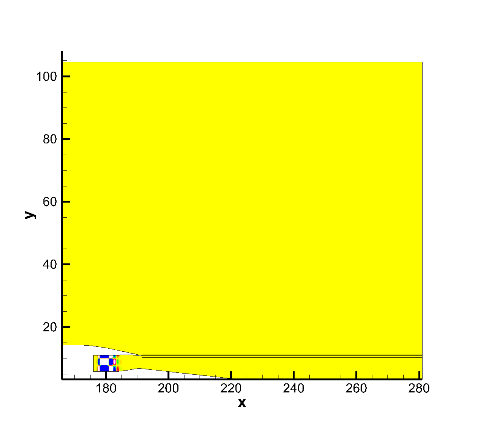
$\omega_4 = 2.6$:



NASA/GE Fan Noise Source Diagnostic Test



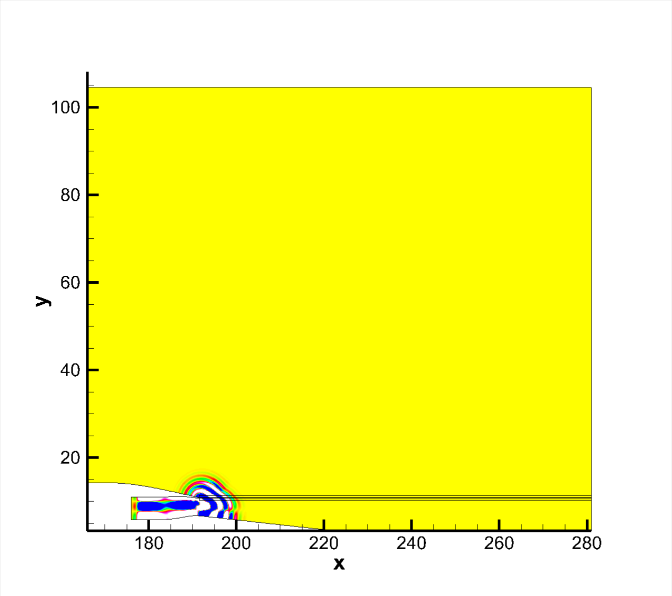
Aft fan exhaust radiation problem



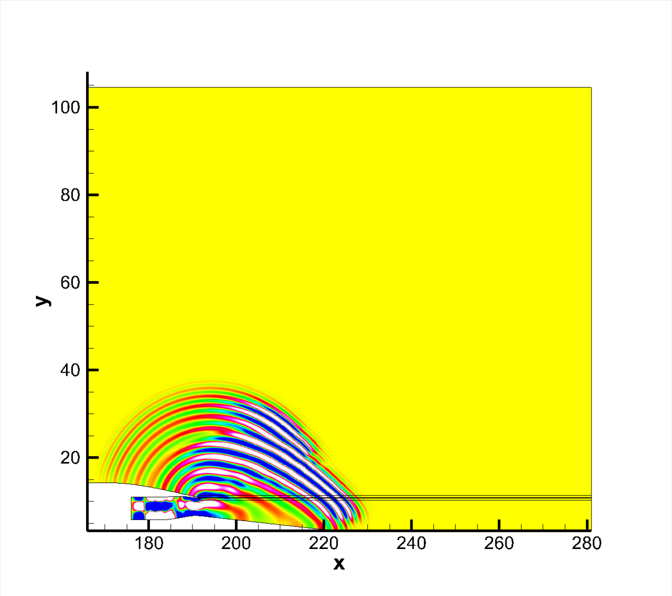
Pressure equation at in-flow region:

$$\frac{\partial p}{\partial t} + \bar{u}_0 \frac{\partial p}{\partial x} + \gamma \bar{p}_0 \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \right) = \phi_{mn}(y,z)\Psi(t)e^{-\sigma(x-x_0)^2}$$

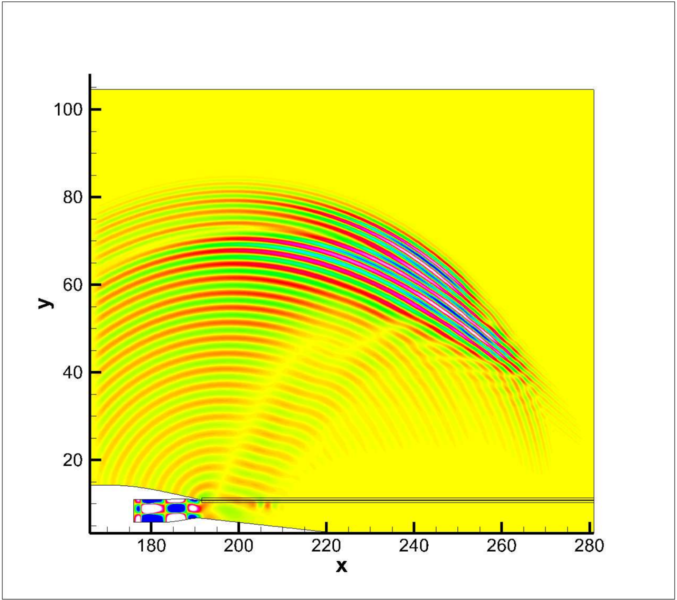
Aft fan exhaust radiation problem



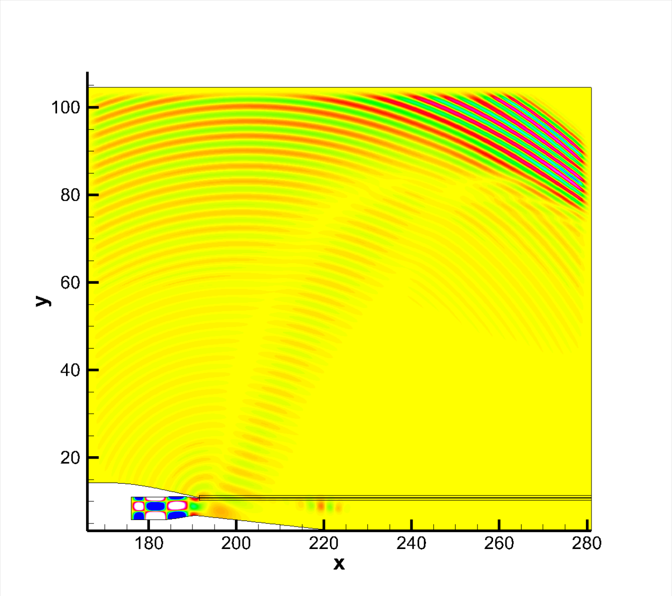
Aft fan exhaust radiation problem



Aft fan exhaust radiation problem

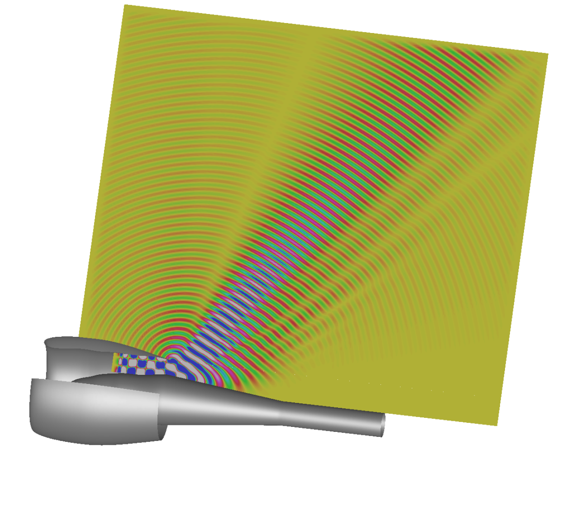


Aft fan exhaust radiation problem



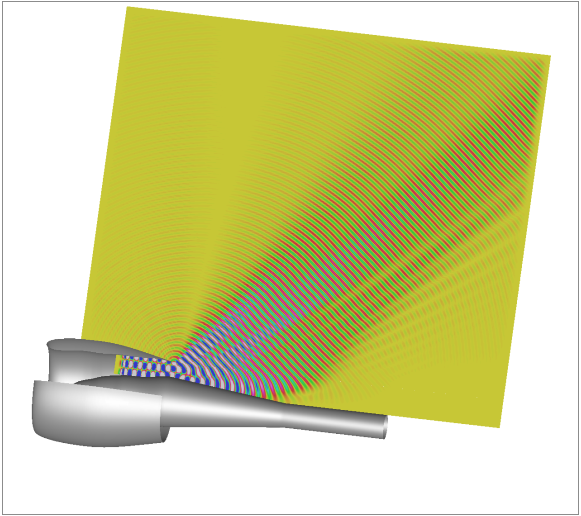
Pressure field obtained by FFT at 2BPF, 61.7% design speed

Mode (-10,2) introduced at source plane inside duct



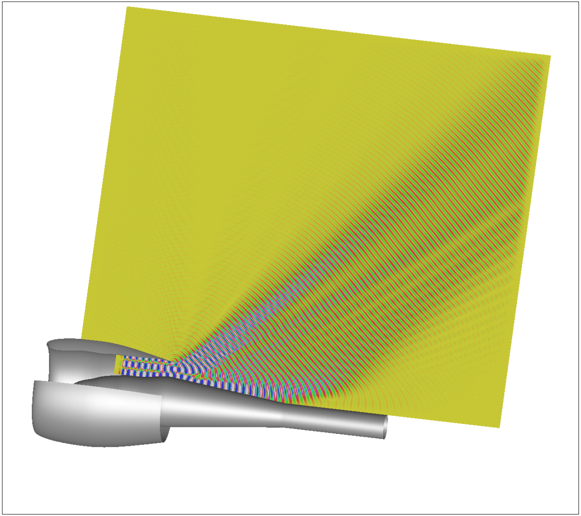
Pressure field obtained by FFT at 3BPF, 61.7% design speed

Mode (-10,2) introduced at source plane inside duct

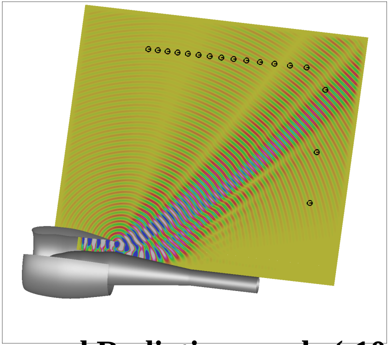


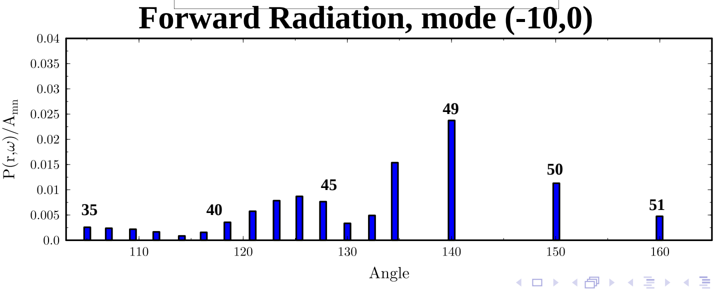
Pressure field obtained by FFT at 4BPF, 61.7% design speed

Mode (-10,2) introduced at source plane inside duct

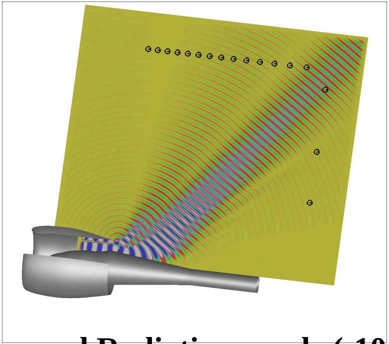


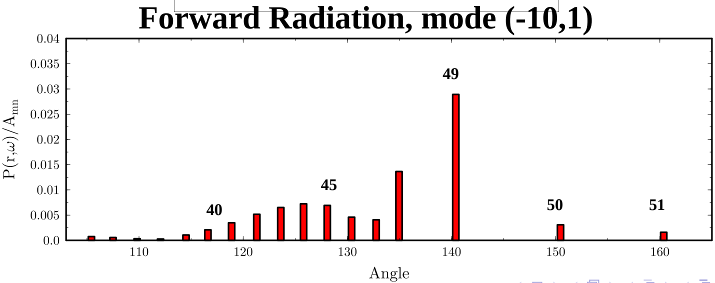
Far-field modal transfer function, forward solution



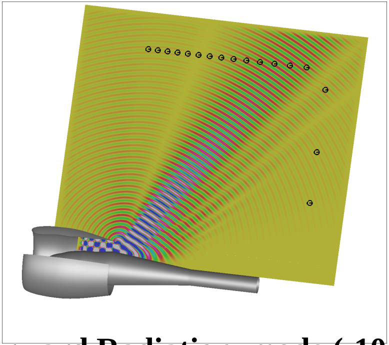


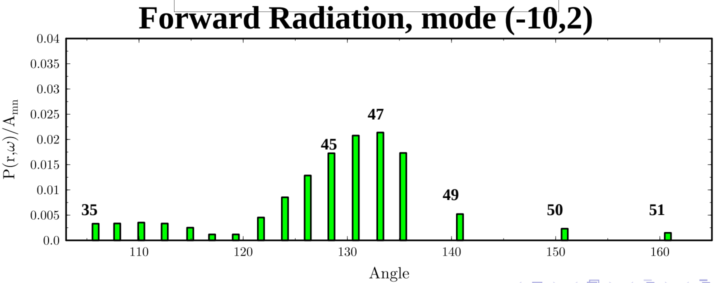
Far-field modal transfer function, forward solution



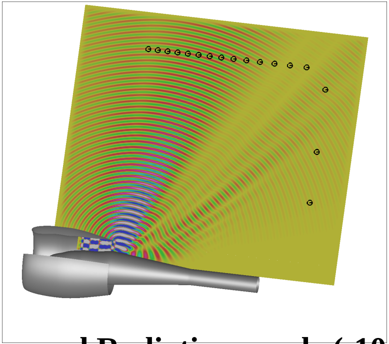


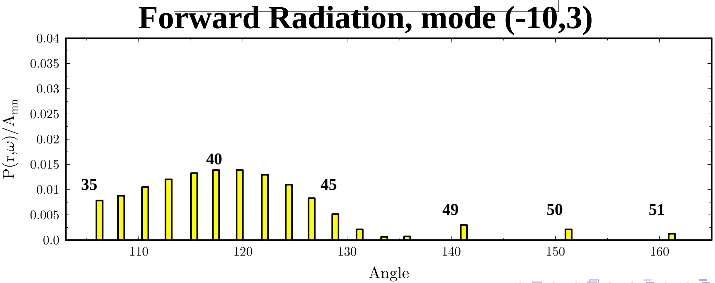
Far-field modal transfer function, forward solution



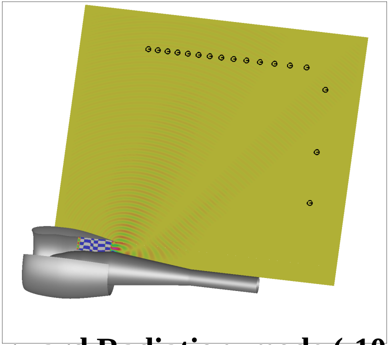


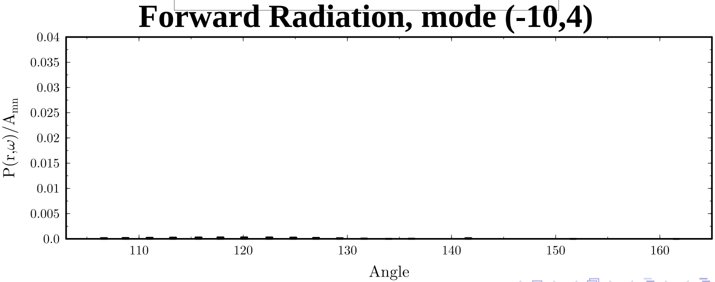
Far-field modal transfer function, forward solution



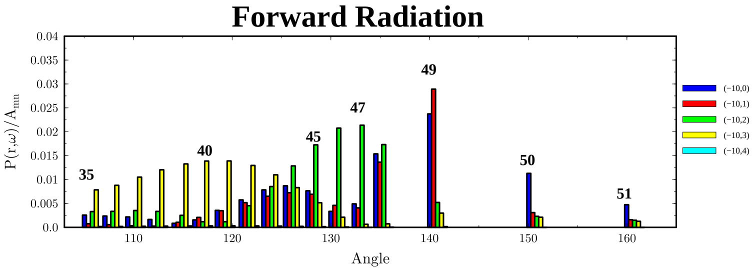


Far-field modal transfer function, forward solution

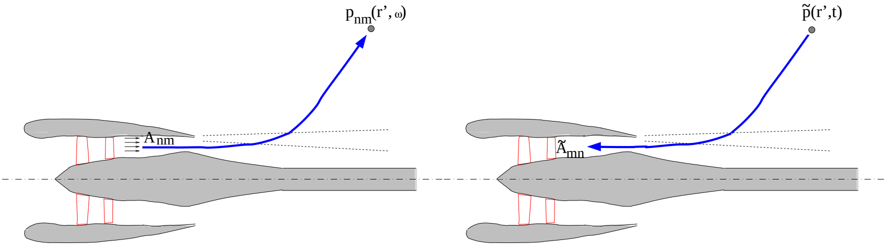




Far-field modal transfer function (2BPF) $\hat{P}_{mn} = \frac{P_{mn}}{A_{mn}}$



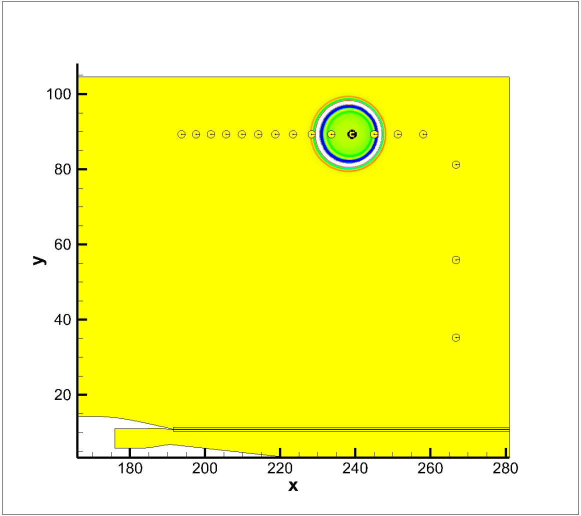
Reciprocity Condition (forward and adjoint problems)



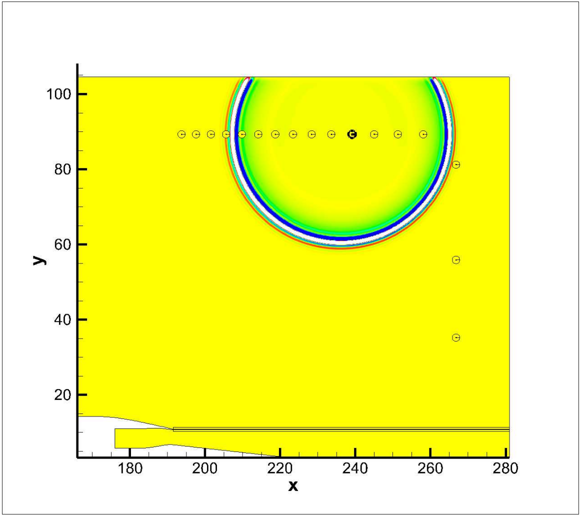
$$\frac{P_{mn}(\boldsymbol{r}', \omega)}{A_{mn}} = \alpha_{mn} \frac{\tilde{A}^*_{mn}}{\tilde{P}(\boldsymbol{r}', \omega)}$$

$$\left(\alpha_{mn} = \int_D \tilde{\phi}^*_{mn} \boldsymbol{A} \phi_{mn} dS = 4\pi \left(\frac{\partial \omega}{\partial k} \right)_{mn} \int_{r_H}^{R} \phi^2_{mn}(r) r dr \right)$$

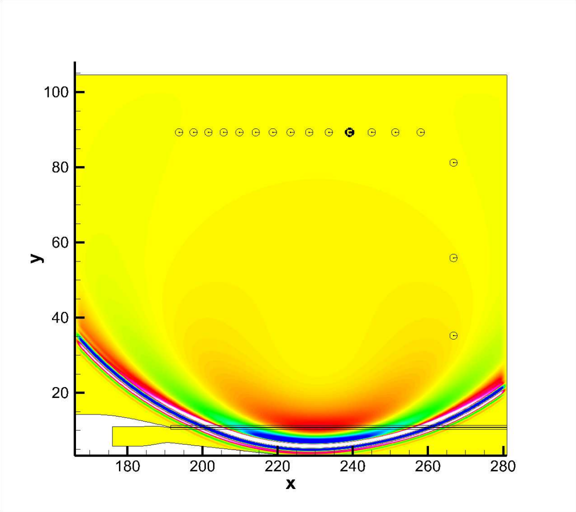
Adjoint solution, time domain



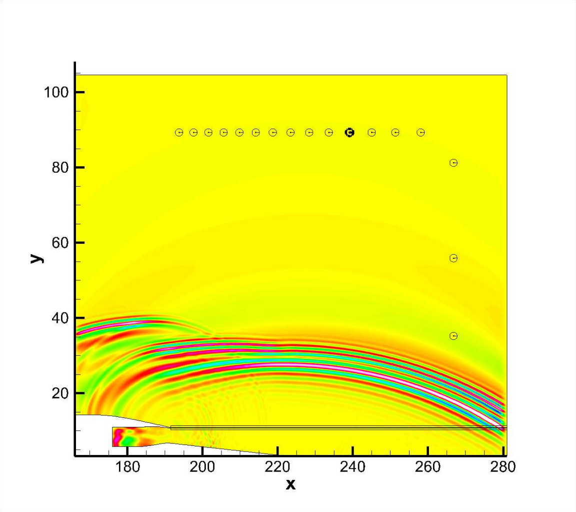
Adjoint solution, time domain



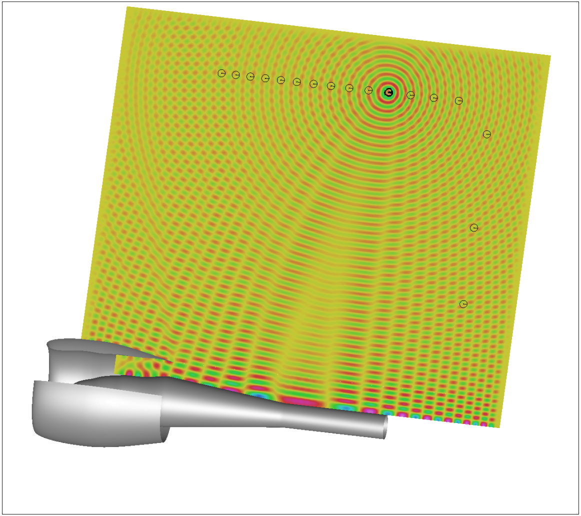
Adjoint solution, time domain



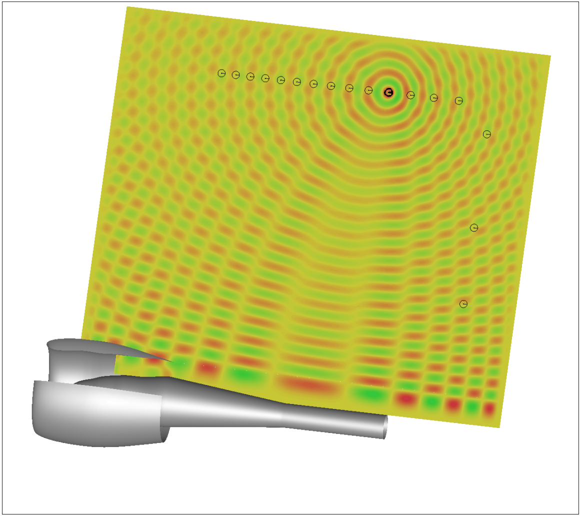
Adjoint solution, time domain



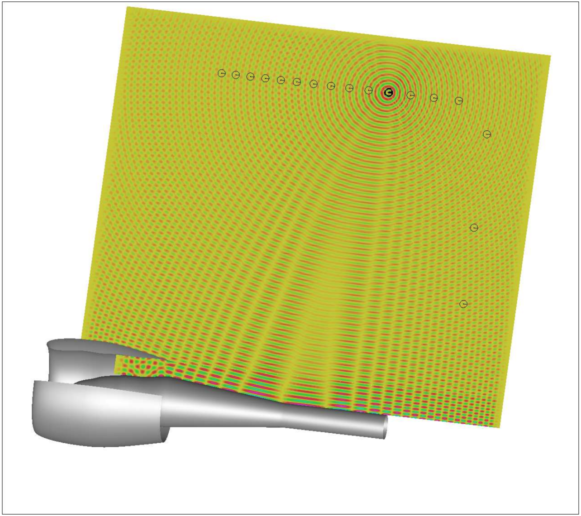
Adjoint solution, frequency domain at 2BPF



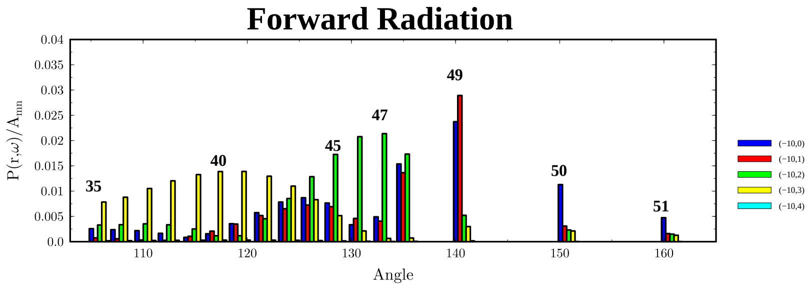
Adjoint solution, frequency domain at BPF

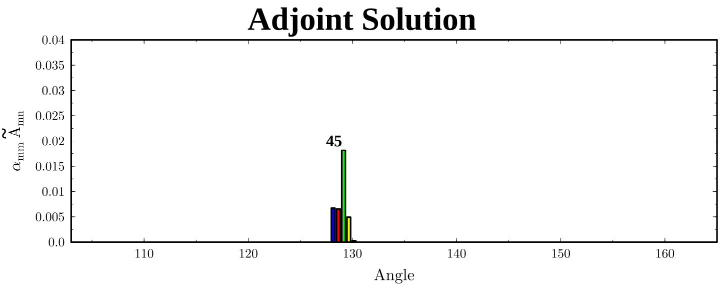


Adjoint solution, frequency domain at 3BPF

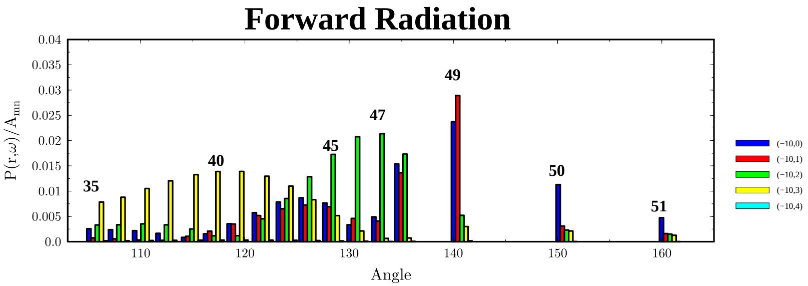


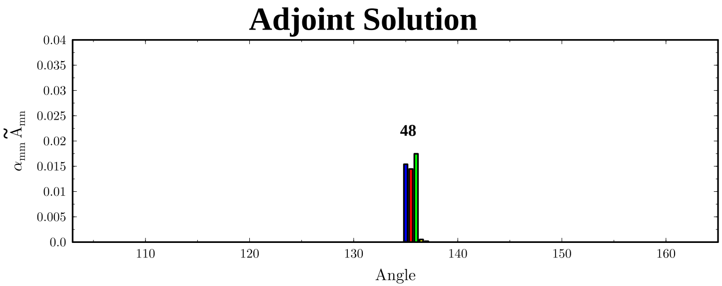
Reciprocity condition (2BPF) $\frac{P_{mn}}{A_{mn}} = \alpha_{mn} \frac{\tilde{A}^*_{mn}}{\tilde{P}}$



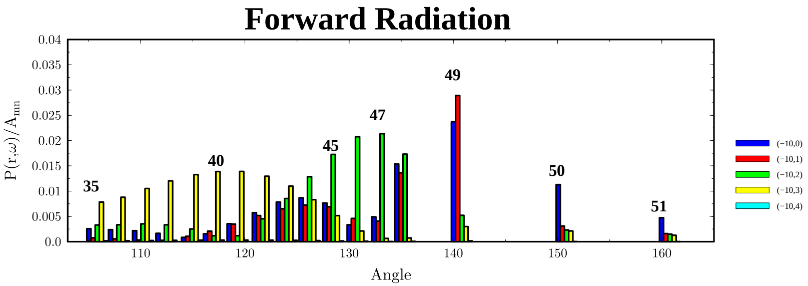


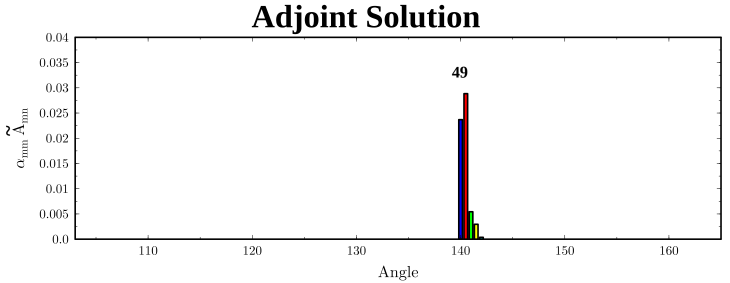
Reciprocity condition (2BPF) $\frac{P_{mn}}{A_{mn}} = \alpha_{mn}\frac{\tilde{A}^*_{mn}}{\tilde{P}}$



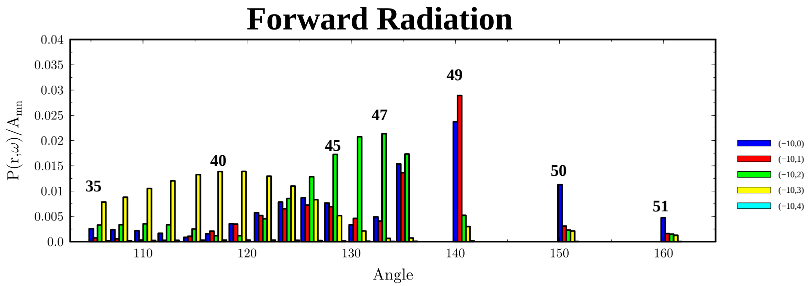


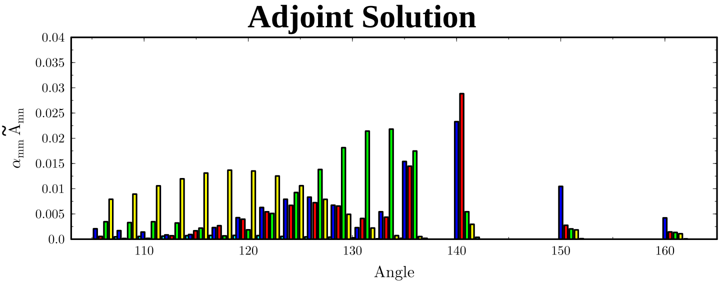
Reciprocity condition (2BPF) $\frac{P_{mn}}{A_{mn}} = \alpha_{mn}\frac{\tilde{A}^*_{mn}}{\tilde{P}}$





Comparison of modal transfer function $\frac{P_{mn}}{A_{mn}} = \alpha_{mn} \frac{\tilde{A}^*_{mn}}{\tilde{P}}$





Modal detection 1

- Far-field pressure

$$p(\boldsymbol{r},\omega) = \sum_{m,n} A_{mn}(\omega)\hat{P}_{mn}(\boldsymbol{r},\omega)$$

$$A_{mn} = A_{mn}^{(r)} + iA_{mn}^{(i)} = \text{Amplitude}$$

$$\hat{P}_{mn} = \text{Modal transfer function}$$

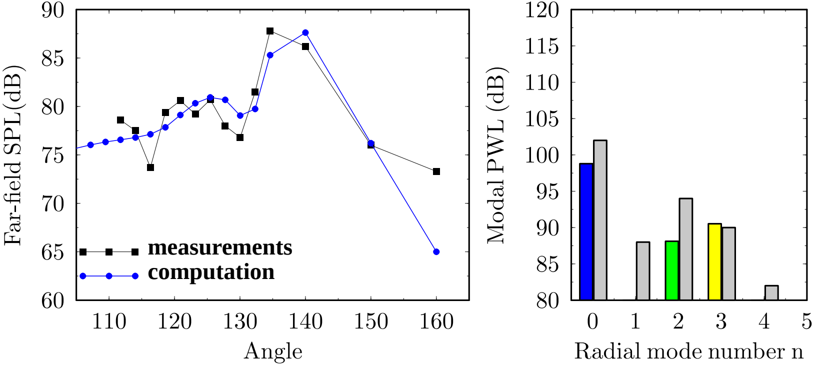
- Assume duct modes have no interference:

$$|p(\boldsymbol{r},\omega)|^2 = \sum_{m,n} |A_{mn}(\omega)|^2 \left|\hat{P}_{mn}(\boldsymbol{r},\omega)\right|^2 \tag{1}$$

- Minimization:

$$\sum_{i=38}^{51} \left(\sum_{m,n} |A_{mn}|^2 \left|\hat{P}_{mn}(\boldsymbol{r}_i,\omega)\right|^2 - P_i^2(\omega) \right)^2 = MIN \tag{2}$$

Far-field SPL



Summary

- Time Domain Wave Packet formulation can be used to eliminate the initial long transient state that is often required in single frequency formulation
- Computational time is reduced due to shortened time duration of the wave packet; the wave packet method is preferred for linear propagation problems even if only solutions at a few frequencies are of interest
- Solution of the adjoint problem provides a useful tool for verifying the numerical results of Euler equations